

Fine structures of Dirac type and their function spaces

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The Fueter-Sce mapping theorem stands as one of the most profound outcomes in complex and hypercomplex analysis, producing hypercomplex generalizations of holomorphic functions. In recent years, delving into the factorization of the second operator appearing in the Fueter-Sce mapping theorem has uncovered its potential to generate novel classes of functions and their respective functional calculi.

The sets of functions obtained from this factorization and the associated functional calculi define the so-called *fine structures on the S -spectrum*. These lectures aim to comprehensively investigate the function theories for the fine structures of Dirac type in the quaternionic framework. We shall briefly discuss polyanalytic functions in the classical complex case and then move to axially harmonic, Fueter regular, and axially polyanalytic functions in the quaternionic case. In particular, we shall discuss new series expansions for these functions, highly nontrivial.

In fact, when considering the hypercomplex realm, specifically the quaternionic or the Clifford setting, extending the concept of complex power series expansion is not immediate, and different Taylor and Laurent expansions appear with different sets of convergence.

We also present the so-called representation formulas for these function spaces; such formulas encode the fundamental properties of the functions and have numerous consequences.

Finally, we shall discuss how the fine structures generalize to higher dimensional algebras.