21st ANNUAL WORKSHOP

on

APPLICATIONS AND GENERALIZATIONS OF COMPLEX ANALYSIS

Booklet of Abstracts

AVEIRO

March 24-25, 2023

Index

GENERAL INFORMATIONS

We welcome you all to the 21st Annual Workshop in Aveiro. The workshop will take place in a hybrid form, with the in-presence events being held at room Sousa Pinto (second floor).

ZOOM link:

https://videoconf-colibri.zoom.us/j/99218921164?pwd=ZGk5MDIvY3BXSWJtcGFUYnRUQzNOQT09

Password: 988192

INTERNET ACCESS

Rooms are available for checking your e-mail trough a computer terminal.

Alternatively, if you possess a personal Laptop with WLAN you can use either the above login or your personal EDUROAM access at your home university.

WHERE TO EAT?

Around the campus there exist several coffee bars where you can have also small meals such as sandwiches, snacks, fruits, ice creams, etc., for prices ranging from 4,10 euros to 7,50 euros.

For lunch or dinner, there exist several possibilities:

- **Cantina de Santiago** Monday to Friday, lunch 11h45 to 14h30 / dinner 18h30 to 20h30, Saturday and Sunday, dinner from 19h to 20h30.
- **Cantina do Crasto**, Monday to Friday, lunch 11h45 to 14h30.
- **Edifício do Snack-bar and Self-Service (ground floor)**, Monday to Friday, lunch 12h to 14h30.
- **Restaurante Universitário (1st floor)**, Monday to Friday, lunch 12h30m to 14h30.

In addition, several restaurants can be found all around the campus our at the town center (at walking distance).

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We wish you all a happy stay and a good and fruitful workshop.

Aveiro, March 22, 2023

The Organizers

Paula Cerejeiras
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ABSTRACTS - 24th March

14:00 *Opening Session*

14:15-15:00 Commutators on Fock spaces

Daniel Alpay
Chapman University, Orange CA, USA

Given a weighted ℓ^2 space with weights associated to an entire function, we consider pairs of weighted shift operators, whose commutators are diagonal operators, when considered as oper- ators over a general Fock space. We establish a calculus for the algebra of these commutators and apply it to the general case of Gelfond-Leontiev derivatives. This general class of operators includes many known examples, like classic fractional derivatives and Dunkl operators. This allows us to establish a general framework which goes beyond the classic Weyl-Heisenberg algebra. Concrete examples for its application are provided.

15:05-15:25 Uniqueness of Calderón problem for complex conductivities in 3D

Ivan Pombo
Universidade de Aveiro, Portugal

In this talk, we present the novel uniqueness result of Calderón problem in 3D for the case of complex conductivities. The idea is based on exploiting the conversion of the conductivity equation to a Dirac system with quaternionic analysis. From here, we study the existence of exponential growing solutions of quaternionic type and obtain the uniqueness from the Alessandrini identity together with their asymptotic behavior.

15:30-15:55 Presentation of virtual laboratories

16:00-16:30 *Coffee-break*

16:35-16:55 On Fundamental Solutions of Higher-Order Space-Fractional Dirac equations

Nelson Faustino
University of Aveiro, Portugal

Starting from the pseudo-differential decomposition $D = (-\Delta)^{\frac{1}{2}}\mathcal{H}$ of the Dirac operator $D = \sum_{j=1}^{n} e_j \partial_{x_j}$ in terms of the fractional operator $(-\Delta)^{\frac{1}{2}}$ of order 1 and of the Riesz-Hilbert type operator $\mathcal{H}$, we will investigate the fundamental solutions of the space-fractional Dirac equation of Lévy-Feller type

$$\partial_t \Phi_\alpha(x,t;\theta) = -(-\Delta)^{\frac{\alpha}{2}} \exp\left(\frac{i\pi\theta}{2}\mathcal{H}\right)\Phi_\alpha(x,t;\theta)$$

involving the fractional Laplacian $-(-\Delta)^{\frac{\alpha}{2}}$ of order α, with $2m \leq \alpha < 2m+2$ ($m \in \mathbb{N}$), and the exponentiation operator $\exp\left(\frac{i\pi\theta}{2}\mathcal{H}\right)$ as the hypercomplex counterpart of the fractional Riesz-Hilbert transform carrying the *skewness parameter* θ, with values in the range $|\theta| \leq \min\{\alpha - 2m, 2m+2-\alpha\}$.

