22^{nd} ANNUAL WORKSHOP

on

APPLICATIONS AND GENERALIZATIONS OF COMPLEX ANALYSIS

Booklet of Abstracts

AVEIRO

March 28-29, 2025

Index

GENERAL INFORMATIONS

We welcome you all to the 22nd Annual Workshop in Aveiro. The workshop will take place in a hybrid form, with the in-presence events being held at room 11.2.21 (second floor).

INTERNET ACCESS

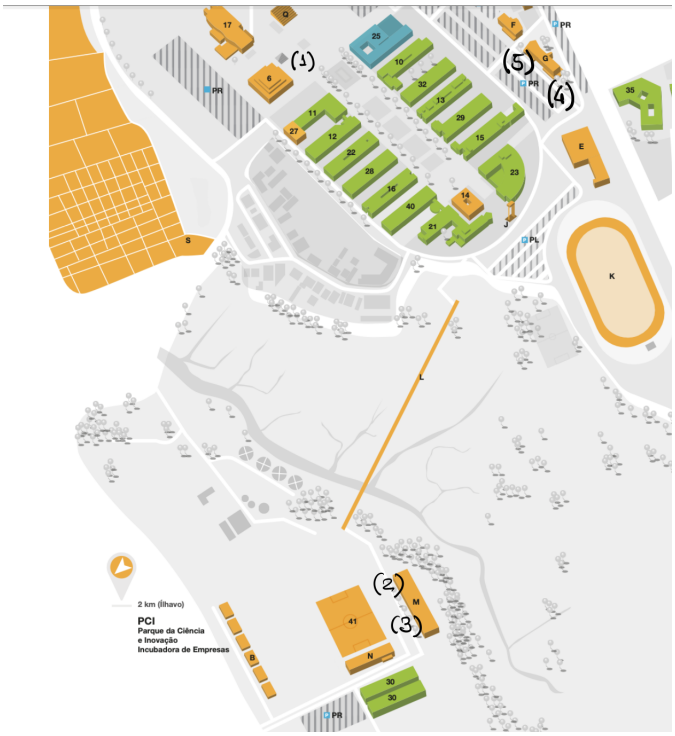
If you possess a personal Laptop with WLAN you can use either the above login or your personal EDUROAM access at your home university.

WHERE TO EAT?

Around the campus there exist several coffee bars where you can have also small meals such as sandwiches, snacks, fruits, ice creams, etc., for prices ranging from 4,90 euros to 9,00 euros.

For lunch or dinner, there exist several possibilities:

- **(1) Cantina de Santiago** Monday to Friday, lunch 11h45 to 14h30 / dinner 18h30 to 20h30, Saturday and Sunday, dinner from 19h to 20h30.
- **(2) Cantina do Crasto**, Monday to Friday, lunch 11h45 to 14h30.
- **(3) Restaurante Vegetariano**, Monday to Friday, lunch 12h30 to 14h30 (on-line booking).
- **(4) Edifício do Snack-bar and Self-Service (ground floor)**, Monday to Friday, lunch 12h to 14h30.
- **(5) Restaurante Universitário (1st floor)**, Monday to Friday, lunch 12h to 14h30.



In addition, several restaurants can be found all around the campus or at the town center (at walking distance).

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We wish you all a happy stay and a good and fruitful workshop.

Aveiro, March 19, 2025

The Organizers

Paula Cerejeiras
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ABSTRACTS - 28th March

14:30 *Opening Session*

14:45-15:30 Functions and operators of the polyharmonic and polyanalytic Clifford fine structures on the S-spectrum

Fabrizio Colombo
Politecnico di Milano, Italy

The spectral theory on the S-spectrum originated to give quaternionic quantum mechanics a precise mathematical foundation and as a spectral theory for linear operators in vector analysis.

This theory has proven to be significantly more general than initially anticipated, naturally extending to fully Clifford operators and revealing unexpected connections with the spectral theory based on the monogenic spectrum, developed over forty years ago by A. McIntosh and collaborators.

In recent years, we have combined slice hyperholomorphic functions with the Fueter-Sce mapping theorem, also called Fueter-Sce extension theorem, to broaden the class of functions and operators to which the theory can be applied. This generalization has led to the definition of what we call the *fine structures on the S-spectrum*, consisting of classes of functions that admit an integral representation and their associated functional calculi.

In this talk, we focus on the fine structures within the Clifford algebra setting, particularly addressing polyharmonic functions, polyanalytic functions, holomorphic Cliffordian functions and their associated functional calculi defined via integral representation formulas.

