CEOC

Centro de Estudos em Optimização e Controlo
(Centre for Research in Optimization and Control)

Research plans 2003-2005

Universidade de Aveiro
Departamento de Matemática

Conteúdo

Structure of CEOC

Coordination

- **Coordinator**: Domingos M. Cardoso
- **Sub-coordinator**: Vitor Neves
- **Secretary**: Eugénio Rocha
- **Project's coordinators**:
  1. **OGT&C**: Domingos M. Cardoso
  2. **MR&MT**: Alexander Plakhov
  3. **C&A**: Rosália Rodrigues
  4. **cotg**: Delfim Torres

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  16. Vitor Neves, PhD
- **PhD students**:
  1. António Pereira, MSc
  2. Deolinda Rasteiro, MSc
  3. João Pedro Cruz, MSc
  4. Paula Rama, MSc
  5. Paulo Gouveia, MSc
  6. Natália Martins, MSc
  7. Ricardo Almeida, MSc

Scientific cooperation

- Ana Maria Carvalho de Almeida, Universidade de Coimbra
- Ana Paula Tomás, Universidade do Porto
- António Guedes de Oliveira, Universidade do Porto
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- Charles Delorme, University of Paris-Sud, Orsay, France
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- Valida Sesadze, Georgian Technical University, Georgia
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Research Projects

1. Optimization, graph theory and combinatorics

1.1 Researcher team

- **Coordinator**: Domingos Moreira Cardoso
- **Researchers**:
  1. Agostinho Agra (PhD, 30%)
  2. Alexander Plakhov (PhD, 20%)
  3. António Batel Anjo (PhD, 30%)
  4. Carlos J. Luz (PhD, 30%)
  5. Cristina Requejo (PhD, 30%)
  6. Domingos Moreira Cardoso (PhD, 40%)
  7. Eugénio Rocha (PhD, 10%)
  8. João Pedro Cruz (MSc, PhD student, 100%)
  9. Natália Martins (PhD student, 50%)
  10. Paula Carvalho (PhD, 40%)
  11. Paula Oliveira (PhD, 30%)
  12. Paula Rama (MSc, PhD student, 35%)
  13. Ricardo Almeida (PhD student, 50%)
  14. Rosa Amélia Martins (PhD, 30%)
  15. Tatiana Tchemisova (PhD, 15%)
  16. Vitor Neves (PhD, 30%)
- **Scientific cooperation**:
  1. António Guedes de Oliveira, Universidade do Porto
  2. Charles Delorme, University of Paris-Sud, Orsay, France
  3. Imme van den Berg, Universidade de Évora

4. Jerzy Szymanski, Adam Mickiewicz University, Poznan, Poland
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8. Luís Gouveia, Universidade de Lisboa
9. Maria João Borges, Instituto Superior Técnico
10. Miguel Constantino, Universidade de Lisboa
11. Mohammad Rostami, Universidade da Beira Interior
12. Rommel Barbosa, Universidade Federal de Goiás, Goiânia, Brazil

1.2 Project's summary

Within this project we deal with analytic, algebraic and combinatorial approaches to several problems in graph theory and combinatorics. Special attention is given to the following topics: spectra of graphs, mainly eigenvectors and eigenvalues of graphs with regularity constraints and their relationship to some subsets of vertices or edges with particular properties; combinatorial properties of the least eigenvalue with applications to the recognition of graphs for which the stability number may be determined by solving a convex quadratic programming problem; improvements on lower and upper bounds on the stability number of graphs, obtained by convex quadratic programming, and comparisons with the Lovász theta number; extensions and applications of the above results to combinatorial optimization. We also intend to study Jordan algebras and their applications to conic optimization (namely, self-concordant barriers and their parameters) and the abnormal extrema phenomenon in nonlinear programming. We expect to obtain results on combinatorics and related topics, namely in multiattribute ranking problems and also on strongly regular graphs, particularly about the existence of the fourth graph of Moore. Stochastic optimization and stochastic approximation algorithms will be developed as well as applications of nonstandard analysis to optimization and control. Furthermore, we intend to provide advanced training in optimization and discrete mathematics and to develop intership programs and research collaboration with industry.

