Lecture 12

Discussion of the sample problems

Instructions

(1) $H = 0$ $\quad \mathcal{H}(\{s\}) = -\overset{\circ}{J} \sum_{\langle ij \rangle} s_i s_j \quad s_i = \pm 1$

$k_B \equiv 1$

$$Z = \sum_{s_1 = \pm 1} \sum_{s_2 = \pm 1} \cdots \sum_{s_N = \pm 1} e^{-\beta \mathcal{H}(\{s\})}$$

$$K \equiv \frac{J}{T} \equiv \beta J$$

$$\{s\} \equiv \{s_1, s_2, \ldots, s_N\}$$

Ways: (1) Directly

or

(2) Using $e^{K s_i s_j} = \cosh K (1 + s_i s_j \tanh K)$

$Z(T) \Rightarrow$ Specific heat $C(T)$

For differentiating over T you may use Mathematica, Maple, etc. if expressions are too cumbersome

Then plot the result:

C vs. T

and find the leading asymptotics:

(a) $C(T)$ at $T \ll J$

(b) $C(T)$ at $T \gg J$

For that again you may use Mathematica, Maple, etc.

2 Build the recursive labeled graph growing under the preferential linking mechanism

As was explained:

$t = 1$:  1

$t = 2$:  1

⋮

$t = 3$ ....

⋮

$t = 4$ .....

Find the degree distribution $P(q)$ at ~~all~~ each of these ~~tin~~ instants

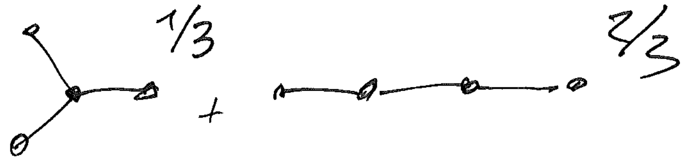
$$P(q,t) = \left\langle \frac{N_q(t)}{N(t)} \right\rangle = \frac{\langle N_q(t) \rangle}{N(t)}$$

$\langle\ \rangle$ – average over all members of the stat. ens. with their stat. weights

$N_q(t)$ – is the number of vertices of degree q of a given member of the ensemble at time t

$N(t)$ – is the number of vertices in a graph at time t. (Here, $N(t)=t$)

E.g. Let



$$P(1) = \frac{\frac{1}{3}\cdot 3 + \frac{2}{3}\cdot 2}{4} = \frac{7}{12}$$

$$P(2) = \frac{\frac{1}{3}\cdot 0 + \frac{2}{3}\cdot 2}{4} = \frac{4}{12}$$

$$P(3) = \frac{\frac{1}{3}\cdot 1 + \frac{2}{3}\cdot 0}{4} = \frac{1}{12}$$

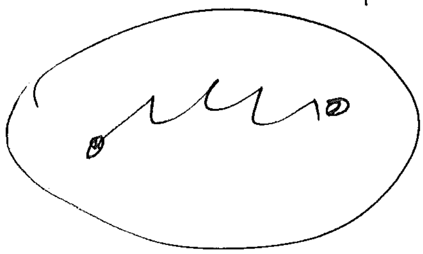
$\frac{1}{12} + \frac{4}{12} + \frac{7}{12} = 1 \rightarrow$ OK

How to find the distribution of the shortest path length:

$$\mathcal{P}(l,t) = \frac{\langle N_l(t) \rangle}{\frac{N(t)[N(t)-1]}{2}}$$

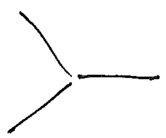
average over the members of ~~the~~ stat. ensemble

This is the number of pairs of nodes in a connected component between which there is some path.



N_l - is the number of the pairs of vertices with the shortest path l.

For the same example:

 $1/3$ +  $2/3$

$$\begin{cases} N_1 = 3 \\ N_2 = 3 \end{cases} \qquad \begin{cases} N_1 = 3 \\ N_2 = 2 \\ N_3 = 1 \end{cases}$$

$$P(1) = \frac{\frac{1}{3}\cdot 3 + \frac{2}{3}\cdot 3}{\frac{4\cdot 3}{2}} = \frac{1}{2}$$

$$P(2) = \frac{\frac{1}{3}\cdot 3 + \frac{2}{3}\cdot 2}{6} = \frac{7}{18}$$

$$P(3) = \frac{\frac{1}{3}\cdot 0 + \frac{2}{3}\cdot 1}{6} = \frac{2}{18}$$

$$\frac{1}{2} + \frac{7}{18} + \frac{2}{12} = 1 \Rightarrow OK$$

(3) Google PageRank

for a given directed graph.

Let $p = 0.15$.

The aim is to find the vector $r_1, \dots, r_N$

If your graph has more than 1 connected components, then

$$\frac{N(N-1)}{2} \rightarrow$$

$$\rightarrow \sum_i \frac{N_i(N_i-1)}{2}$$

i labels different connected components

substitute in the above formulas

You may first estimate $\{r_i\}$ by formulas for uncorrelated graphs:

For that, ~~find $N_{q_{in} q_{out}}$ numbers of vertices with given q_{in} and q_{out}.~~

use Eq. (11.6) from "Lectures on Complex Nets"

$\Rightarrow (r_1^{(0)}, r_2^{(0)} \ldots r_N^{(0)})$

start iterations (11.5) from this vector.

How many iterations you will need to obtain a sufficiently precise result?

Compare this result with $\{r_i^{(0)}\}$.

The code may be written in any language, even in Mathematica.

How to prepare yourself for the exam?

(1) Solve 3 sample problems in detail, completely!

(2) For each of these 3 problems prepare programs/codes in Mathematica, Maple, C++, etc.

(3) Have everything prepared to explore quite similar problems in the test, namely, similar problems but for different graphs, so these your programs will be absolutely necessary.