Asymmetric Cryptography

SIO

João Paulo Barraca

universidade de aveiro
deti departamento de eletrónica, telecomunicações e informática

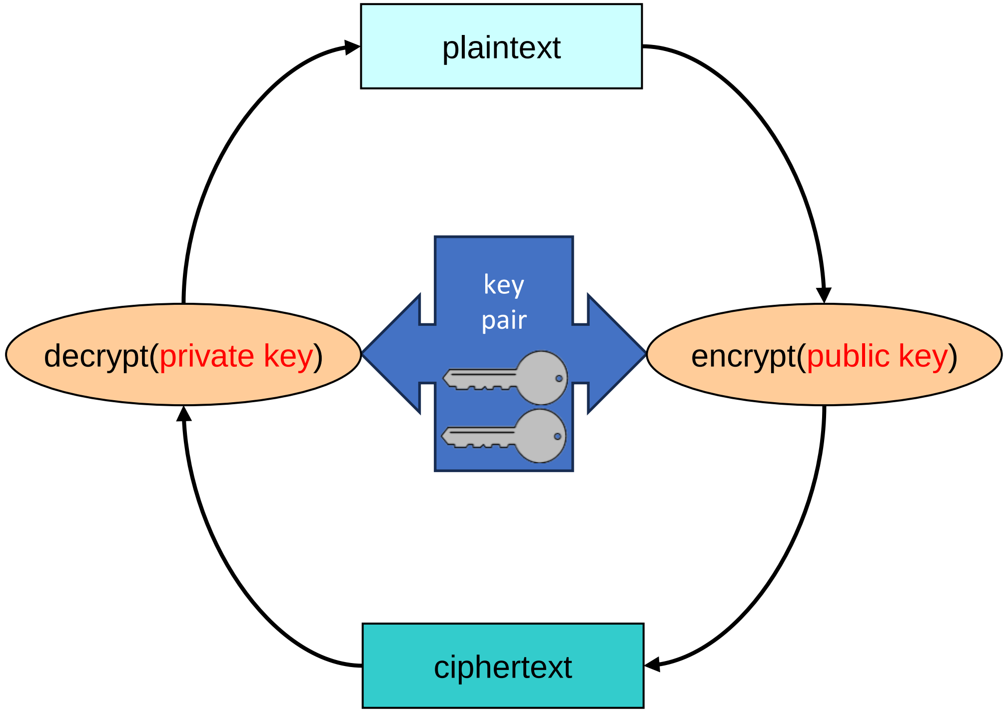
Asymmetric (Block) Ciphers

- Use key pairs
  - **One private key**: personal, not transmittable
  - **One public key**: available to all

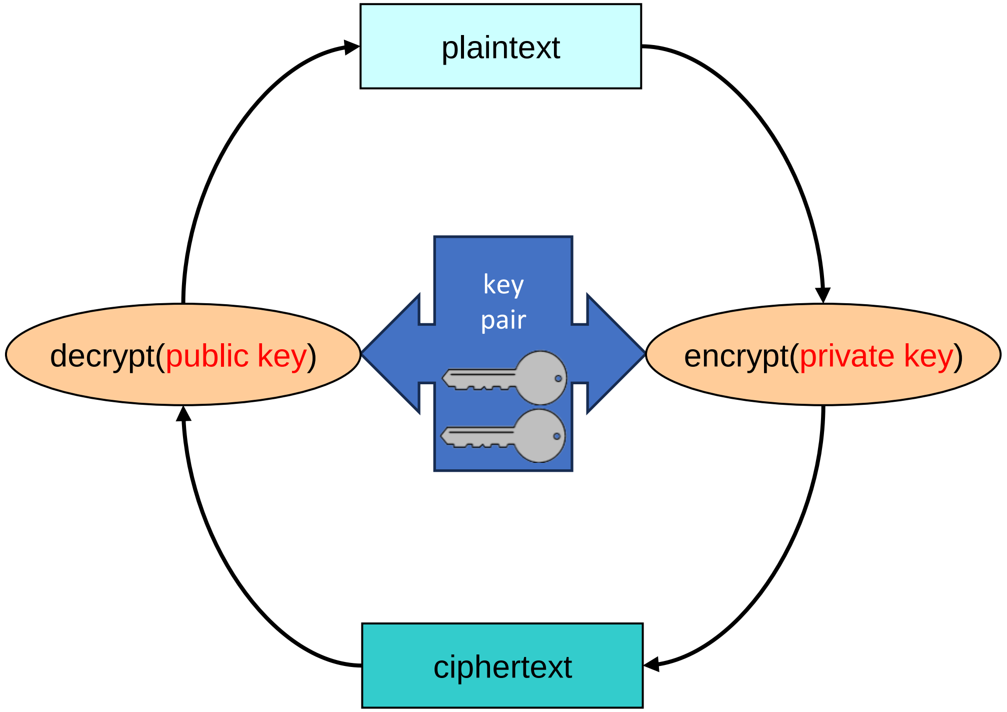
- Allow
  - Confidentiality without any previous exchange of secrets
  - Authentication
    - Of contents (data integrity)
    - Of the data origin (source authentication, or digital signature)

