Departamento de Electrónica e Telecomunicações da Universidade de Aveiro

Sistemas Multimédia

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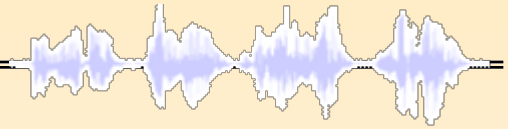
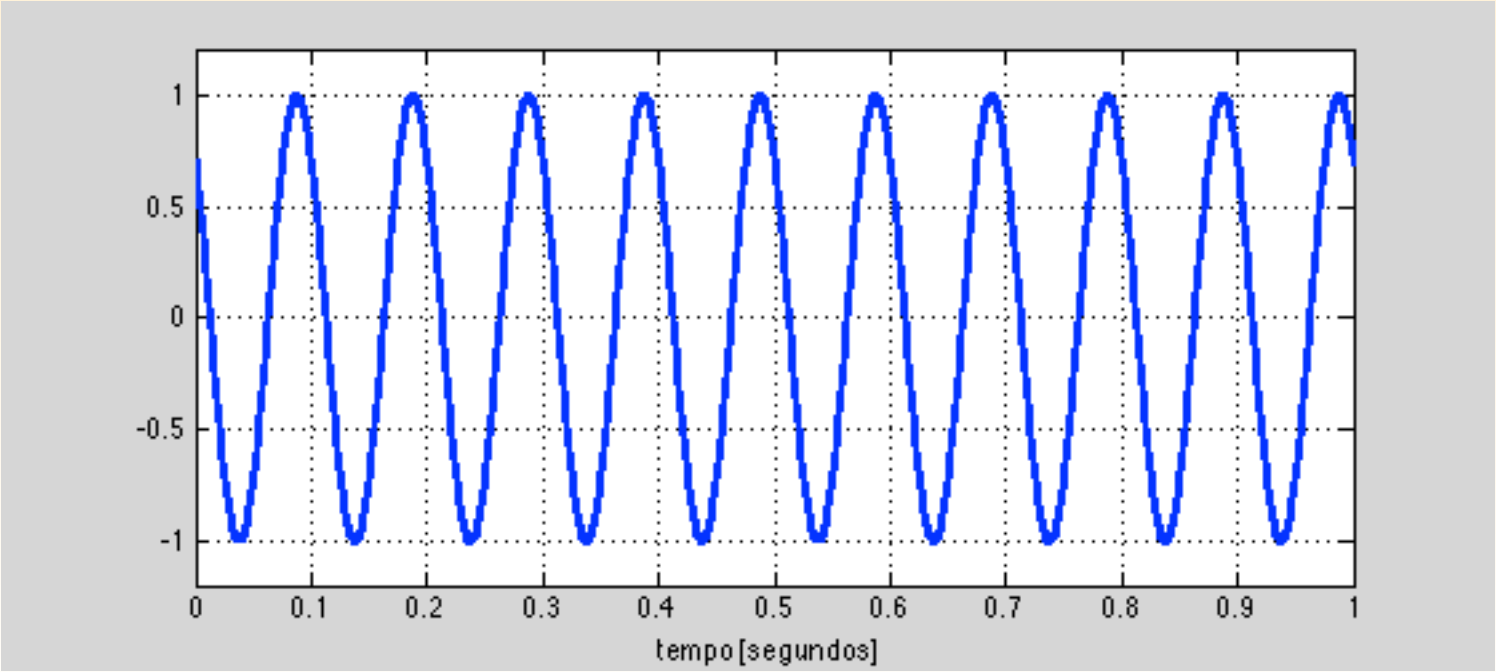
Telecomunicações e Informática