1.3 Project's description objectives

The primary goals of this project include carrying out research and providing advanced training in optimization, graph theory and combinatorics. Aiming to capture the interest of Portuguese students and junior researchers to discrete mathematics, a Portuguese book on this topic will be published (as far as we know this will be the first book in this language). Additionally, in co-operation with other groups and universities, a MSc course on discrete mathematics will be prepared. The main research objectives cover *graph spectra*, *continuous optimization and graphs*, *combinatorics and related topics*, *stochastic optimization and*

stochastic approximation and *nonstandard optimization techniques*. We search for practical implementation of results of our work and for further development of internship programs in cooperation with industry.

1. **Objectives on graph spectra**. We expect to obtain results about adjacency and Laplacian eigenvectors and eigenvalues of graphs with regularity constraints and about their relations with the existence of subsets of vertices and subsets of edges with regularity properties.

2. **Objectives on continuous optimization and graphs**. Recognition of graphs with convex-QP stability number (that is, for which the stability number may be determined by convex quadratic programming), improvements on the determination of lower and upper bounds on the stability number of graphs obtained by convex quadratic programming and extensions to combinatorial optimization problems. Furthermore we intend to apply Jordan algebras on self-concordant barriers over symmetric cones and to study necessary and sufficient conditions for abnormal extrema without constraint qualification.

3. **Objectives on combinatorics and related topics**. Results about the existence of the fourth graph of Moore. Conditions under which a ranking solution is a linear extension of a quasi-order extension of a preference weighted sum relation defined by a multiattribute ranking problem. Concordance graphs and concordance pairs of linear orders. Application of specialization orders and digital topologies in shape reorganization in digitalized image.

4. **Objectives on stochastic optimization and stochastic approximation**. Algorithms for the stochastic optimal path problem. Algorithms of accelerated convergence, aiming to achieve better performance by speeding up the transient stage in stochastic approximation.

5. **Objectives on nonstandard optimization techniques**. To apply hyperfinite discretization and non-standard hull methods in critical point theory as well as to control in differentiable manifolds.

1.4 Relationship with the state of the art

1. **Graph spectra**. Spectra of graphs is an intensive research area with a few results on graphs with regularity constraints [4, 12]. A graph with a (k, τ)-regular set (a set of vertices S inducing a k-regular graph such that each vertex out of S has τ neighbors in S) is an example of a graph with regularity constraints. Several practical problems can be model by (k, τ)-regular sets. For instance, a graph G is Hamiltonian if and only if its line graph has a $(2, 4)$-regular set inducing a connected graph. Another example, a graph has a perfect matching if and only if its line graph has a $(0, 2)$-regular set.

2. **Continuous optimization and graphs**. The recognition of graphs with convex quadratic stability number [6] still remains an open problem. Furthermore only a few computational experiments about lower and upper bounds on the stability number of graphs determined by convex quadratic programming were done [19, 20]. On the other hand, the relationship of convex quadratic bounds for the stability number with the Lovasz theta number [18] still needs clarification. Several approaches of conic optimization using Jordan algebras [11] can be simplified and some extensions to combinatorial problems can be obtained . Concerning the abnormal extrema [1] there are no necessary and sufficient conditions for the occurrence of this kind of phenomena.

3. **Combinatorics and related topics**. There is a very short family of strongly regular graphs with parameters $(n, k; 0, 1)$, with 3 known graphs. It is not known whether there exists one additional strongly regular graph with parameters $(3250, 57; 0, 1)$ designated fourth graph of Moore [9]. There are several results related with its possible existence. On the other hand, validation techniques in multiattribute ranking problems using weak order extensions of posets are not common on the literature. Furthermore, there are several open problems related with concordance graphs [8] which are the comparability graphs of posets of dimension two with particular combinatorial properties. Specialization orders and digital topologies is a hot topic with several applications in image processing and image recognition [17].

4. **Stochastic optimization and approximation**. The stochastic optimal path problem is being studied since 1969. However, only a few theoretical results are known, even though there is a recognizable applicability to telecommunication networks. The proposed mathematical model maximizes the expected value of an utility function over a directed network, where the distances related with the arcs are real random variables following gaussian distributions.
   A stochastic approximation algorithm with step size adaptation was proposed and its convergence was studied [2]. Now, an algorithm with geometric step size adaptation is proposed and its convergence is proved.

5. **Nonstandard optimization techniques**. Applications of nonstandard analysis to critical point theory have not yet been made; in 1993 and 1994 Tuckey ([35]) and Rubio ([24]) have used hyperfinite techniques in variational calculus to treat Young's generalized curves and turbulence problems and in order to obtain mild generalizations of variational principles. Attempts at a nonstandard analysis of manifolds have been made since the late 1970s until 2000, with mixed success, by Stroyan (study of curvature in [29]), Neves (topology on spaces of smooth functions in [22]), Schlesinger (dynamical systems and twistor theory in [28]) and Hertrich-Jeromin (griding of surfaces in [14]).

