

The Nature of Complex Networks

Sergey N. Dorogovtsev

University of Aveiro

José F. F. Mendes

University of Aveiro

OXFORD
UNIVERSITY PRESS

For those whom we love

Gentlemen! As is custom among our kind, we do not
plunge headlong into folly on the orders of a single madman,
but act according to our own collective madness!

Assassin's Creed IV: Black Flag

Preface

The authors of this book are scientific workers in the field of complex networks. This is an explosively developing multidisciplinary area, which involves practically everything, natural and artificial, in the world, and we, theoretical physicists, explore it, armed with the standard tools of statistical physics. What we are doing is uncovering the actual simplicity and beauty of complex systems including the random complex networks. The book is about the key ideas and concepts of this field which is already called the science of networks. The highly heterogeneous networked systems, which we explore, typically have no governing authority, no command centre taking decisions and issuing orders. Nonetheless, even without orchestration, these remarkably widespread systems function very effectively and reliably. Moreover, the absence of centralization in favour of distributed self-organization is vital for their existence and robustness.

Thanks to numerous tips one can appreciate the pace and scale of efforts to surveil, control, protect, and manipulate society, in essence, large social and information networks. The sweeping reset of our ‘highly interconnected and interdependent world’, induced and concerted by the Covid-19 pandemic (Schwab and Malleret, 2020), is forcefully amplifying these exertions. The big problem now is how to control large complex systems that is impossible to govern. A related fundamental problem is how to structure and efficiently process the collected big data. Similarly impressive and costly efforts are under way to engineer and manage large cellular, technological, deep-learning, and many other networks. These extensive works demonstrate how important it is to understand in detail the principles of organization and function of highly heterogeneous and highly structured random networked systems, that is the nature of complex networks.

In 2003 we published the first reference book in this research area, the ‘*Evolution of Networks: From Biological Nets to the Internet and the WWW*,’ which essentially focused on evolving networks. Our new, more dense and advanced, book touches upon a much wider range of networks and networked systems paying particular attention to the directions that emerged after 2002–2003. Writing a book is about asking good questions. So, keeping the book compact, we consider only the issues and key results

PREFACE

that, we believe, will not lose their value in the future. To avoid drowning in serious mathematics we have to present a number of basic theorems from graph theory without even sketching the ideas of their proofs. Furthermore, for the sake of brevity we do not include in the book chapters devoted to numerous applications and empirical material. The reader will find detailed discussions of these matters in a number of comprehensive books and reviews, which we cite.

The researchers studying complex networks will acquire from this concise modern book a number of new issues and ideas, not yet touched upon in other reference volumes. We propose a statistical mechanics view of random networks based on the concept of statistical ensembles, but approaches and methods of modern graph theory, concerning random graphs, overlap strongly with statistical physics. Hence mathematicians have a good chance to discover interesting things in our book, even though it does not contain mathematical proofs and trades off rigour for comprehension, brevity, and relevance. We expect that this book will be useful for undergraduate, master, and PhD students and young researchers from multidisciplinary studies, physics, computer science, and applied mathematics wishing to gain a serious insight into the principles of complex networks. The book can be used as a text in university courses on complex networks. We propose to determined students not only a brief trip to the land of complex networks but an option to stay there forever. For a more elementary and concise introduction to the science of networks, we refer most impatient readers to the text of one of us, S. N. Dorogovtsev, ‘Lectures on Complex Networks.’

We are deeply indebted to our friends and colleagues all over the world for their encouragement and advice, first and foremost to Gareth Baxter, who has read and commented on the manuscript, Alexander Goltsev, Ginestra Bianconi, Pavel Krapivsky, Rui Americo da Costa, Gábor Timár, João Gama Oliveira, Nahid Azimi-Tafreshi, KyoungEun Lee, António Luís Ferreira, Sooyeon Yoon, Bruno Coutinho, Marta Santos, Nuno Araujo, Ricardo Jesus, Marian Boguñá, Maria Ángeles Serrano, Dmitri Krioukov, Edgar Wright, Romualdo Pastor-Satorras, Tiago Peixoto, Jorge Pacheco, Francisco Santos, Peter Grassberger, Mark Newman, Byungnam Kahng, Zdzislaw Burda, Michel Bauer, Raissa D’Souza, Doochul Kim, Mikko Alava, Hyunggyu Park, Petter Holme, Kimmo Kaski, Hawoong Jeong, Satu Elisa Schaeffer, José Ramasco, János Kertész, Jae Dong Noh, László Barabási, Shlomo Havlin, Matteo Marsili, Bartłomiej Waclaw, Geoff Rodgers, Malte Henkel, Davide Cellai, James Gleeson, Alexander Povolotsky, Alexander

PREFACE

Zyuzin, Santo Fortunato, Vittoria Colizza, Alessandro Vespignani, Alex Arenas, Guido Caldarelli, Filippo Radicchi, Adilson Motter, Sid Redner, Yimir Moreno, Maxi San Miguel, Alain Barrat, Marc Barthelemy, Agata and Piotr Fronczak, Konstantin Klemm, Lenka Zdeborová, Zoltán Toroczkai, Alexei Vázquez, Florent Krzakala, Valentina Guleva, Jesus Gomez-Gardeñes, Bosiljka Tadic, Piet Van Mieghem, Dima Shepelyansky, Sergey Maslov, Ernesto Estrada, Bruno Gonçalves, Serguei Nechaev, Alexander Gorsky, Ronaldo Menezes, Hans Herrmann, Nelly Litvak, Andrei Raigorodskii, and Dafne Bandeira. We are enormously bound to Alexander Nikolaevich Samukhin (1953–2018), our friend and collaborator, who showed us the essence and logic of statistical mechanics and its mathematical apparatus. Our book would be impossible without his friendship and tuition. Finally, our warmest thanks to the editorial and production staff at Oxford University Press for their guidance, encouragement, and patience.