Operations of an asymmetric cipher

Confidentiality



Authenticity



Use cases: confidential communication

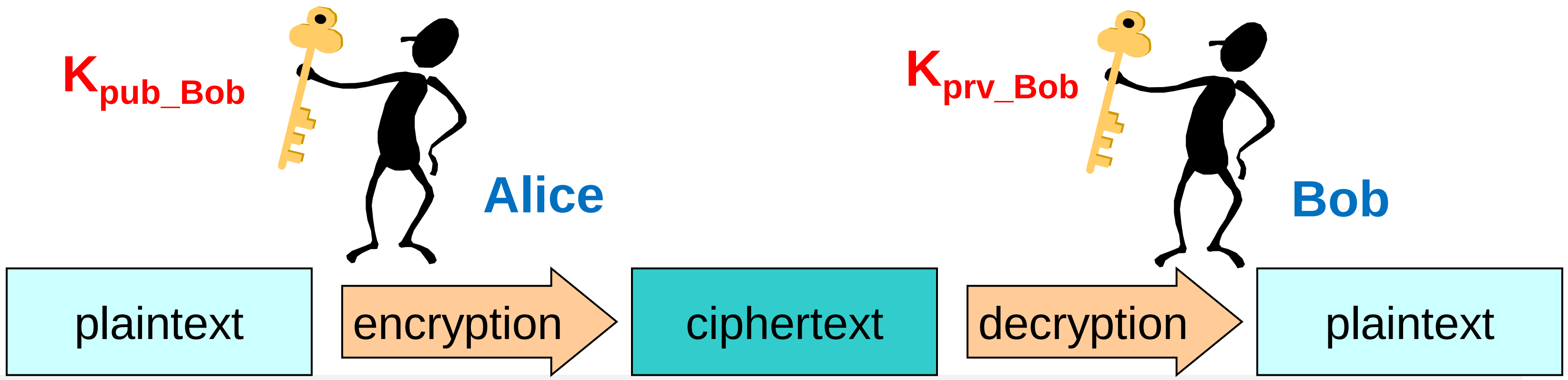
- Secure communication with a target (Bob)
  - Alice encrypts plaintext **P** with Bob's public key K_{pub_Bob}

    Alice: $C = \{P\}K_{pub\_Bob}$

  - Bob decrypts cyphertext **C** with his private key K_{prv_Bob}

    Bob: $P' = \{C\}K_{prv\_Bob}$

  - **P'** should be equal to **P** (requires checking using integrity control)
  - K_{pub_Bob} needs to be known by Alice



Use cases: authenticated communication

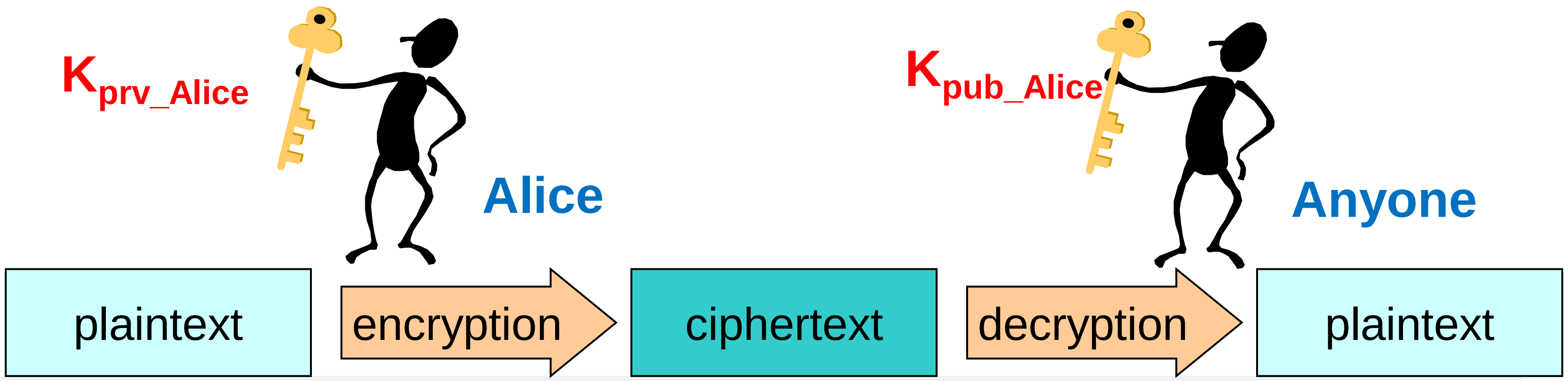
- Authenticate the communication from Alice
  - Alice encrypts plaintext **P** with her private key K_{prv_Alice}