Such model problem permits us to obtain hypercomplex counterparts for the fundamental solutions of higher-order heat-type equations $\partial_t F_M(x,t) = \kappa_M (\partial_x)^M F_M(x,t)$ ($M = 2,3,\ldots$) in case where the even powers resp. odd powers $D^{2m} = (-\Delta)^m$ ($M = \alpha = 2m$) resp. $D^{2m+1} = (-\Delta)^{m+\frac{1}{2}}\mathcal{H}$ ($M = \alpha = 2m+1$) of D are being considered.

17:00-17:20 A singular integral operator with a non-Carleman shift and conjugation

Rui Marreiros
Universidade do Algarve, Portugal

On the Hilbert space $\widetilde{L}_2(\Gamma)$, $\Gamma = \mathbb{R}, \mathbb{T}$, the singular integral operator with a non-Carleman shift and conjugation $K = P_+ + (aI + AC)P_-$ is considered, where $P_\pm$ are the Cauchy projectors, $A = \sum_{j=0}^{m} a_j U^j$, a, a_j, $j = \overline{1,m}$, are continuous functions on Γ, U is the shift operator and C is the operator of complex conjugation. We obtain some estimates for the dimension of the kernel of the operator K.

17:25-17:45 Harmonic and polyanalytic functional calculi on the S-spectrum for unbounded operators

Stefano Pinton
Politecnico di Milano, Italy

Harmonic and polyanalytic functional calculi have been recently defined for bounded commuting operators. Their definitions are based on the Cauchy formula of slice hyperholomorphic functions

and on the factorization of the Laplace operator in terms of the Cauchy-Fueter operator $\mathcal{D}$ and of its conjugate $\overline{\mathcal{D}}$. Thanks to the Fueter extension theorem when we apply the operator $\mathcal{D}$ to slice hyperholomorphic functions we obtain harmonic functions and via the Cauchy formula of slice hyperholomorphic functions we establish an integral representation for harmonic functions. This integral formula is used to define the harmonic functional calculus on the S-spectrum. Another possibility is to apply the conjugate of the Cauchy-Fueter operator to slice hyperholomorphic functions. In this case, with a similar procedure we obtain the class of polyanalytic functions, their integral representation and the associated polyanalytic functional calculus. In this talk I will explain how to extend the harmonic and the polyanalytic functional calculi to the case of unbounded operators and to prove some of the most important properties. These two functional calculi belong to so called fine structures on the S-spectrum in the quaternionic setting. Fine structures on the S-spectrum associated with Clifford algebras constitute a new research area that deeply connects different research fields such as operator theory, harmonic analysis and hypercomplex analysis.

This talk is based on a joint work with F. Colombo and A. De Martino.

17:50-18:10 Bergman and Hardy spaces in the octonionic monogenic and slice monogenic setting

`Sören Kraußhar`
`Universität Erfurt, Germany`

In this talk we give some further insight on the theoretical background of octonionic Bergman and Hardy spaces in the context of octonionic monogenic and slice monogenic functions. The non-associativity of the octonions implies that neither octonionic monogenic nor slice monogenic functions have the algebraic structure of an O-module. Consequently, there is no direct analogue of a Cauchy-Schwarz inequality neither a Fischer-Riesz representation theorem in the octonionic setting. In the first part of the talk we explain how some of the fundamental problems in defining a reproducing kernel can be overcome in the non-associative setting by looking at the real part of an appropriately defined para-linear octonion-valued inner product. In our approach the presence of a weight factor of norm 1 in the definition of the inner product is an intrinsic new ingredient in the octonionic setting. Also in the Bergman case these intrinsic weight factors are crucial. With our approach we also manage to address the slice monogenic octonionic setting. We conclude by presenting formulas for the slice monogenic reproducing kernels for the unit ball, the right octonionic half-space and strip domains bounded in the real direction.