15:35-15:55 The H^∞ functional calculus for Clifford operators

Francesco Mantovani
Politecnico di Milano, Italy

In this talk we introduce the H^∞-functional calculus for unbounded bisectorial operators in a Clifford module over the algebra $\mathbb{R}_n$. This work is based on the universality property of the S-functional calculus, which shows its applicability to fully Clifford operators. While recent studies have focused on bounded operators or unbounded paravector operators, we now investigate unbounded fully Clifford operators and define polynomially growing functions of them. We first

generate the ω-functional calculus for functions that exhibit an appropriate decay at zero and at infinity. We then extend this calculus to functions with a constant value at zero and at infinity. Finally, using a subsequent regularization procedure, we can define the H^∞- functional calculus for the class of regularizeable functions, which in particular include functions with polynomial growth at infinity and if T is injective also functions with polynomial growth at zero.

This talk is based on a joint work with F. Colombo and P. Schlosser.

16:00-16:30 *Coffee-break*

16:35-16:55 On a singular integral operator with two shifts and conjugation

Rui Marreiros
Universidade do Algarve, Portugal

On the Hilbert space $\tilde{L}_2(\mathbb{T})$ the singular integral operator with two shifts and conjugation $K = P_+ + \left[aI + \left(\sum_{j=0}^{m} a_j U_\alpha^j\right) U_\beta C\right] P_-$ is considered, where $P_\pm$ are the Cauchy projectors, $a, a_j, j = \overline{0, m}$, are continuous functions on the unit circle $\mathbb{T}, U_\alpha$ and U_β are non-Carleman and Carleman shift operators, respectively, both preserving the orientation on $\mathbb{T}$, and C is the operator of complex conjugation. An estimate for the dimension of the kernel of the operator K is obtained.

17:00-17:20 CK-extension for Dirac-type operators with non-constant coefficients and its applications

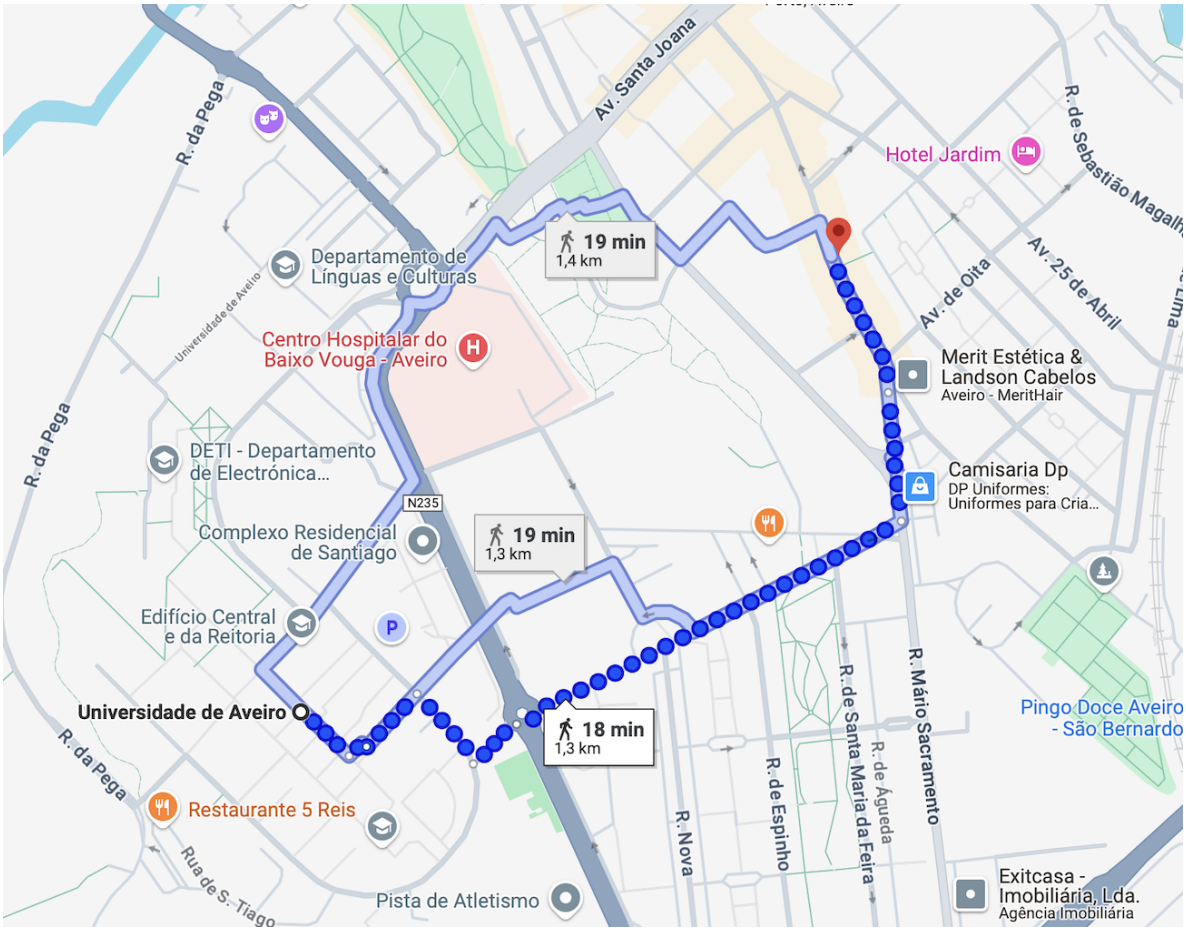
Kira Morozova
Universidade de Aveiro, Portugal