    Alice: C = {P}K_{prv_Alice}

  - Anyone can decrypt cyphertext **C** with Alices' Public key K_{pub_Alice}

    Anyone: P'= {C}K_{pub_Alice}

  - If **P'** = **P**, then **C** is Alice's signature of **P**
  - K_{pub_Alice} needs to be known by the message verifiers



Asymmetric ciphers

Issues

- Advantages
  - They are a fundamental authentication mechanism
  - They allow to explore features that are not possible with asymmetric ciphers
- Disadvantages
  - Performance: 2 or 3 orders of magnitude over AES
  - Very inefficient and memory consuming: Large keys
- Problems
  - Trustworthy distribution of public keys: how to know if the public key is the correct one?
  - Lifetime of key pairs: How to make sure that we can deal with lost/deprecated/leaked keys?

Asymmetric ciphers

Overview

- Approaches: complex mathematic problems
  - **Discrete logarithms** of large numbers
  - **Integer factorization** of large numbers
- Most common algorithms
  - RSA
  - ElGamal
  - Elliptic curves (ECC)
- Other techniques with asymmetric key pairs
  - Diffie-Hellman (key agreement)

RSA

Rivest, Shamir, Adelman, 1978

- Keys: Private: (d, n) Public: (e, n)

- Public key encryption (confidentiality) of **P**
  - **$C = P^e \bmod n$**
  - **$P = C^d \bmod n$**

- Private key encryption (authenticity) of **P**
  - **$C = P^d \bmod n$**
  - **$P = C^e \bmod n$**

P, C are numbers!
Message is converted to/from numbers

$0 \leq P, C < n$

RSA

Rivest, Shamir, Adelman, 1978

- Computational complexity: **Discrete logarithm** and **Integer factoring**

- Key selection
  - Large **n** (hundreds or thousands of bits)
  - **n = p × q** with **p** and **q** being large (secret) prime numbers
  - Chose an **e** co-prime with **(p-1) × (q-1)**
  - Compute **d** such that **e × d ≡ 1 (mod (p-1) × (q-1))**
  - Discard **p** and **q**
  - The value of **d** cannot be computed out of **e** and **n**
    - Only from **p** and **q**

coprime → gcd(a, b) = 1

× → multiplication

mod → modulo operation

≡ → modular congruence

a ≡ b mod n iff rem(a,n) = rem(b,n)

Playing with RSA

- $p = 5$ $q = 11$ (prime numbers)
  - $n = p \times q = 55$
  - $(p-1) \times (q-1) = 40$
- $e = 3$ (public key = e, n)
  - Coprime of 40
- $d = 27$ (private key = d, n)
  - $e \times d \equiv 1 \pmod{40}$ -> $d \times e \bmod 40 = 1$ -> $(27 \times 3) \bmod 40 = 1$
- For a message to encrypt, P = 26   (notice that $P, C \in [0, n-1]$)
  - $C = P^e \bmod n$ = $26^3 \bmod 55 = 31$
  - $P = C^d \bmod n$ = $31^{27} \bmod 55 = 26$

Hybrid Encryption

- Combines symmetric with asymmetric cryptography
  - Use the best of both worlds, while avoiding problems
  - Asymmetric cipher: Uses public keys (but it is slow)
  - Symmetric cipher: Fast (but with weak key exchange methods)

- Method:
  - Obtain K_{pub} from the receiver
  - Generate a random K_{sym}
  - Calculate **$C1 = E_{sym}(K_{sym}, P)$**
  - Calculate **$C2 = E_{asym}(K_{pub}, K_{sym})$**
  - Send **C1 + C2**
    - C1 = Text encrypted with symmetric key
    - C2 = Symmetric key encrypted with the receiver public key
      - May also contain the IV