Universidade de Aveiro

Sinusóide : tempo e frequência

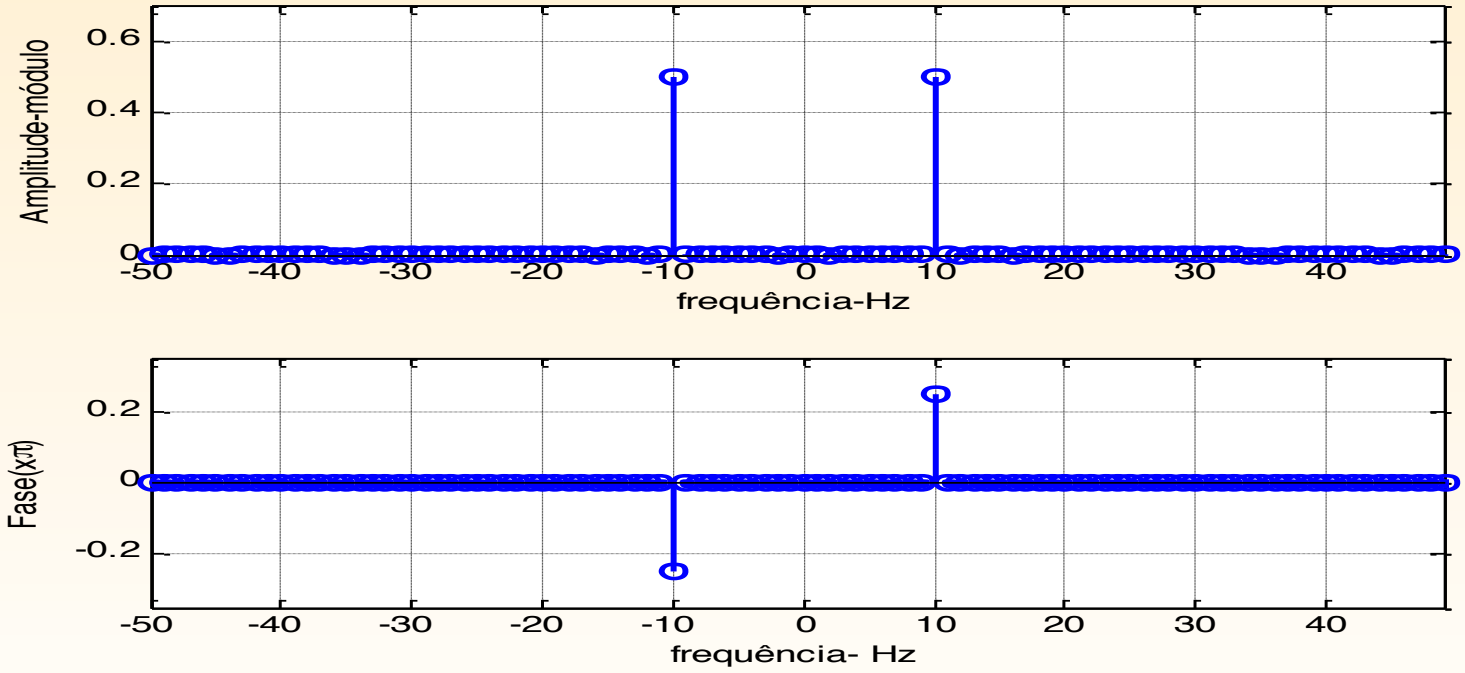


$$x(t) = \cos(2\pi 10t + \pi/4)$$

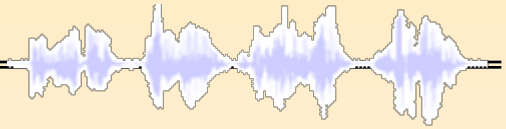


Sinusóide – Representação Espectral

$$x(t) = \frac{A}{2} e^{j\varphi} e^{j2\pi ft} + \frac{A}{2} e^{-j\varphi} e^{-j2\pi ft}$$