One of the big current challenges in Mathematics is the study of PDEs with distributional coefficients. This is due to the fact that the product of two distributions is ill-defined, even the concept of Colombeau distributions being not general enough. Recently, there has been a new development in this topic due to the introduction of a new type solutions called "very weak solutions". This term was coined in 2015 by C. Garetto and M. Ruzhansky in their paper "Hyperbolic second order equations with non-regular time dependent coefficients". In this presentation will introduce the concept of "very weak solutions" and how they relate to classical solutions. Then, we will discuss them from the point of view of Dirac-type operators with distributional coefficients by constructing the corresponding CK-extension and deriving the appropriate energy estimates.

17:25 -17:45 TBA

Mihaela Vajiac
Chapman University, CA Orange, USA

19:30 *Conference dinner* Restaurante Espeto do Sul



ABSTRACTS - 29th March

10:00-10:45 On the complex Robin Laplacian

James Kennedy
Universidade de Lisboa, Portugal

We will discuss properties of the Robin Laplacian on a bounded Lipschitz domain Ω, where the parameter α appearing in the boundary condition

$$\frac{\partial u}{\partial \nu} + \alpha u = 0$$

on $\partial\Omega$ (ν being the outward-pointing unit normal to Ω) is complex and hence the operator is not self-adjoint. Besides giving an overview of basic well-posedness and generation properties of the operator, we will be interested in the behaviour of its spectrum and numerical range as functions of α. The spectrum consists of complex eigenvalue curves which are (possibly multivalued) meromorphic functions of α, and which also enjoy a duality with the spectrum of the (complex) Dirichlet-to-Neumann operator (DNO).

We will give sharp trace-type inequalities which show that the numerical range must lie in a parabolic region in $\mathbb{C}$; this, in particular, leads to information about the potential rate of divergence of the eigenvalues as $\alpha \to \infty$, including eigenvalue bounds which are new even in the case of real α. We will also exploit the duality with the DNO to illustrate how the actual behaviour of the eigenvalues is, in more than one sense, complex.

This talk will largely be based on joint work with Sabine Bögli and Robin Lang.

10:50-11:20 *Coffee-break*

11:25-11:45 Lower Bounds for Dyadic Square Functions of indicator functions of sets

Natanael Alpay
University of California Irvine, CA Irvine, USA

We prove that for any Borel measurable subset $A \subset [0,1]$, the inequality $\|S_2(\mathbb{1}_A)\|_1 \geq I(|A|)$ holds, where I denotes the Gaussian isoperimetric profile. This improves upon the classical lower

bound $\|S_2(\mathbb{1}_A)\|_1 \gtrsim |A|(1-|A|)$ by a factor of $\sqrt{\log \frac{1}{|A|(1-|A|)}}$. In addition, we study lower bounds for the α-norm of $S_1(\mathbb{1}_A)$, and we obtain a threshold behavior around $\alpha = 1$. We show that

$$\|S_1(\mathbb{1}_A)\|_1 \geq \min\{|A|, 1-|A|\} \log_2 \frac{1}{\min\{|A|, 1-|A|\}},$$

and that this bound is sharp at points $|A| = 2^{-k}$ or $|A| = 1 - 2^{-k}$ for every nonnegative integer k. For each fixed $\alpha \in (0,1)$, we further establish that $\|S_1(\mathbb{1}_A)\|_\alpha \geq \min\{|A|, 1-|A|\}$, with the decay rate $|A|$, as $|A| \to 0$, being optimal.

11:50-12:10 Quadratic estimates for the H^∞-functional calculus

Peter Schlosser
Graz University of Technology, Austria

For unbounded, sectorial operators T one defines the *ω-functional calculus*

$$f(T) := \frac{1}{2\pi} \int_{\partial U \cap \mathbb{C}_J} S_L^{-1}(s,T) ds_J f(s). \tag{1}$$

Here f is a slice hyperholomorphic function which decays at $s = 0$ and $s = \infty$ fast enough, such that the above integral exists. Although the operator T is unbounded, the decay of f leads to a bounded operator $f(T) \in \mathcal{B}(V)$.