1.5 Tasks

1.5.1 Graph spectra

- **Task duration - months**: 36
- **persons*month**: 28,8
  - **Research team**: Domingos M. Cardoso (15%), Paula Rama (35%) and Rosa Amélia Martins (30%).
- **Expected results** (max. 100 words): Characterization of (k, τ)-regular sets using adjacency and Laplacian eigenvectors and eigenvalues. Spectral implications of the existence of (k, τ) regular sets in graphs. Spectral results on graphs with equitable and/or almost equitable partitions. Characterization of graphs whose minimum eigenvalue does not decrease when an edge or a node is deleted.
- **Task description** (max. 200 words): The task is based on the study of adjacency and Laplacian eigenvectors and eigenvalues of graphs and on their relations with particular combinatorial structeres. The study also covers graph products and its eigenvalue and eigenvector implications. Particular attention will be given to the minimum eigenvalue of several families of graphs which we call adverse (these are graphs where the minimum eigenvalue does not change when any vertex or the neighborhood of any vertex is deleted).

1.5.2 Continuous optimization and graphs

- **Task duration - months**: 36
- **persons*month**: 41,4
  - **Research team**: Agostinho Agra (30%), Carlos J. Luz (30%), Cristina Requejo (30%), Domingos M. Cardoso (10%) and Tatiana Tchemisova (15%).
- **Expected results** (max. 100 words): Characterization of graphs with convex quadratic stability number. Polynomial time recognition of this type of graphs in particular families. Applications of the results about graphs with convex-QP stability number to the determination of maximum matchings by convex quadratic programming. Improvements on lower and upper bounds on the stability number of graphs determined by convex quadratic programming. Relations between this upper bounds and the Lovasz theta number. Relations between the optimal parameter of a self-concordant barrier over a symmetric cone, the Carathéodory number of the cone and the rank of an underlying Euclidean Jordan algebra and their extensions to combinatorial problems. Optimality conditions for abnormal extrema.

- **Task description** (max. 200 words): Along this task, graphs for which we have not been able to recognize whether they are or not graphs with convex-QP stability number are studied. Particular families, with particular subsets of vertices will be identified. Namely the recognition of these graphs is related to the existence of (k, τ)-regular sets, with $k = 0$ and τ equal to the symmetric value of the minimum adjacency eigenvalue. This research also aims at obtaining effective computable upper and lower bounds for the stability number of graphs leading to the determination of these upper and lower bounds by convex quadratic programming. We study optimal parameters of self-concordant barriers in conic optimization, using Euclidean Jordan algebras, trying to extend the obtained results to combinatorial optimization and graph theory. Abnormality phenomena will be studied from the point of view of geometric control theory where the computation of Morse index and nullity of abnormal extremal plays a crucial role in the verification of the optimality conditions.

1.5.3 Combinatorics and related topics

- **Task duration - months**: 36
- **persons*month**: 27
  - **Research team**: Alexander Plakhov (10%), Domingos M. Cardoso (15%), Eugénio Rocha (10%) e Paula Carvalho (40%).
- **Expected results** (max. 100 words): Construction techniques of strongly regular graphs and results about the existence of the fourth graph of Moore, more specifically: assuming that the fourth graph of Moore exists, to obtain results about the existence of some subsets of vertices with particular regularity properties. Linear and weak-order extensions of quasi-orders defined by weighted sums of binary preference relations in the framework of multiattribute ranking problems and validation techniques for multiattribute ranking solutions. Estimation of the portion of concordance pairs among all possible pairs of linear orders defined over a set X and estimation of the portion of concordance graphs among all graphs with vertex set X. Developments in digital topology with applications of specialization orders on image processing and image recognition.
- **Task description** (max. 200 words): There are several models for the construction of strongly regular graphs, namely by means of integer programming or the definition of a family of partitions of a particular set. The properties of this family will be studied within the framework of poset theory. Special attention will be given to the Moore graphs short family of diameter 2, namely to the possible existence of the fourth graph of Moore. Note that this family has only 3 known graphs (which are strongly regular with valency 2, 3 and 7) and possibly only one more (the fourth graph of Moore, with valency 57). A weak order extension of a quasi-order

(that is a reflexive and transitive binary relation) defined by the transitive closure of weighted sum binary preference relations on multiattribute ranking problems can be used for validation of ranking results. Digital topological spaces which are discrete spaces of points, usually defining computer images, combined with specialization orders, become a crucial tool for image processing and image recognition techniques.