Randomization of asymmetric encryptions

- RSA is a deterministic algorithm: equal messages result in equal outputs

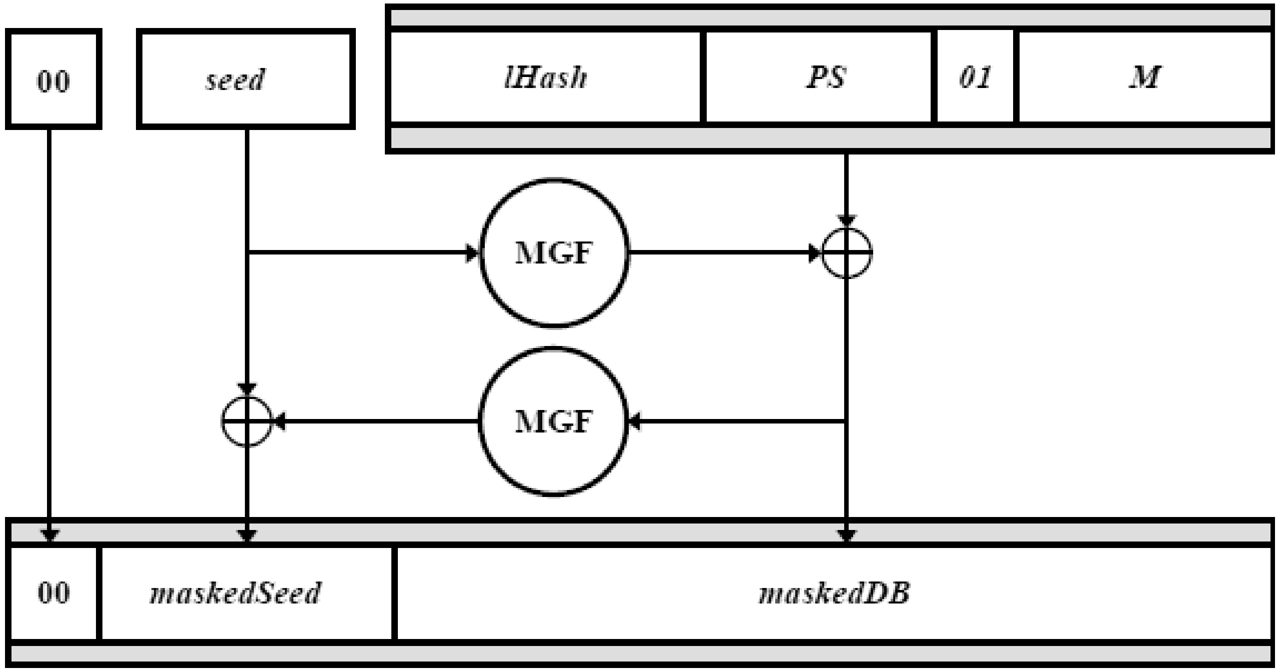
- What we need: Non-deterministic result of asymmetric encryptions
  - **N** encryptions of the same value, with the same key, should yield N different results
  - **Goal: prevent the trial & error discovery of encrypted values**

- Approaches
  - Concatenation of value to encrypt with two values
  - A fixed one (for integrity control)
  - A random one (for randomization)

Randomization of asymmetric encryptions

OAEP (Optimal Asymmetric Encryption Padding)

- `iHash`: digest over Label
- `seed`: random value
- PS: zeros
- M: plaintext
- MGF: Mask Generation Function
  - Similar to Hash, but with variable size



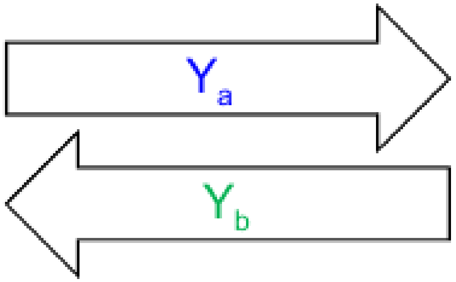
Diffie-Hellman Key Agreement (1976)

q (large prime)

α (primitive root mod q)

a = random

$Y_a = \alpha^a \bmod q$



b = random

$Y_b = \alpha^b \bmod q$

$K_{ab} = {Y_b}^a \bmod q$

$K_{ba} = {Y_a}^b \bmod q$

$K_{ab} = K_{ba}$

Diffie-Hellman Key Agreement (1976)

a = random

$Y_a = \alpha^a \bmod q$

$K_{ac} = Y_c^{\,a} \bmod q$

Y_a →

← Y_c

c = random

$Y_c = \alpha^c \bmod q$

$K_{ca} = Y_a^{\,c} \bmod q$

$K_{cb} = Y_b^{\,c} \bmod q$

← Y_b

Y_c →

b = random

$Y_b = \alpha^b \bmod q$

$K_{bc} = Y_c^{\,b} \bmod q$

Elliptic Curve Cryptography (ECC)