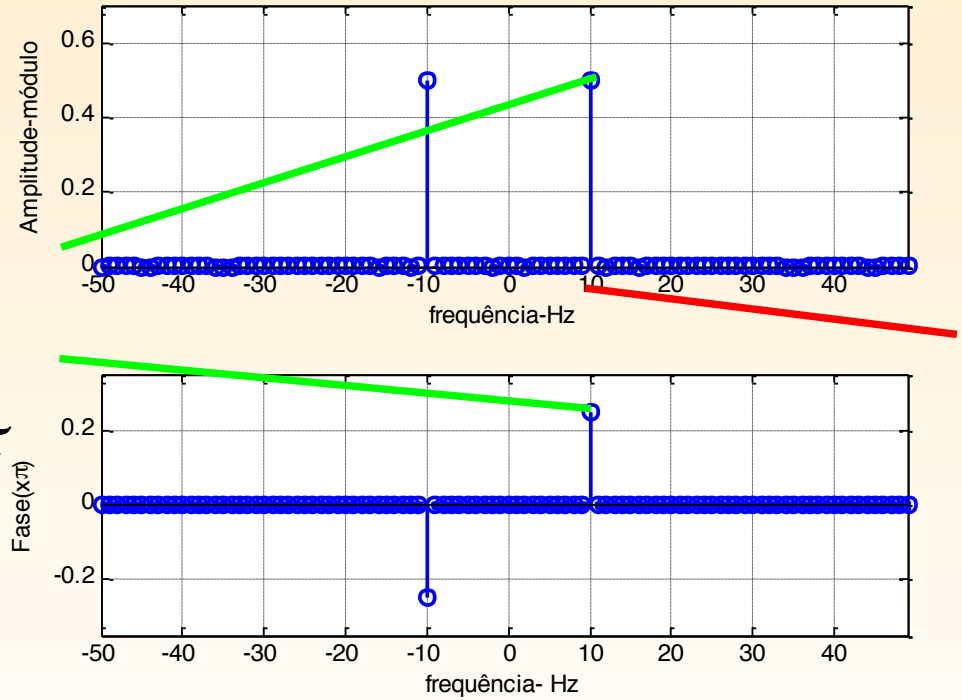
Sinusóide – Representação Espectral



$$\frac{A}{2}e^{j\varphi} \Rightarrow$$

$$\frac{A}{2} = 0.5$$

$$\varphi = 0.25\pi$$



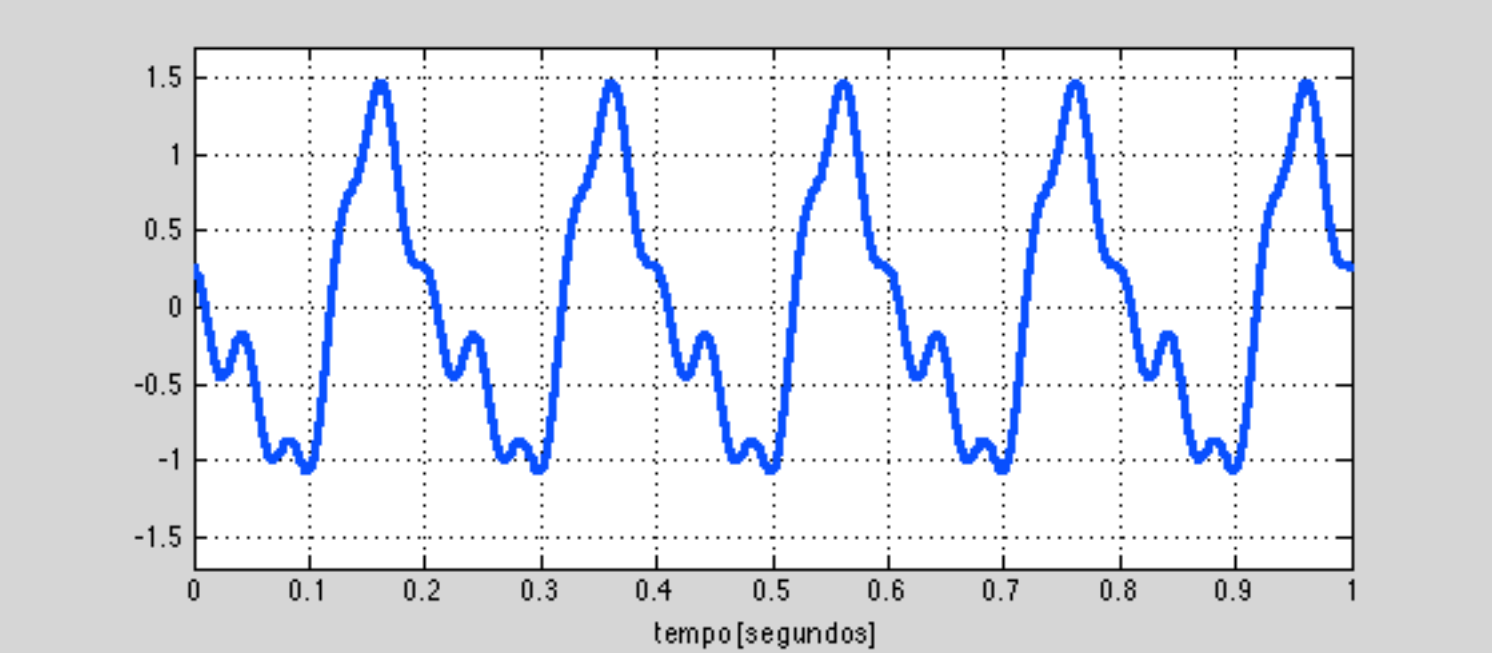
$$e^{j2\pi ft} \Rightarrow$$

$$f = 10Hz$$

$$\Rightarrow 1\cos(2\pi 10t + 0.25\pi)$$

Soma de sinusóides

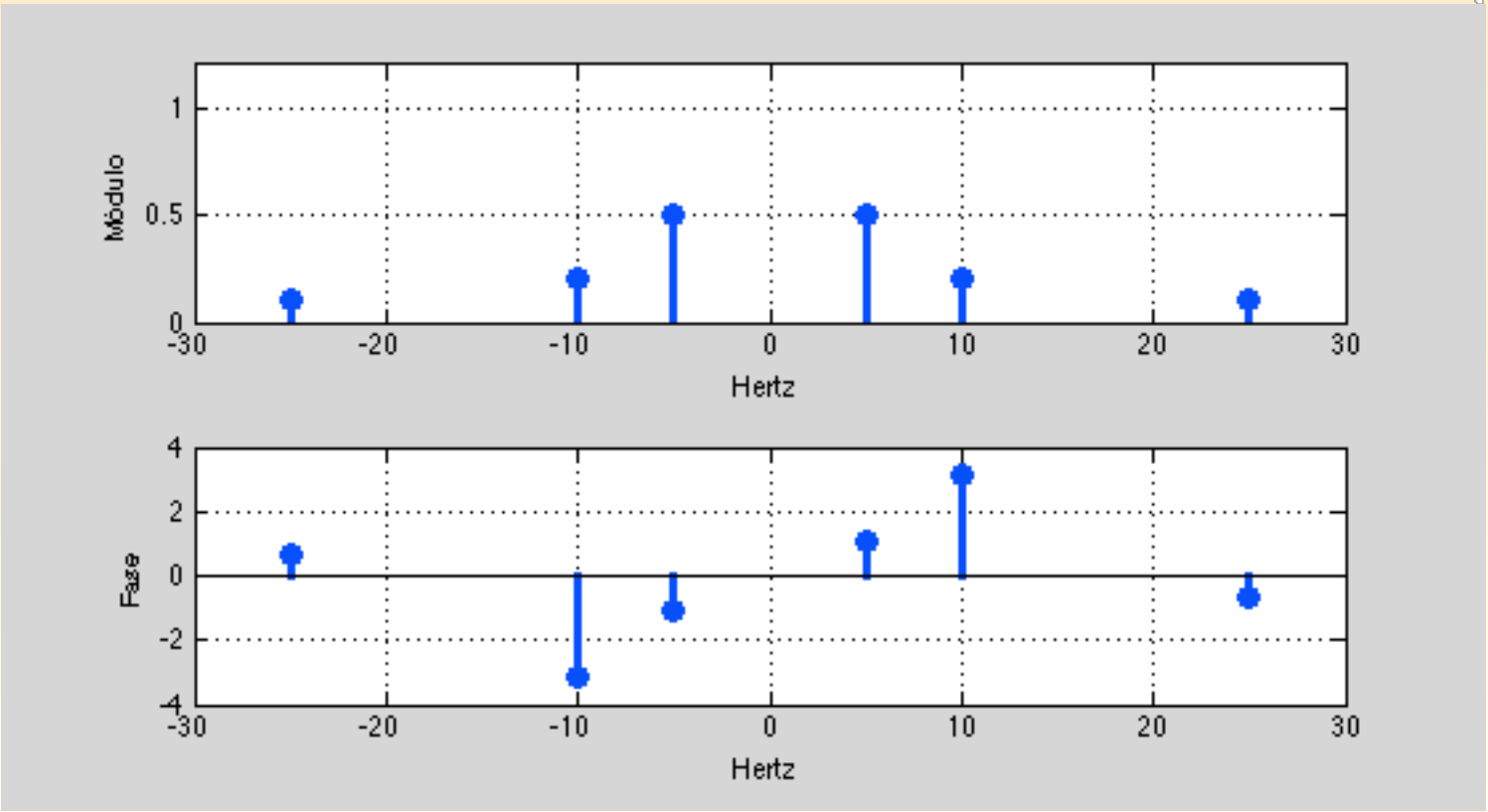
$$x(t) = A_0 + \sum_{k=1}^{N} A_k \cos(2\pi f_k t + \phi_k) \qquad x(t) = \sum_{k=-N}^{N} a_k e^{j2\pi f_k t} \qquad a_k = \frac{A_k}{2} e^{j\phi_k}$$



Quantas sinusóides tem o sinal?