1.5.4 Stochastic optimization and stochastic approximation

- **Task duration - months**: 36
- **persons*month**: 61,2
  - **Research team**: Alexander Plakhov (10%), António Batel Anjo (30%), João Pedro Cruz (100%), Paula Oliveira (30%).
- **Expected results** (max. 100 words): Optimization models and algorithms for network problems with random costs on arcs, nodes and both. A stochastic approximation algorithm with geometric step size adaptation is based on the heuristic techniques of step size adaptation known from the literature. It allows one to speed up convergence at the expense of precision of determining the solution.
- **Task description** (max. 200 words): The research on stochastic optimization is based on the study of models for maximizing the expected value of an utility function over a directed random network, where the cost related to the arcs are real random variables following a given distribution. The results on stochastic approximation will be obtained using methods of probability theory, in particular of martingale theory and of stochastic approximation theory.

1.5.5 Nonstandard optimization techniques.

- **Task duration - months**: 36
- **persons*month**: 23,4
  - **Research team**: Vítor Neves (30%), Natália Martins (40%), Ricardo Almeida (50%).
- **Expected results** (max. 100 words): Nonstandard characterizations of Palais-Smale type sequences or nets and their properties. Establishment of properties of internal functions with different kinds of smoothness. Existence and location of critical points of internal functionals. Existence and classification of internal solutions to ODEs with discontinuous linearities. Characterization of submanifolds of finite dimensional Euclidean spaces by means of their approximation by grids of infinitesimal width wherein discretely differentiable functions act. Embeddings of infinite dimensional manifolds in hyperfinite dimensional Euclidean spaces by means of

nonstandard hulls of internal finite maps. Peano and Charathéodory type existence theorems for internal ODEs in locally convex spaces.

- **Task description** (max. 200 words): Construction of non-standard analogs to Palais-Smale condition and applications of Mountain Pass like theorems to Differential Equations. Development of a notion of Differentiable Manifold based on the concept of infinitesimal transformation, that encompasses infinite dimensional manifolds modelled on locally convex spaces. Application to Dynamical Systems.

2. Problems of minimal resistance and problems of mass transfer

2.1 Researcher team

- **Coordinator**: Alexander Plakhov
- **Researchers**:
  1. Alexander Plakhov (PhD, 20%)
  2. Delfim Torres (PhD, 10%)
  3. Leslie Bajuelos (PhD, 5%)
  4. Tatiana Tchemisova (PhD, 10%)
- **Scientific cooperation**:
  1. Anatoly Stepin, Moscow State University
  2. Vladimir Levin, Central Institute of Economics and Mathematics, Moscow
  3. Thomas Lachand-Robert, Savoie University, France
  4. Diogo Gomes, Instituto Superior Técnico
  5. Giuseppe Buttazzo, University of Pisa, Italy

2.2 Project's summary

To study the problem of minimal resistance for the bodies moving in a rarefied medium, as well as to study the exactly solvable problems of mass transfer and to investigate the relation between finite- and infinite-dimensional transport problems.

2.3 Project's description objectives

Study of the classical (Newtonian) problem of the body of least resistance in various classes of bodies: non-convex axisymmetric and non-axisymmetric bodies (two- and three-dimensional cases, with and without restrictions imposed on the maximal number of collisions of particles with the body). Minimization of specific resistance of infinite surfaces. Minimization of resistance to bodies moving in a medium of positive temperature, herein including bodies which perform both translational and rotational motions, and of rotating bodies with rough surface. Relationship between the problem of minimal resistance of non-convex rotating bodies and the one-dimensional problem of mass transfer. Exactly solvable problems of mass transfer; relationship between linear transport and mass transfer problems. The problem of finding the pressure generated by a given non-parallel flux of particles and the inverse problem of determining density of the flux generating a given pressure distribution.