- Elliptic curves are specific functions
  - They have a generator (G)
  - A private key K_{prv} is an integer with a maximum of bits allowed by the curve
  - A public key K_{pub} is a point $(x,y) = K_{prv} \times G$
  - Given K_{pub}, it should be hard to guess K_{prv}

- Curves
  - NIST curves (15)
    - P-192, P-224, P-256, P-384, P-521
    - B-163, B-233, B-283, B-409, B-571
    - K-163, K-233, K-283, K-409, K-571

Other curves

- Curve25519 (256 bits)
- Curve448 (448 bits)

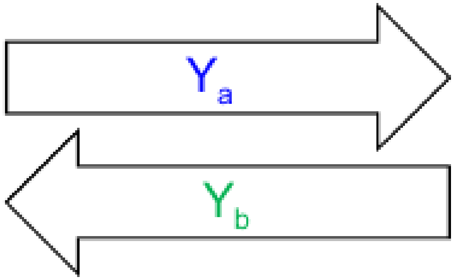
ECDH: DH with ECC

ECC curve → G

a = random

$Y_a = a\,G$

$K_{ab} = a\,Y_b$



$K_{ab} = K_{ba}$

b = random

$Y_b = b\,G$

$K_{ba} = b\,Y_a$

ECC public key encryption

Combines hybrid encryption with ECDH

- Obtain K_{pub_recv} from the receiver
- Generate a random K_{prv_send} and the corresponding K_{pub_send}
- Calculate $K_{sym} = K_{prv\_send}\ K_{pub\_recv}$

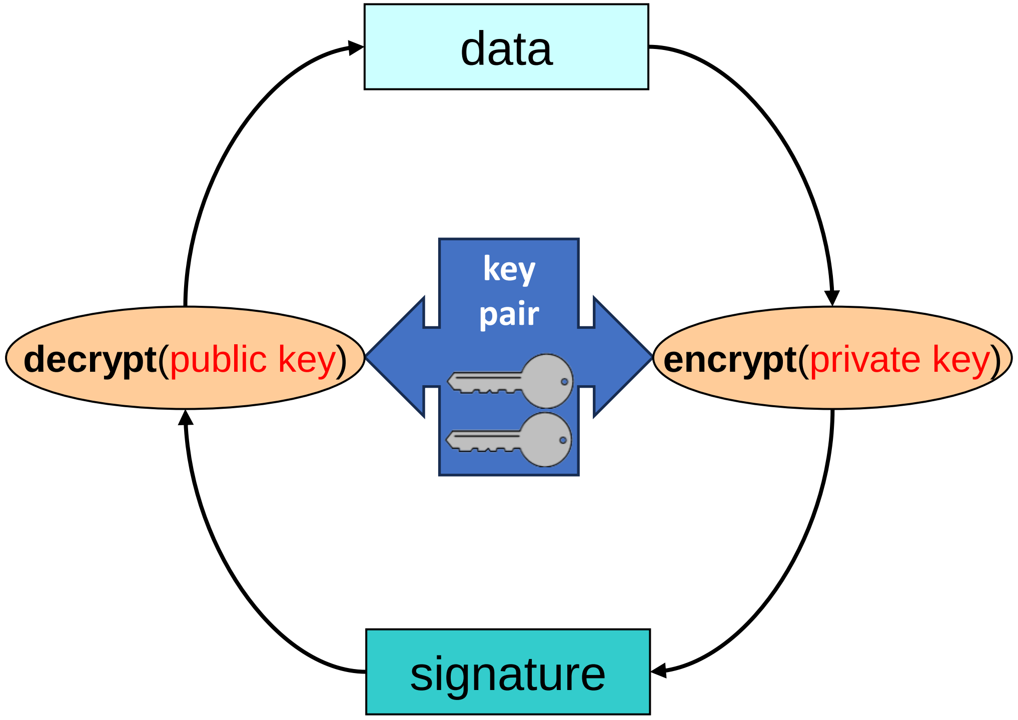
- $C = E(\ P,\ K_{sym}\ )$

- Send $C + K_{pub\_send}$
- Receiver calculates $K_{sym} = K_{pub\_send}\ K_{prv\_recv}$

