Complementos de Física Estatística

Preliminary program

The main aims of this supplementary course on statistical physics are:

(1) to provide students with more deep understanding of basic statistical mechanics, its concepts, and its direct applications, (2) to improve students' abilities of solving simple problems on various statistical physics topics, (3) to provide the basic mathematical apparatus of modern statistical mechanics, (4) to touch upon issues of statistical physics, which are beyond the scope of the main course of this discipline, and (5) to touch upon the modern problems of statistical physics, related, mostly, with complex networks.

* The method: Material should be mostly presented as a chain of problems with repetitions on higher and higher level.

Introduction. The organization and the idea of statistical physics. Equilibrium and non-equilibrium systems. The history of statistical physics and of related branches of mathematics. The first insight into complex networks. The Internet and the Web. The mathematics of statistical physics. Randomness. Random and deterministic systems.

Mathematical methods of statistical physics. Slowly and rapidly decreasing (increasing) functions in physics. Asymptotic series. Analytic functions and the steepest descent method. Generalized functions. The delta function and the Kronecker symbol. Methods of calculation of integrals and sums in physics. Asymptotic estimates. Factorial and its asymptotics. The gamma function. Various ways of derivation of the Stirling formula.

Basic notions of combinatorics. Events. Elementary events. Space of events. Different kinds of choice and selections. Permutations. Binomial formulas. The notion of probability. Compatible and incompatible events. Statistics of events. Summation of probabilities. Conventional probability. Independent events. Bayes' theorem. Combinatorial approach to Bose, Fermi, and Maxwell/Boltzmann statistics. Bose, Fermi, and Maxwell/Boltzmann distributions for finite systems. Random variables.

Probability distributions. Averages. Moments of distributions. Convolutions of distributions. Generating functions in theoretical physics and mathematics. The notion of dispersion of a Correlations and independent random variables, correlation functions. Tests and distributions of successes and failures. Binomial distribution and its limiting cases - Poisson and Normal distributions. Theory of errors. Limiting theorems for probabilities: the law of big numbers and the central limiting theorem in various formulations. Heavy tailed distributions in statistical physics.

The fundamental notion of statistical ensemble. Statistical weights. Partition functions. Examples. Random networks. Erdos-Rnyi random graphs. $G_{n,p}$ model. Percolation. Equilibrium and growing networks. Small worlds. The configuration model and hidden variables model of complex networks. Entropy, temperature, free energy, and other thermodynamic potentials.

Microcanonical, canonical, and grand canonical ensembles. Examples. Disordered systems. Annealed and quenched disorder. Thermodynamic average and averaging over quenched disorder. The idea of the replica trick. The idea of Gibbs. Bose, Fermi, and Maxwell/Boltzmann distributions in statistical mechanics.

Exact calculation of partition functions and thermodynamic quantities in simple spin systems. The Ising model on simple clusters. The solution of the one-dimensional Ising model in zero magnetic field. Boundary conditions. The Ising model on perfect trees. The one-dimensional Ising model with magnetic field. How to calculate susceptibility and correlations. Derivatives of the partition function over local fields. Description of the two- and higher-dimensional Ising models. The lattice gas model, relation to the Ising model.

Physics of critical phenomena. The mean field theory for the Ising model. Various approaches to the derivation of the mean field equations. The mean field free energy. The effective Hamiltonian. A variational approach. Variational derivatives. The main results of the mean field theory. Continuous phase transitions. Upper and lower critical dimensions.

Microscopic and phenomenological approaches in physics. The Landau theory of continuous critical phenomena. The order parameter. The Landau free energy. A second order phase transition versus a first order one. Critical singularities in the Landau theory. Critical fluctuations and correlations in the phenomenological description. Landau-Ginzburg functional and its minimization.

Derivation of the Euler-Lagrange equations. Analysis of the one-dimensional Landau-Ginzburg functional. Fluctuation corrections to mean field values. The Ginzburg-Levanyuk parameter and the criterion of the validity of the mean-field theory. Gaussian versus non-Gaussian critical fluctuations. Relaxation of the order parameter. Antiferromagnets and spin glasses.

Evolving statistical ensembles. Equilibrium versus non-equilibrium statistical physics. Master equation approach. Relaxation processes. Small deviations from equilibrium. Random walks and diffusion in one-, two-, and highly dimensional systems. Markov processes. Langevin equation and Fokker-Planck equation. Fluctuation dissipation theorem. Various specific stochastic processes. Neural networks.

Self-organized criticality. Sand piles. Forest-fire models. Exponential growing networks. The preferential attachment mechanism. Scale-free networks. Clustering. Condensation phenomena in complex networks. Generic topology of directed networks. Cooperative models defined on complex networks. The SIS-model and contact processes.

Bose-Einstein condensation. Perspectives and research fronts of statistical physics.

[1] S. N. Dorogovtsev, *Lectures on Complex Networks* (Oxford University Press, Oxford, 2010).

[2] S. N. Dorogovtsev, A. V. Goltsev, and J. F. F. Mendes, Critical phenomena in complex networks, Rev. Mod. Phys. **80**, 1275 (2008); Secs. I–III.

[3] P. L. Krapivsky, S. Redner, and E. Ben-Naim, *A Kinetic View of Statistical Physics* (Cambridge University Press, Cambridge, 2010); files: `http://physics.bu.edu/~redner/896/`

[4] J. Mathews and R. L. Walker, Mathematical Methods of Physics (Benjamin, New York, 1964).

[5] W. Feller, An Introduction to Probability Theory and its Applications (Wiley & Sons, New York, 1956), v. 1.

[6] L. D. Landau and E. M. Lifshits, Statistical Physics.

[7] T. FlieBbach, Curso de Física Estatística (Lisboa, 2000).

[8] E. J. S. Lage, Física Estatística (Lisboa, 1995).

[9] S. R. A. Salinas, Introdução à Física Estatística (EDUSP, So Paulo, 1997).

[10] J. P. Casquilho and P. I. C. Teixeira, Introdução à Física Estatística (IST Press, Lisboa, 2011).

[11] J. Sethna, Statistical Mechanics: Entropy, Order Parameters and Complexity (Oxford University Press, Oxford, 2006).