Computational Logic
2024/25

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Outline

LOGIC IN COMPUTATION

DIFFERENT PERSPECTIVES ON THE NAME "COMPUTATIONAL LOGIC"

- Theory of Computation: Mathematical logic as a fundamental pillar for defining Computer Science as a "Science". E.g.
  - Foundations of Algorithms, complexity, computability, ...
- Constraint Programming
- Logic Programming
- In this Course: **(Modal) Logic as a foundation to support the rigorous development process of systems**

MOTIVATIONS

ANDRE PLATZER

... there is probably no other area where the gap is more noticeable between the tremendous complexity of the systems we can built and the modest size of systems that we can analyse

Logical Analysis of Hybrid Systems, 2012

Looking at Current Engineering Practices

The quality and reliability of

most computational systems are ensured by **test-oriented** development methodologies.

Edsger Dijkstra (Turing Award 1972)

Program testing can be used to show the presence of bugs, but never to show their absence!

(1969)

Looking at the Practice of Classical Engineering ...

e.g., in Mechanics, mathematics is used

- as an **unambiguous language** to express requirements
- to support **rigorous modeling**
- to support **validation and verification** tasks

Would it be plausible to build a bridge using a test-oriented approach?

There are computational systems with the same level of criticality...

Logic in Computing

Formal logics are useful mathematical tools
for

- modeling
- reasoning, and
- verification

of complex systems.

Logic is often referred to as
The calculus of computer science

The Role of Formal Logics in This Context

Is it possible to take advantage of the methods and results of Logic for the formal analysis and development of systems?

Modal Logic (the focus of this course)

- A topic in Logic that has been studied since Aristotle to express notions of necessity/possibility.
- The relational semantics by Kripke (1960s, 20th century) opened up a vast domain of applications for the formalism. Kripke models are based on **transition systems**.
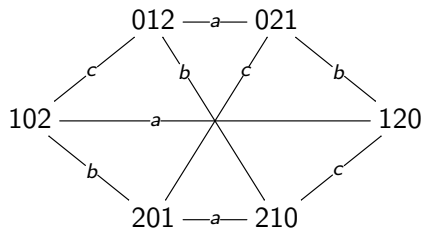
Processes Are Transition Systems

Two Coffee Machines
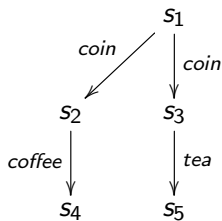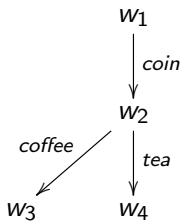
Multi-Agent Knowledge Systems Are Transition Systems
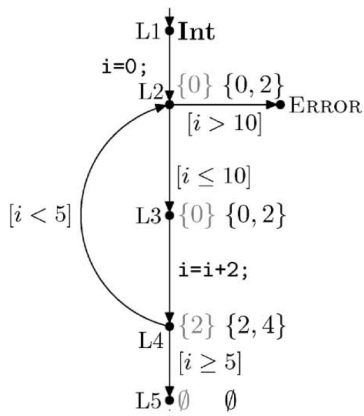
The Envelope Game

E.g. in state 012: Ana has the envelope with 0, Bob has the one with 1, and Clara has the one with 2.

Programs are transition systems


```
int i = 0;
do {
    assert(i <= 10);
    i = i+2;
} while (i < 5);
```

L1 • Int

i=0;

L2 • $\{0\}$ $\{0,2\}$ •—• Error

$[i > 10]$

$[i \leq 10]$

L3 • $\{0\}$ $\{0,2\}$

i=i+2;

$[i < 5]$

L4 • $\{2\}$ $\{2,4\}$

$[i \geq 5]$

L5 • $\emptyset$ $\emptyset$

Outline

Syllabus

- **Review of Propositional and First-Order Logic**
- **Modal Logic**
  - Syntax and semantics
  - Definability in Modal Logic
  - Bisimulations and Modal Invariance
  - Hennessy-Milner Theorem
  - Calculus - correctness and completeness
- **Variants**
  - Multi-modal
  - Temporal Logics
  - Dynamic Logic and Floyd-Hoare Calculus

Information

My Contact

- madeira@ua.pt

Scheduling Office Hours

- Any convenient time

Outline

1 Why a Computational Logic Course?

2 Course Structure

3 Assessment Method

Assessment

The course assessment will consist of three distinct components:

- **Two written tests** (T_1 on **02/04/25** and T_2 on **28/05/2025**)
- **Project** (P) – Each student will be assigned a specific topic in the field (a logic or verification tool) to explore throughout the semester. The assessment of this component will be based on the project presentation **(04/06/2025)** and the submission of a final report.

Final Grade Calculation:

$$\text{Final Grade} = 0.3 * T_1 + 0.4 * T_2 + 0.3 * P$$

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