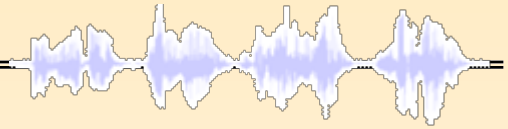
Qual é o valor das frequências das sinusóides?

Soma de sinusóides



Quantas sinusóides tem o sinal?
Qual é o valor das frequências das sinusóides?
Amplitude? Fase?

Soma de sinusóides: leitura dos gráficos

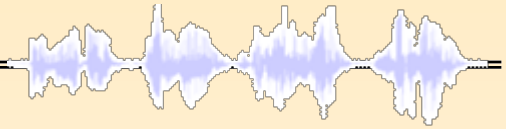


***F* (Hz)**	***a***	***φ***
-25	0.1	π/5
-10	0.2	-π
-5	0.5	-π/3
5	0.5	π/3
10	0.2	π
25	0.1	-π/5

N=3 (sinusóides)

$$x(t) = \cos\left(2\pi 5t + \frac{\pi}{3}\right) + 0.4\cos\left(2\pi 10t + \pi\right) + 0.2\cos\left(2\pi 25t - \frac{\pi}{5}\right)$$

Séries de Fourier

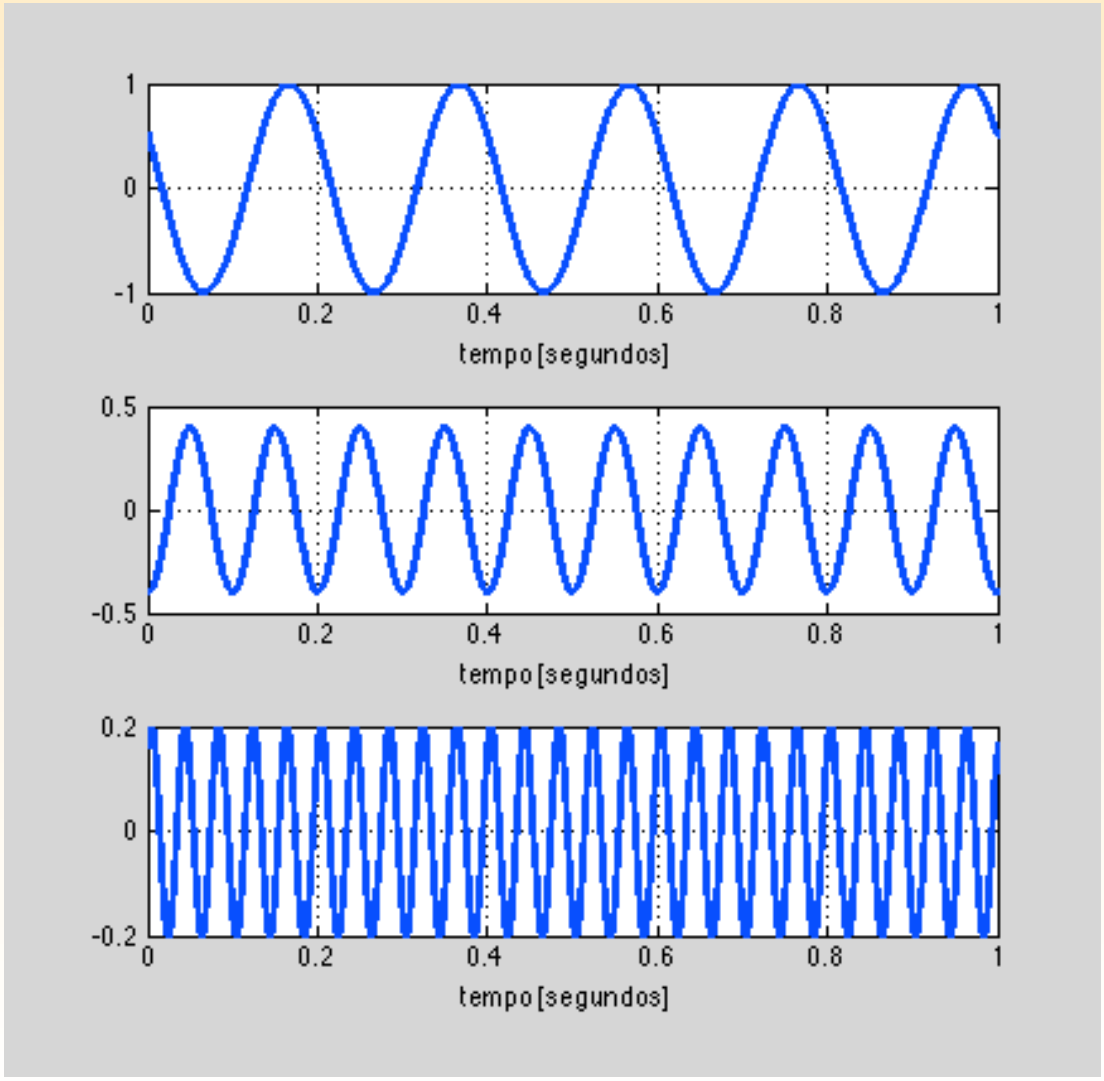


Qualquer sinal de período T pode ser representado como uma soma de sinusóides cuja frequência é um múltiplo de $F=1/T$.

$$x(t) = A_0 + \sum_{k=1}^{N} A_k \cos(2\pi kFt + \phi_k)$$

$$x(t) = \sum_{k=-N}^{N} a_k e^{j2\pi kFt} \qquad a_k = \frac{A_k}{2} e^{j\phi_k}$$