Moreover, for functions f which are (polynomially) growing at $s = 0$ and $s = \infty$, one can generalize (1) to the so called *H^∞-functional calculus*

$$f(T) := e(T)^{-1}(ef)(T), \tag{2}$$

where e is a certain regularizer function. Due to the growth of the function f,this definition now leads to unbounded, closed operators $f(T)$.

The main question adressed in this talk will be: In which cases does the H^∞-functional calculus (2) give a bounded operator? I will give a necessary and sufficient condition on the operator T, called *Quadratic estimates*, such that $f(T) \in \mathcal{B}(V)$ and

$$\|f(T)\| \leq \|f\|_\infty,$$

for every bounded holomorphic function f.

12:15-12:35 The S-resolvent estimates for the Dirac operator on hyperbolic and spherical spaces

Simão Lucas
Politecnico di Milano, Italy

In this talk, we present a novel approach to studying Dirac operators on manifolds, which makes use of the spectral theory based on the S-spectrum. Specifically, we examine in detail the cases of the Dirac operator $\mathcal{D}_H$ on the hyperbolic space and the Dirac operator $\mathcal{D}_S$ on the spherical space, where these operators, and their squares $\mathcal{D}_H^2$ and $\mathcal{D}_S$, can be written in a very explicit form. In fact, let T denote a (right) linear Clifford operator, the S-spectrum is associated with a second-order polynomial in the operator T, that is, with the operator defined as $Q_s(T) := T^2 - 2s_0T + |s|^2$. This allows us to associate to the Dirac operator boundary conditions that can be of Dirichlet type but also of Robin-like type.

This is joint work with I. Beschastnyi (Centre Inria d'Université Côte d'Azur, France), F. Colombo (Politecnico di Milano, Italy), and I. Sabadini (Politecnico di Milano, Italy).

12:40-14:55 *Lunch-break*

15:00-15:45 Fujita exponent on Hörmander vector fields

Marianna Chatzakou
University of Ghent, Belgium

$$\begin{cases} u_t(t,x) - \Delta u(t,x) = f(u(t,x)) \\ u(0,x) = u_0(x)\,, \end{cases}$$

on a general manifold for Hörmander sum of squares. In the particular case where $f(u) = u^p$ we obtain the so-called critical Fujita exponent. The above problem with an extra time-dependent nonlinearity term will also be discussed in the setting of a unimodular Lie group. In both cases, our approach relies on the heat kernel estimates of the involved operators, while in the first case it also relies on recent advances concerning global lifting theorems.

My talk will be based on two joints works, one of which is still in progress, with A. Kassymov and M. Ruzhansky.

15:50-16:10 Generalized partial-slice monogenic functions in the octonionic case

Irene Sabadini
Politecnico di Milano, Italy

The concept of generalized partial-slice monogenic function was introduced over Clifford algebras. The present talk discusses how to extend the study of generalized partial-slice monogenic functions from the associative case of Clifford algebras to non-associative alternative algebras, such as octonions. The new class of functions encompasses the regular functions over octonions and slice regular functions, indeed both appear in the theory as special cases. In this non-associative setting of octonions, we shall set up results such as identity theorem, Representation Formula, Cauchy (and Cauchy-Pompeiu) integral formula, maximum modulus principle, Fueter polynomials, Taylor series expansion. Although the study is limited to the case of octonions, it is clear from the statements and the arguments in the proofs that the results hold more in general in real alternative algebras.

Joint work with Z. Xu.

16:15-16:45 *Coffee-break*

16:50-17:10 Some more news on the octonionic Bergman projection and the octonionic Π-operator

Sören Kraußhar
Universität Erfurt, Germany

In this talk we explain the difficulties caused by the non-associativity that arise in attempting to define a Bergman kernel and a related Bergman projection of square integrable functions satisfying the octonionic Cauchy-Riemann equation. Already in the Stokes formula there appears an associator term which actually leads to two versions of the Cauchy and the Borel-Pompeiu formula. In the definition of a Bergman projection these associator terms play again a crucial role and they lead to different possible versions. We shall see that a rather canonical version of the Bergman projection is obtained when implementing a certain intrinsic octonionic weight factor of unit length. After that, we briefly discuss an octonionic version of the T-operator and the Π-operator. If time permits we round off this talk by giving a few applications to PDE arising in the octonionic context.

Joint work with M. Ferreira, M.M. Rodrigues and N. Vieira.

[1] R.S.Kraußhar, M. Ferreira, N. Vieira and M.M. Rodrigues. The Teodorescu operator and the Π-operator in octonionic analysis and some applications. Journal of Geometry and Physics 206, 105328, 2024.

17:15-17:35 Time-dependent Dirac operators with distributional coefficients

Narciso Gomes
Universidade de Cabo-Verde, Cabo-Verde

17:40 *Closing Session*