Joint work with F. Colombo and I. Sabadini.

[1] F. Colombo, R.S. Kraußhar, I. Sabadini: Octonionic monogenic and slice monogenic Hardy and Bergman spaces, submitted for publication (February 2023).

19:30 *Conference dinner*

ABSTRACTS - 25th March

9:30-10:15 Boundary value problems for first-order elliptic operators with compact and non-compact boundary

Lashi Bandara
Brunel University London, United Kingdom

The index theorem for compact manifolds with boundary, established by Atiyah-Patodi-Singer in the mid-70s, is considered one of the most significant mathematical achievements of the 20th century. An important and curious fact is that local boundary conditions are topologically obstructed for index formulae and non-local boundary conditions lie at the heart of this theorem. Consequently, this has inspired the study of boundary value problems for first-order elliptic differential operators by many different schools, with a class of induced operators adapted to the boundary taking centre stage in formulating and understanding non-local boundary conditions.

That being said, much of this analysis has been confined to the situation when adapted boundary operators can be chosen self-adjoint. Dirac-type operators are the quintessential example. Nevertheless, natural geometric operators such as the Rarita-Schwinger operator on 3/2-spinors, arising from physics in the study of the so-called Delta baryon, falls outside of this class. Analytically, this requires analysis beyond self-adjoint operators. In recent work with Bär, the compact boundary case is handled for general first-order elliptic operators, using spectral theory to choose adapted boundary operators to be invertible bi-sectorial. The Fourier circle methods present in the self-adjoint analysis are replaced by the bounded holomorphic functional calculus, coupled with pseudo-differential operator theory and semi-group techniques. This allows for a full understanding of the maximal domain of the interior operator as a bounded surjection to a space on the boundary of mixed Sobolev regularity, constructed from spectral projectors associated to the adapted boundary operator. Regularity and Fredholm extensions are also studied.

For the noncompact case, a preliminary trace theorem as well as regularity theory are handed by resorting to the case with compact boundary. This necessitates deforming the coefficients of the interior operator in a compact neighbourhood. Therefore, even for Dirac-type operators, allowing for fully general symbols in the compact boundary case is paramount. Under slightly stronger geometric assumptions near the noncompact boundary (automatic for the compact case) and when the interior operator admits a self-adjoint adapted boundary operator, an upgraded trace theorem mirroring the compact setting is obtained. Importantly, there is no spectral assumptions other than

self-adjointness on the adapted boundary operator. This, in particular, means that the spectrum of this operator can be the entire real line. Again, the primarily tool that is used in the analysis is the bounded holomorphic functional calculus.

10:20-10:40 Symbolic computation applied to the factorization concept

Ana C. Conceição
CEAFEL, Universidade do Algarve

In our work we use the computer algebra system Wolfram Mathematica to implement analytical algorithms, developed by us, within the operator theory. The design of our algorithms is focused on the possibility of implementing on a computer all, or a significant part, of the extensive symbolic and numeric calculations. The main goal of this talk is to present operator theory algorithms related to the factorization concept, that have applications in the study of the invertibility of singular integral operators, in the analysis of the spectrum and in the computation of the kernel of certain classes of operators. Several nontrivial examples computed with the algorithms are presented.

This is a joint work with Jéssica C. Pires.

10:45-11:15 *Coffee-break*

11:20-11:40 A construction of the fundamental solution for the Leutwiler-Weinstein operator

Heikki Orelma
Tampere University, Tampere, Finland

In this talk, we will discuss a simple way to find the fundamental solution for the Leutwiler-Weinstein equation. We discuss a general setting and then we find "radial-type" solutions for the equation. Then we use the one of the radial-type solutions for a candidate for the fundamental solution and evaluate the equation in distribution sense. Our aim is to give the construction as elementary level as possible. Only usual distribution theory with multivariable calculus are needed. This is a joint work with prof. em. S.-L. Eriksson.