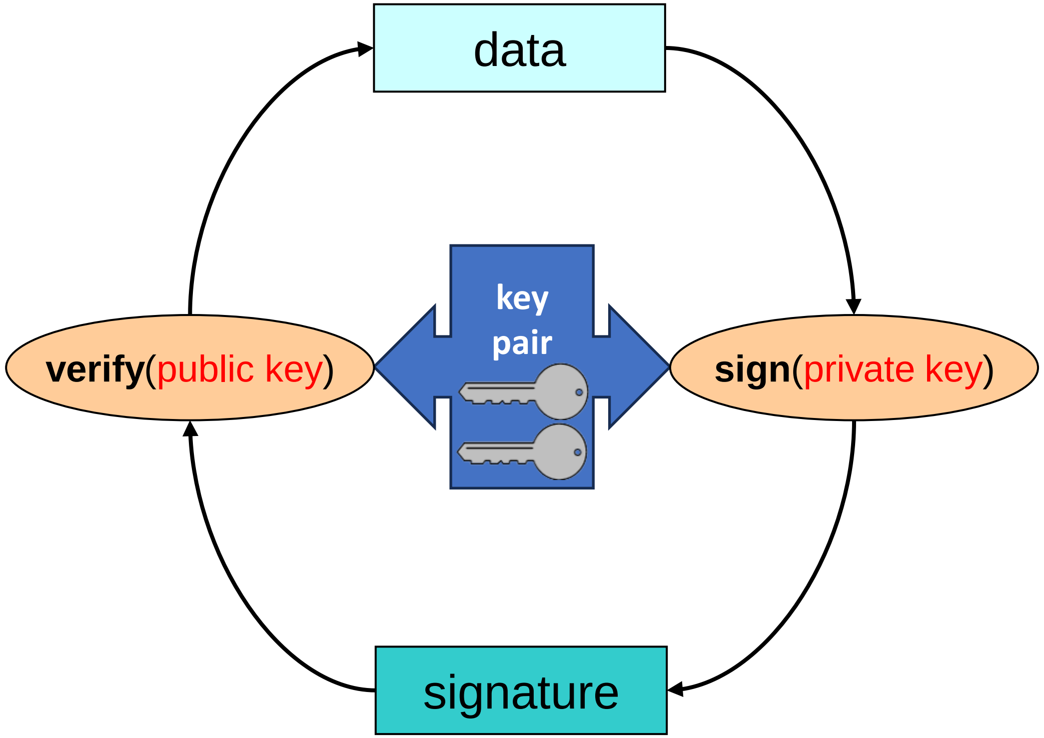
- $P = D(\ C,\ K_{sym}\ )$

Digital signatures

Encrypt/Decrypt (RSA)



Sign/Verify (ElGamal, EC)



Operations with Private Keys

- Authenticate the contents of a document
  - Ensure its integrity (it was not changed)

- Authenticate its author
  - Ensure the identity of the creator/originator

- Prevent repudiation of the encrypted payload
  - Non-repudiation
  - Genuine authors cannot deny authorship
    - Only the identified author could have generated a given payload
    - Because only the author has the private key

Digital signatures

- Authenticate the contents of a document
  - Ensure its integrity (it was not changed)
- Authenticate its author
  - Ensure the identity of the creator/originator
- Prevent repudiation of signatures
  - Non-repudiation property
  - Genuine authors cannot deny authorship
    - Only the identified author could have generated a given signature

Practical Considerations

- Encryption with private key is vital for authentication
  – Only the author can make it, everyone can verify it

- But… sending secure authenticated texts will require two (slow) encryptions
  – Remember: Asymmetric ciphers are slow and inefficient

- Preferred Approach: **Encrypt Hash(T), creating Digital Signatures**

Digital Signatures

- Approaches
  - Digest function of the Text (only for performance)
  - Asymmetric encryption/decryption or signature/verification