Aveiro
August 2021

S.N.D.
J.F.F.M.

Contents

1	First Insight	1
1.1	Statistical mechanics view of random systems	1
1.2	History of networks	4
1.3	Small and large worlds	10
1.4	First structural characteristics of networks	13
1.5	Equilibrium versus growing trees	15
1.6	Real-world networks	19
2	Graphs	23
2.1	Types of graphs	23
2.2	Graphicality	29
2.3	Representations of graphs	30
2.4	Walks, paths, and cycles	36
2.5	Triangles	38
2.6	Cliques	40
2.7	Betweenness centrality	41
2.8	Connectivity	43
2.9	Spectra of graphs	44
3	Classical Random Graphs	55
3.1	$G(N, p)$ model	55
3.2	Erdős–Rényi model	57
3.3	Cycles and clustering	58
3.4	Giant connected component	60
3.5	Finite connected components	64
4	Equilibrium Networks	70
4.1	The configuration model	70
4.2	Local tree-likeness	72
4.3	Generating maximally random networks	77
4.4	Hidden variables	78
4.5	Annealed networks and graphons	80
4.6	Clustering, cycles, and cliques	81
4.7	Correlations	83

CONTENTS

4.8	Cut-offs and rich club phenomenon	91
4.9	Motifs in networks	94
4.10	Equilibrium network ensembles	96
4.11	The case of triangles	99
4.12	Weighted networks	100
4.13	Random geometric graphs	104
4.14	Small-world networks	105
4.15	Networks embedded in metric spaces	109
5	Evolving Networks	113
5.1	Random recursive trees	113
5.2	Preferential attachment	116
5.3	Origin of preferential attachment	123
5.4	Condensation phenomenon	126
5.5	Berezinskii–Kosterlitz–Thouless (BKT) transition	130
5.6	Accelerated growth and densification	132
5.7	Transitions in dense networks	134
5.8	Growth and decay	135
5.9	Power of choice	137
5.10	Evolving weighted networks	137
5.11	Evolution preserving degrees	139
5.12	Evolution of simplicial complexes	140
5.13	Deterministic graphs	144
6	Connected Components	146
6.1	Giant connected component	146
6.2	Site and bond percolation	152
6.3	Message passing	155
6.4	Beyond tree-likeness	164
6.5	Bow-tie structure of directed networks	168
6.6	Finite connected components	171
6.7	k -cores	175
6.8	Dynamics of pruning	187
6.9	s -cores in weighted networks	194
6.10	k -connected components	195
6.11	Giant component in correlated networks	197
6.12	k -clique percolation	198
6.13	Explosive percolation	202
6.14	Largest component in finite networks	207

6.15 Cores and controllability	210
7 Epidemics and Spreading Phenomena	216
7.1 Bootstrap percolation	216
7.2 The Watts model	219
7.3 Main epidemic models	223
7.4 The SIS model and contact process	224
7.5 The SIR, SI, and SIRS models	233
7.6 Epidemic outbreaks	237
7.7 Epidemics in a metapopulation	243
7.8 Rumour spreading	246
7.9 Opinion formation: The voter model	248
7.10 Propagation of memes	253
8 Networks of Networks	258
8.1 Networks with vertices and edges of different types	258
8.2 Mutually connected components	259
8.3 Avalanches and cascading failures	272
8.4 Other percolation problems	274
8.5 Dynamical systems on multilayer networks	281
9 Spectra and Communities	284
9.1 Adjacency matrix spectra	284
9.2 Laplacian matrix spectra	290
9.3 Localization	299
9.4 Stochastic block model	307
9.5 Modularity	308
9.6 Detection of communities	314
9.7 Overlapping communities	323
10 Walks and Search	326
10.1 Diffusion and random walks on networks	326
10.2 Greedy routing	334
10.3 Navigability	337
10.4 Google PageRank	342
11 Temporal Networks	345
11.1 The concept of a temporal network	345
11.2 Random walks on temporal networks	348
11.3 Epidemic spreading	351

CONTENTS

12 Cooperative Systems on Networks	356
12.1 The Ising model	356
12.2 Critical phenomena	360
12.3 Games on networks	363
12.4 Phase synchronization	365
13 Inference and Reconstruction	374
13.1 Finding the root of a growing tree	374
13.2 Finding missing links	378
14 What's Next?	380
Further reading	382
Appendix A Adjacency Matrix for Hypergraphs	388
Appendix B Spectra of Symmetric Normalized Laplacians of Sample Graphs	390
Appendix C Generating Functions	392
Appendix D Hyperscaling Relations for Percolation	397
Appendix E Degree Distribution of a Damaged Network	401
Appendix F Non-backtracking Matrix	402
Appendix G Treating General Interdependent Networks	406
References	410
Index	440