11:45-12:30 Towards quantum limits for subelliptic operators

Véronique Fischer
University of Bath, United Kingdom

We discuss the recent developments of semi-classical and micro-local analysis in the context of nilpotent Lie groups and for sub-elliptic operators. In particular, we give an overview of pseudo-differential calculi recently defined on nilpotent Lie groups as well as of the notion of quantum limits in the Euclidean and nilpotent cases.

12:35-14:25 *Lunch-break*

14:30-15:15 A mathematical time machine to visit the past, present, and future of the theory of pseudo-differential operators.

Duván Cardona Sanchez
Ghent University, Ghent, Belgium

Harmonic analysis is the study of the Fourier transform in different group structures. The microlocal analysis is a modern PDE technique for investigating linear differential operator properties using Fourier analysis. Spectral theory analyses the structure and the geometric information of the eigenvalues/eigenspaces of an operator. But, with the glasses of the theory of pseudo-differential operators, they are the same thing. Let us drive the mathematical time machine to visit the past, present, and future of the theory of pseudo-differential operators. We will review the impact of this theory, some open problems, as well as some of its modern developments.

15:20-15:40 On some hyperholomorphic Theta functions

Irene Sabadini
Politecnico di Milano, Italy

In this talk we discuss a generalization of theta series in the context of the slice monogenic function theory in $\mathbb{R}^{n+1}$ where me make use of the so-called $*$-exponential function in a hypercomplex variable. We introduce a suitable generalized Poisson summation formula in this framework and we apply an properly adapted Fourier transform. As a direct application we prove a transformation formula for slice monogenic theta series. We also investigate their transformation behavior. Finally, we discuss a possible extension to the monogenic case.

The talk is based on joint work with F. Colombo, R.S. Krausshar.

15:45-16:15 *Coffee-break*

16:20-16:40 The Fresnel integral technique in the space $\mathcal{A}_1(\mathbb{C})$, with an application to Superoscillations

Peter Schlosser
Politecnico di Milano, Italy

Superoscillation is an optical (or quantum mechanical) phenomenon, where waves (or wave functions) of low frequency interact in a way that the resulting superposition has a larger frequency than any of its components. Mathematically, this phenomenon is described by a sequence of functions of the form

$$F_n(x) = \sum_{l=0}^{n} C_l(n) e^{ik_l(n)x} \overset{n\to\infty}{\longrightarrow} e^{ikx}, \tag{1}$$

which are linear combinations of exponentials with frequencies $|k_l(n)| \leq 1$, but converge to some exponential with frequency $k > 1$.

The main question is: How does this phenomenon behave during the time evolution with respect to the Schrödinger equation, i.e. when interacting with some external potential?

For this question it turns out to be natural to consider the functions F_n and also the convergence in (1) in the space of exponentially bounded entire functions

$$\mathcal{A}_1(\mathbb{C}) = \left\{ F \in \mathcal{H}(\mathbb{C}) \;\middle|\; \exists A, B \geq 0 : |F(z)| \leq A e^{B|z|} \right\}.$$

The key ingredient will be the the Fresnel integral technique applied to the Green's function representation of the solution

$$\Psi(t,x) = \int_{\mathbb{R}} G(t,x,y) F(y),$$

of the Schrödinger equation with initial condition F.

16:45-17:05 Monogenic Ridgelet transform and its implementation

João Costa
University of Aveiro, Portugal

17:10-17:30 Phase retrieval in quaternionic analysis

Haipan Shi
Hebei Normal University, China

In this talk we discuss the phase retrieval problem in the quaternionic setting, that is, the possibility to reconstruct quaternionic signals from the magnitudes of their fast quaternionic Fourier transform measurements.

17:35 *Closing Session*