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*On the various definitions of cyclic operads*

We view cyclic operads as structures combining operations that have only (named) entries and no distinguished output. Starting from a contravariant (and non-skeletal) version $S : \mathbf{Bij}^{op} \to \mathbf{Set}$ of Joyal's species of structures, partial compositions and identities are defined, as done, say, by Markl in the appendix of [1]. This leads to a natural combinator syntax . But we found it convenient to introduce as well a $\lambda$-calculus-style syntax, called $\mu$-syntax, that allows a crisp and economical formulation of the laws to be satisfied. Instead of dealing only with operators $f \in S(X)$, the $\mu$-syntax involves two kinds of expressions:

$$c ::= \langle s|t\rangle \mid f\{t_x|x \in X\} \quad \text{and} \quad s,t ::= x \mid \mu x.c\,,$$

called *commands* (which mimick operators themselves, with no entry selected), and *terms* (representing operators with one selected entry), respectively, these being subject to the following set of equations:

$$\langle s|t\rangle = \langle t|s\rangle, \quad \langle \mu x.c|s\rangle = c[s/x] \quad \text{and} \quad \mu x.\langle x|y\rangle = y\,.$$

We prove that the set of commands of our syntax, quotiented by the given equations, is in one-to-one correspondence with the set of unrooted trees with nodes decorated by operations and half-edges labeled by names, thereby proving the equivalence between the partial (or biased) presentation and the (unbiased) definition of (cyclic) operads as algebras over a monad. Our proof makes use of rewriting. The equations of the $\mu$-syntax give rise to a (non-confluent) critical pair

$$c_1[\mu x.c_2/y] \leftarrow \langle \mu y.c_1|\mu x.c_2\rangle \rightarrow c_2[\mu y.c_1/x]\,.$$

The distinct normal forms of a command correspond in a natural way to enumerations of the nodes of the corresponding tree.

In addition, we also discuss two monoidal-like definitions, guided by the "microcosm principle" of Baez (like Fiore did for ordinary symmetric operads and dioperads): according to the first one, a cyclic operad is a pair $(S,\ \nu : S\triangle S \to S)$ where $S\triangle T = (\partial S) \otimes (\partial T)$, and where $\nu$ commutes (in an appropriate sense) with the "associativity-like" isomorphism

$$(S\triangle T)\triangle U + T\triangle(S\triangle U) + (T\triangle U)\triangle S \cong S\triangle(T\triangle U) + (S\triangle U)\triangle T + U\triangle(S\triangle T)\,.$$

The second one will be presented in the talk.

References:

[1] M. Markl, Modular envelopes, OSFT and nonsymmetric (non-$\Sigma$) modular operads, arXiv:1410.3414.

[2] M. Fiore, Lie Structure and Composition, CT2014 slides, http://www.cl.cam.ac.uk/∼mpf23/talks/CT2014.pdf.

*Joint work with Pierre-Louis Curien.

[3] E. Getzler, M. Kapranov, Cyclic operads and cyclic homology (Geom., Top., and Phys. for Raoul Bott), International Press, Cambridge, MA, 1995, 167-201.