Harmónicos do sinal

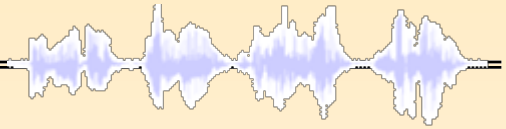


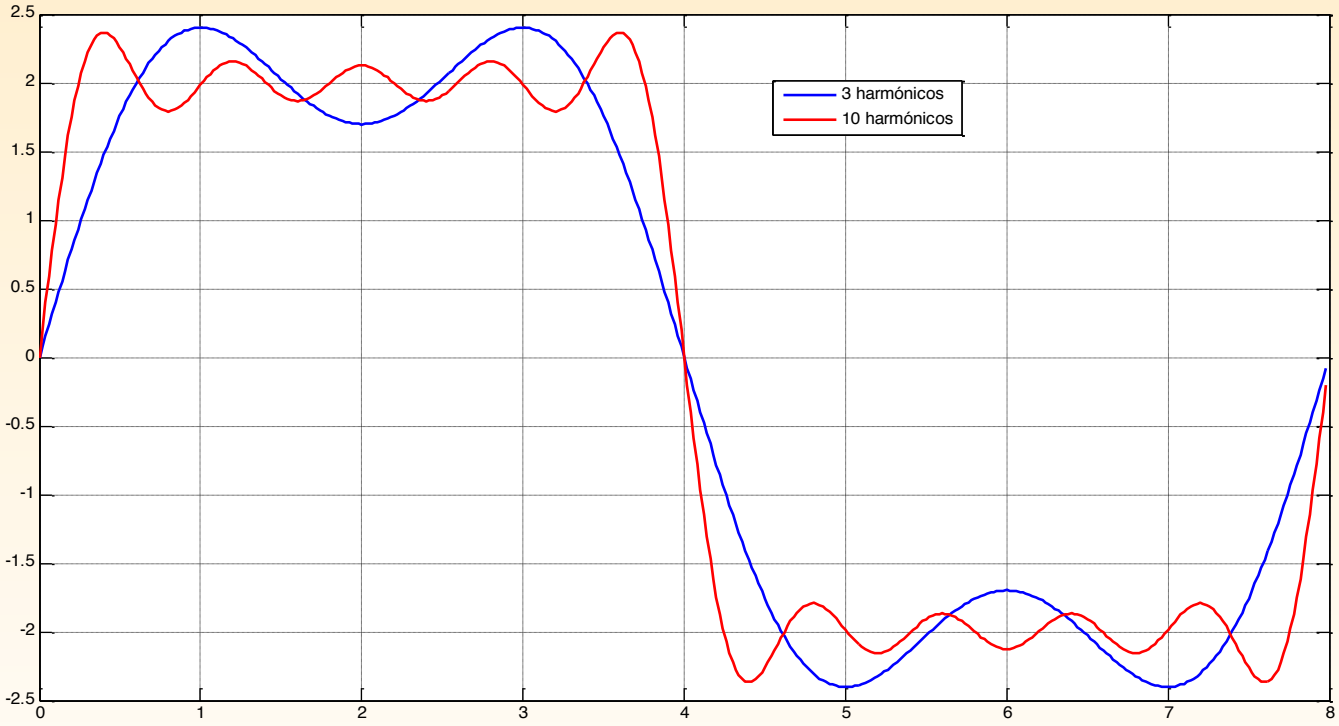
$k = \pm 1$

$k = \pm 2$

$k = \pm 5$

Recontrução do sinal

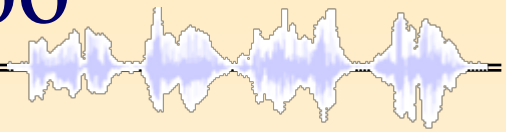




Soma dos harmónicos

Demonstração Interactiva

Conteúdo de frequência de um sinal discreto no tempo



Ficheiro wav

N amostras
e
F_a- frequência de amostragem

→ FFT(Fast Fourier Transform)
N amostras na frequência

N impar

$$k = 0,1,\cdots\frac{N-1}{2}$$

$$\Rightarrow f = k\frac{F_a}{N}$$

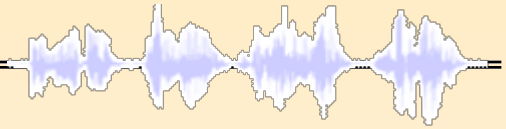
N par

$$k = 0,1,\cdots\frac{N}{2}$$

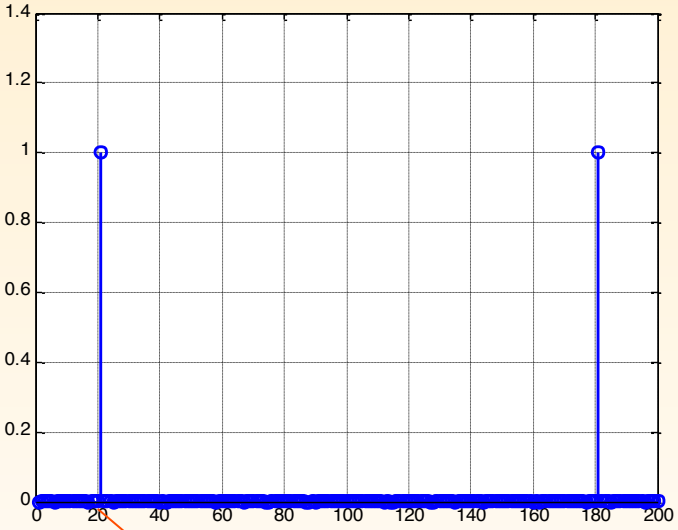