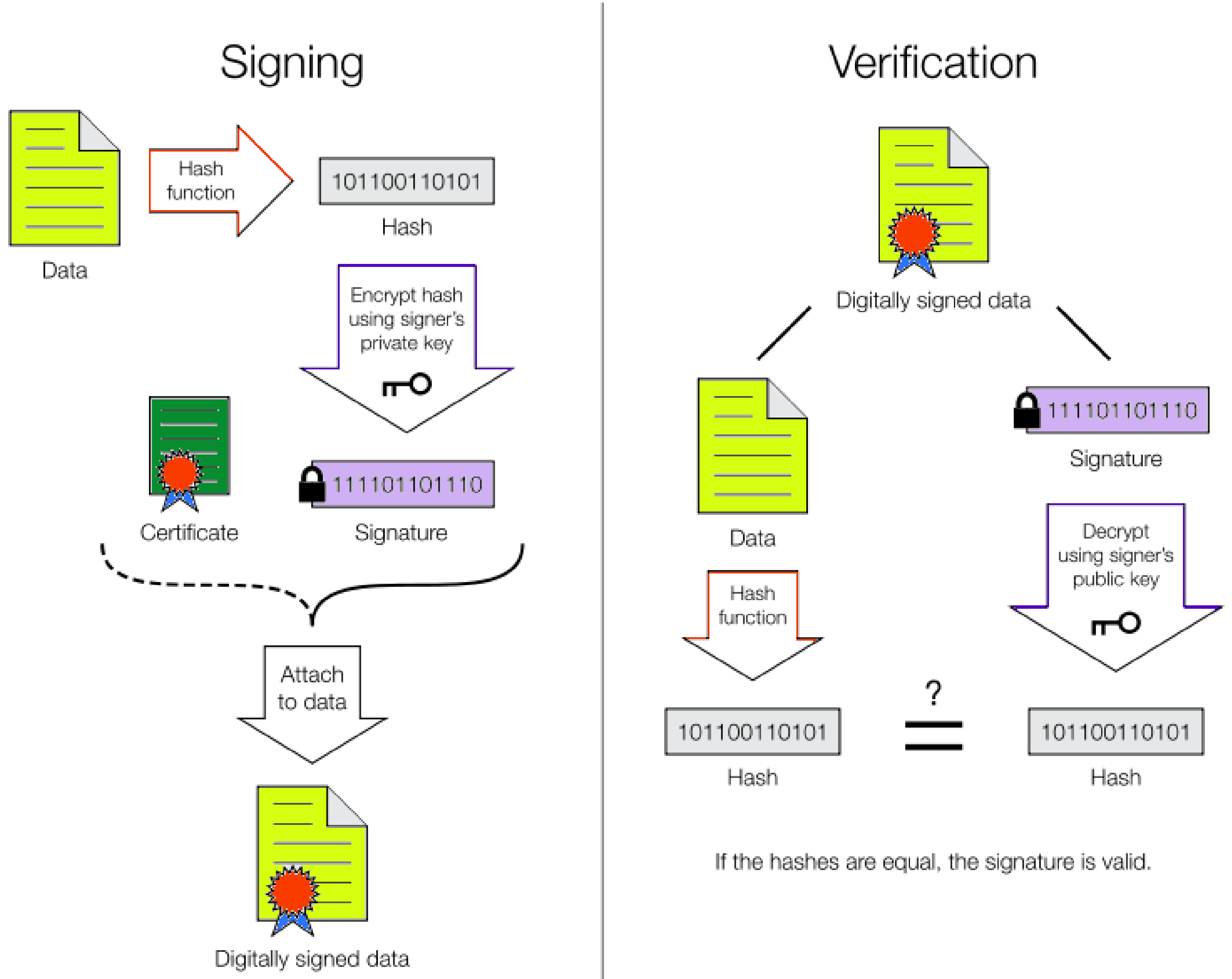
Signing:

$$A_x(doc) = info + E(K_x^{-1}, digest(doc + info))$$
$$A_x(doc) = info + S(K_x^{-1}, digest(doc + info))$$
$$info = \text{signing context, signer identity, } K_x$$

Verification:

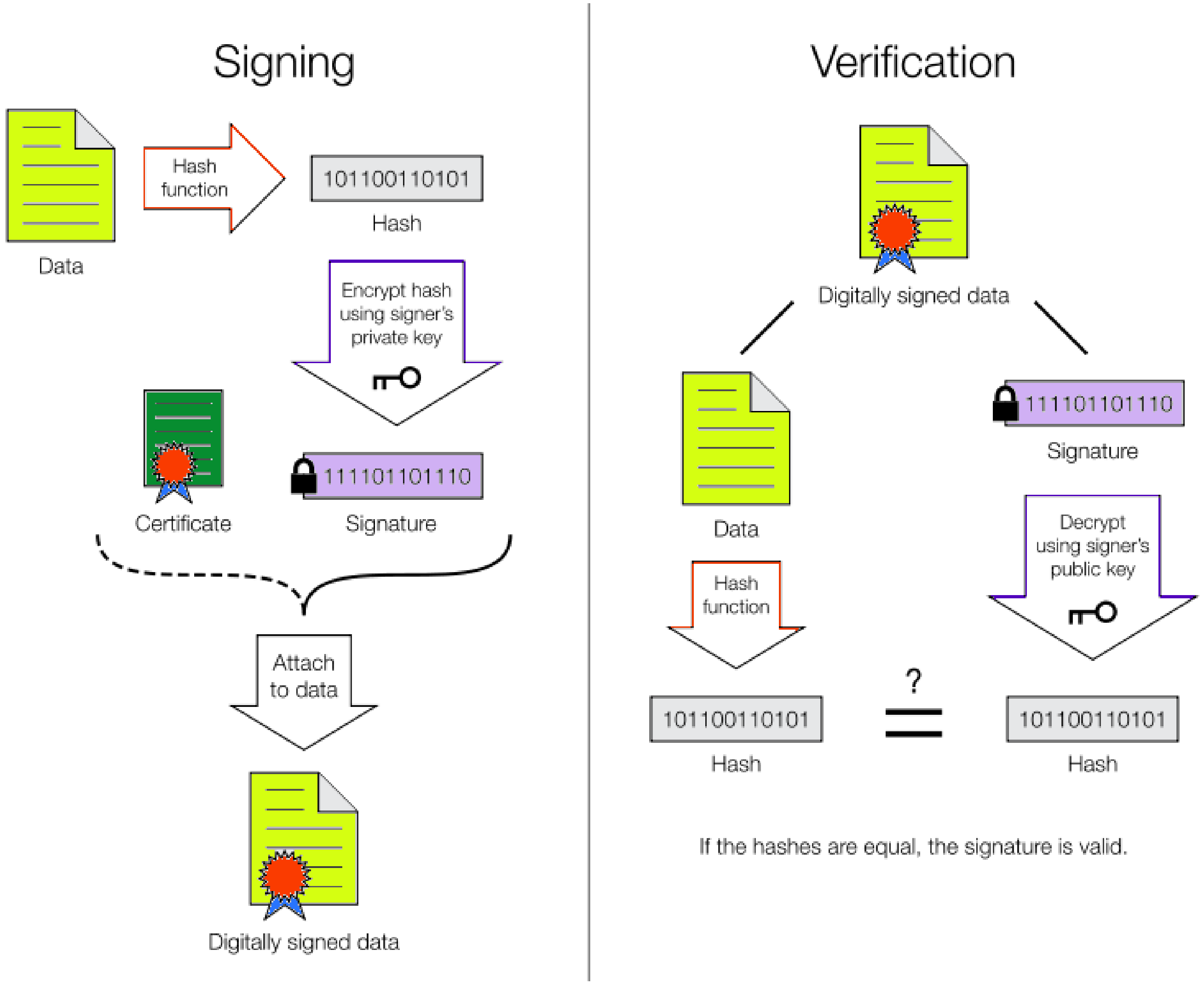
$$D(K_x, A_x(doc)) \equiv digest(doc + info)$$
$$V(K_x, A_x(doc), doc, info) \rightarrow \text{True / False}$$

Encryption / decryption signatures



wikipedia, http://en.wikipedia.org/wiki/Digital_signature

Encryption / decryption signatures



Digital Signature on a mail message

Multipart content, signature w/ certificate

```
From - Fri Oct 02 15:37:14 2009
[…]
Date: Fri, 02 Oct 2009 15:35:55 +0100
From: User A <usera@domain.com>
MIME-Version: 1.0
To: User B <userb@domain.com>
Subject: Teste
Content-Type: multipart/signed; protocol="application/x-pkcs7-signature"; micalg=sha1; boundary="------------ms050405070101010502050101"

This is a cryptographically signed message in MIME format.

--------------ms050405070101010502050101
Content-Type: multipart/mixed;
 boundary="------------060802050708070409030504"

This is a multi-part message in MIME format.
--------------060802050708070409030504
Content-Type: text/plain; charset=ISO-8859-1
Content-Transfer-Encoding: quoted-printable

Corpo do mail

--------------060802050708070409030504—
--------------ms050405070101010502050101
Content-Type: application/x-pkcs7-signature; name="smime.p7s"
Content-Transfer-Encoding: base64
Content-Disposition: attachment; filename="smime.p7s"
Content-Description: S/MIME Cryptographic Signature

MIAGCSqGSIb3DQEHAqCAMIACAQExCzAJBgUrDgMCGgUAMIAGCSqGSIb3DQEHAQAAoIIamTCCBUkwggSyoAMCAQICBAcnIaEwDQYJKoZIhvcNAQEFBQAwdTELMAkGA1UEBhMCVVMxGDAWBgNV
[…]
KoZIhvcNAQEBBQAEgYCofks852BV77NVuww53vSxO1XtI2JhC1CDlu+tcTPoMD1wq5dc5v40Tgsaw0N8dqgVLk8aC/CdGMbRBu+J1LKrcVZa+khnjjtB66HhDRLrjmEGDNttrEjbqvpd2QO2
vxB3iPTlU+vCGXo47e6GyRydqTpbq0r49Zqmx+IJ6Z7iigAAAAAAAA==
--------------ms050405070101010502050101--
```

Digital Signatures at kernel.org