$$\Rightarrow f = k\frac{F_a}{N}$$

Outros valores de *k* : frequências negativas

Exemplo

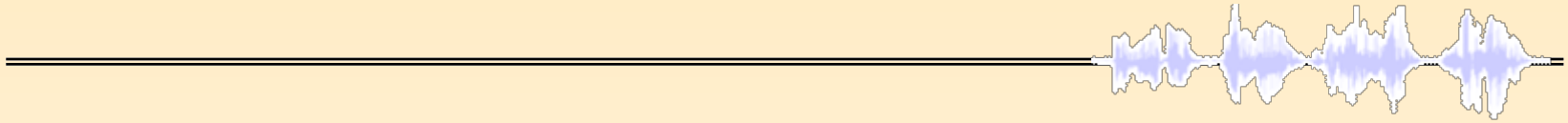


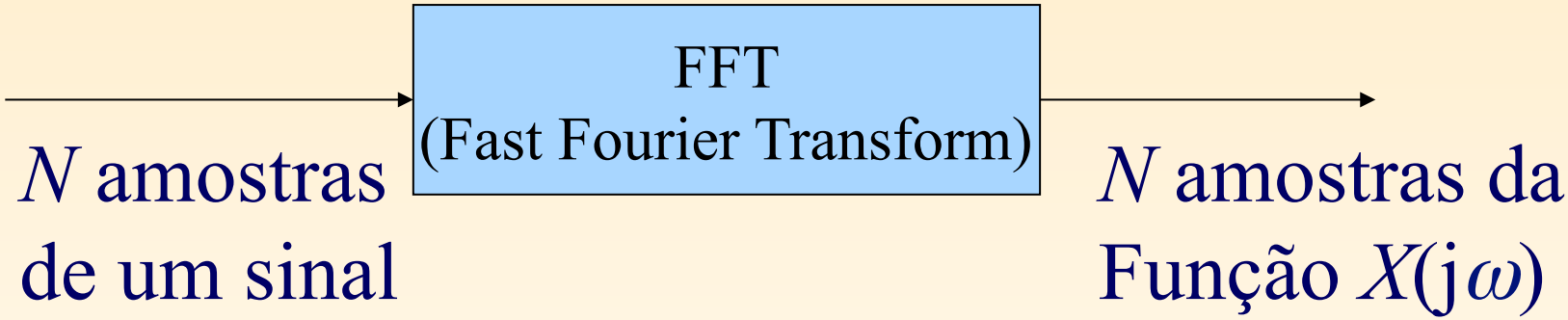
Sinal com N=200 amostras e F_a=100 Hz



$$k = 20 \Rightarrow f = 20\,\frac{100}{200} = 10\,Hz$$

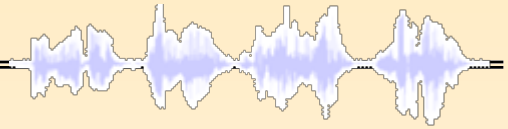
Algoritmos: Top 10





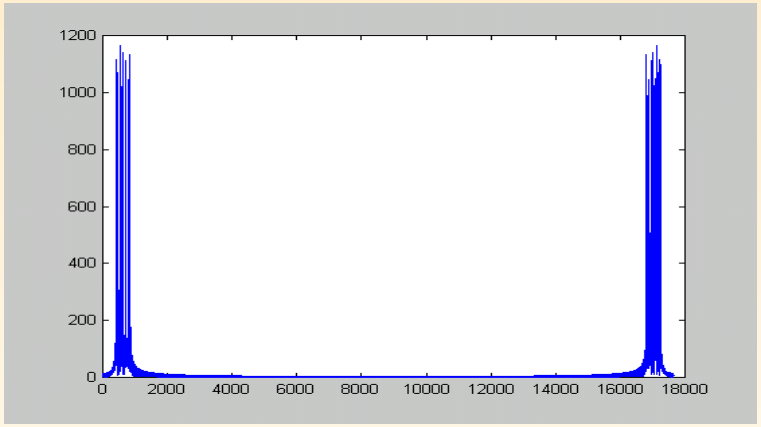
Algoritmo de 1965 em oitavo lugar na lista de
http://www.cecm.sfu.ca/~jborwein/algorithms.html
e os autores de alguns
http://x86.cs.duke.edu/courses/fall06/cps258/references/topten.pdf

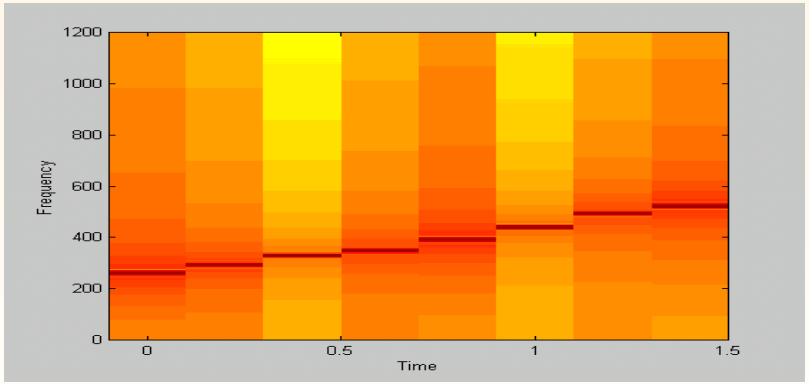
Espectrograma – análise em sub-segmentos



Uma escala musical sintética

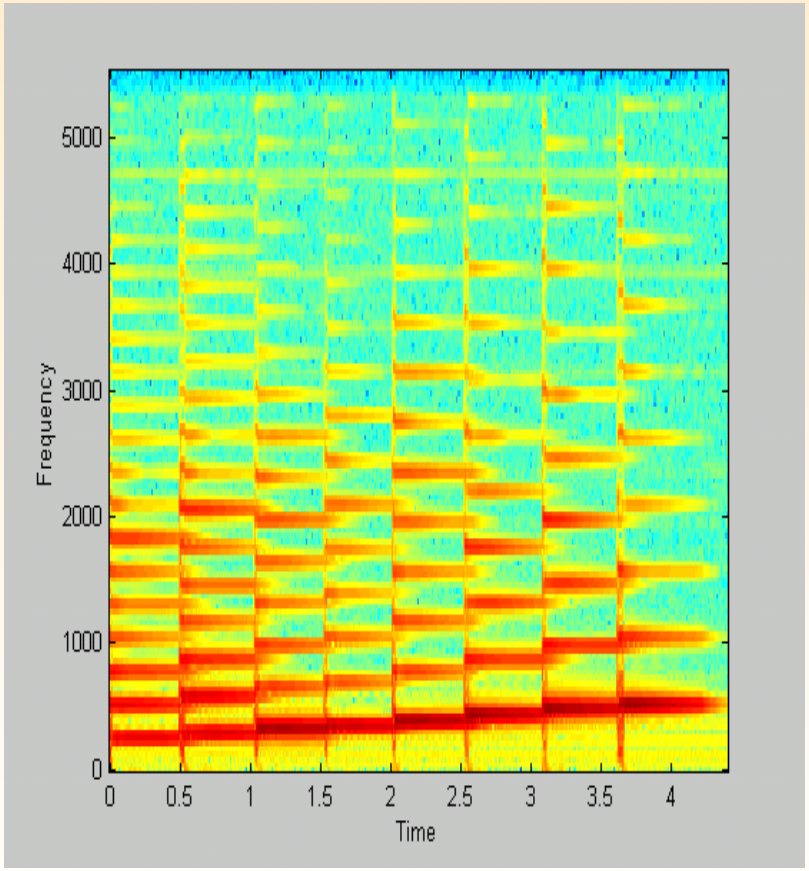
```
f=[262 294 330 349 392 440
   494 524]
fa=11025;
t=0:1/fa:0.2;
xa=[];
for i=1:length(f)
   x=cos(2*pi*f(i)*t);
    xa=[xa x];
end
plot(...)
spectrogram(xa,...)
```



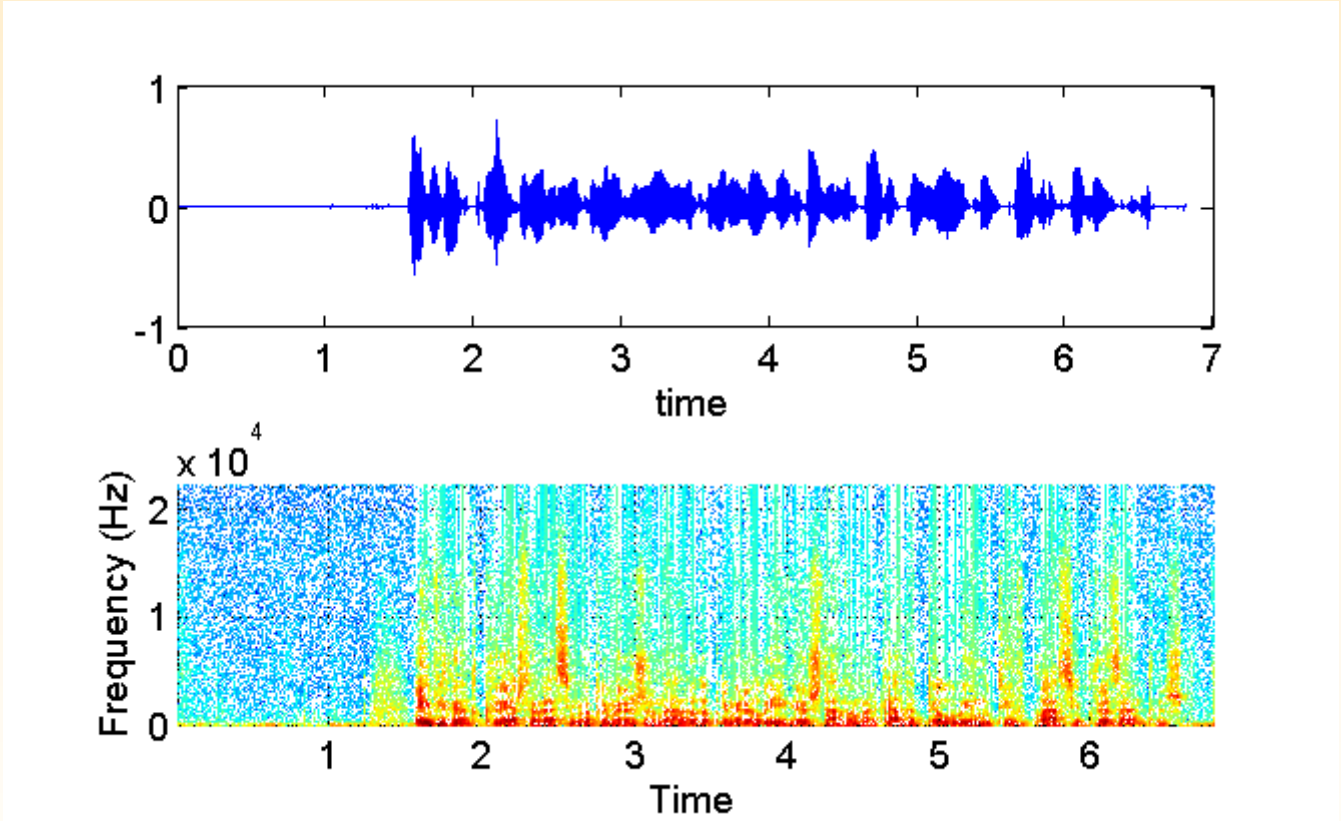


Espectrograma de um Instrumento Musical

No caso de um insrumento musical verifica-se que cada nota é composta por um conjunto de harmónicos com uma frequência múltipla da fundamental.

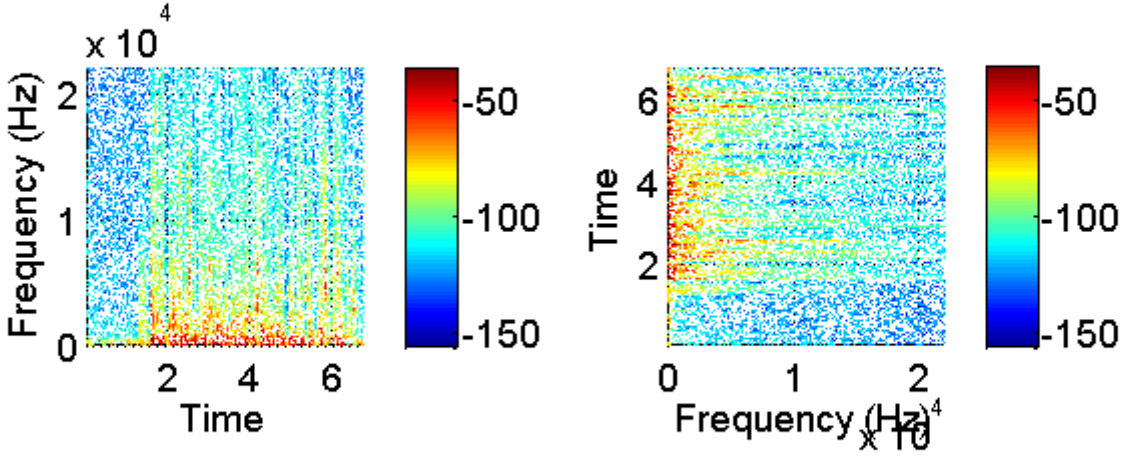


Exemplo de um Sinal de Voz



Sinal de voz: N=300001 e Fa=44100 Hz

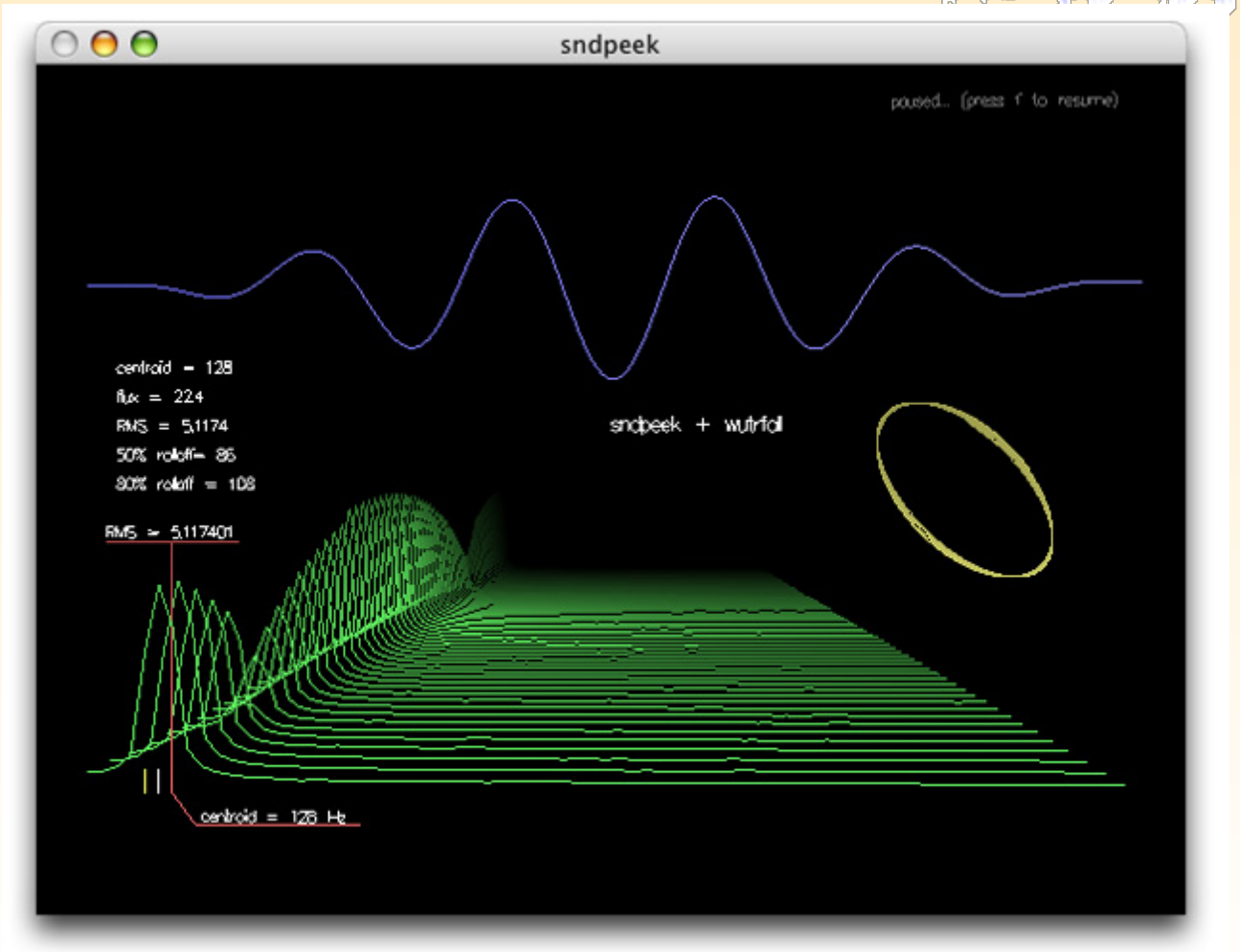
Espectrograma: duas representações



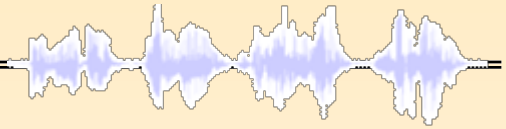
spectrogram(x,256,128,256,44100,'yaxis')_ ou
spectrogram(x,256,128,256,44100)

Demo – sndpeek

http://soundlab.cs.princeton.edu/software/sndpeek/



Bibliografia



- James H. McClellan, "Signal Processing First", Prentice Hall, 2003. (Capítulo 3)
- Signal Processing First Website: http://www.ieeta.pt/dspfirst/contents/index.htm