2.4 Relationship with the state of the art

Since the early 1990th, the interest to Newton's problem of the body of minimal resistance revived. By means of calculus of variations, interesting and unexpected results have been obtained for the classes of non-axisymmetric and/or non-convex bodies (recall that Newton stated his problem for convex axisymmetric bodies). See papers of Buttazzo, Kawohl, Lachand-Robert, and others. Throughout this study, it was assumed that every particle of the medium collides with the body at most once. On the contrary, the most part of our research is devoted to the case of multiple collisions of particles with the body. This problem seems to be more realistic; it is found to be closely related to the billiard theory and the mass transfer problem. Newton assumed the collisions to be absolutely elastic and the particles to be immovable. Horstmann et al (1999) considered the case of non-elastic collisions, which is physically more relevant. We expect to advance in this direction, considering resistance in a medium with temperature motion of particles. The theory of mass transfer is rapidly growing last years; yet, the number of exactly solvable problems, even in one dimension, is relatively small. Also, the relation between finite-dimensional transport problems and the mass transfer problems is not completely understood. Our aim is to discover new exactly solvable problems, as well as to represent the mass transfer problem as the limiting case of finite-dimensional transport problem. The problems of resistance of rotating and/or rough bodies were not studied. Yet, they may be of interest when constructing artificial satellites.

2.5 Tasks

2.5.1 Minimal resistance and problems of mass transfer

- **Task duration - months**: 36
- **persons*month**: 16,2
  - **Research team**: Alexander Plakhov (20%), Delfim Torres (10%) and Leslie Bajuelos (5%) Tatiana Tchemisova (10%).
- **Expected results** (max. 100 words): Solution of Newton's problem in the classes of non-convex axisymmetric and non-axisymmetric bodies, in two and three dimensions. Zero-resistance solution. Solution of the problem in arbitrary dimension for bodies containing a half-space. The body of minimal resistance in a medium of positive temperature; limiting cases of zero and of infinite temperature. Estimation of resistance of convex and non-convex rotating bodies, in two and three dimensions, and of rotating bodies with rough surface. Solution of the direct and inverse problems of non-parallel flux of particles. Exact solutions of a one-dimensional mass transport problem with cost function having an interval of convexity and an interval of concavity. Find exact results of some geometrical two-dimensional mass transfer problems.

- **Task description** (max. 200 words): The results on Newton's problem of minimal resistance will be obtained using methods of billiard theory, of mass transfer, and of optimal control theory. It is planned to obtain exact solutions on mass transfer in two dimensions, using methods of finite-dimensional transport problem.

3. Computability and algorithms

3.1 Research team

- **Coordinator**: Maria Rosália Dinis Rodrigues
- **Research Team**:
  1. Maria Rosália Dinis Rodrigues (30%)
  2. Antonio Leslie Bajuelos Dominguez (25%)
  3. António Ferreira Pereira (100%)
- **Scientific cooperation**:
  1. Ana Paula Tomás, Universidade do Porto
  2. Manuel Abellanas Oar, Universidad Poltécnica de Madrid, Spain
  3. Ana Maria Carvalho de Almeida, Universidade de Coimbra

3.2 Project's Summary

This project is situated in the general area of Theoretical Computer Science. It comprises the work developed by two senior researchers and four PhD students (two to start in 2004/2005) from CEOC and other R&D units. The ongoing research concerns the topics of: Complexity of Problem Instances, Algorithms and Heuristics for Computational Geometry and Quantum Computation.

3.3 Project's Description Objectives

1. **Overall Objectives**. The underlying motivation for this project is the establishment of a solid basis for the creation of a research group in the Computational Mathematics area. There are several reasons for this resolution: the University of Aveiro has stated the need for the development of Informatics and Computer Science research groups, the Mathematics Department has expressed the need for research in Computer Science and the FCT's Mathematics Evaluation Panel has regularly pointed out the need for research in Computational Mathematics within Portuguese R&D units.

   We believe that the existence of this project can play an important role in the motivation and recruitment of young researchers in this area.

2. **Specific Objectives**.

   - Objectives on Complexity Theory: the search for a theoretical framework for the classification of the Instances of NP-hard Problems, in terms of their computational cost.

- Objectives on Computational Geometry: to develop approximation algorithms and heuristics for the Minimum Vertex Guard Problem, to study de Vertex π-Floodlights Problem for Orthogonal Polygons and to characterize the visibility graphs of certain classes of polygons.
- In the field of Quantum Computation, our work focus on the design of quantum arithmetic circuitry, particularly on the use of generalized redundant number systems, as well as on the problem of quantum computing simulation.

3.4 Relationship with the state of the art

1. **Complexity Theory**. Although an enormous amount of work has been published on the classification of problems in terms of their Computational Complexity, very little is know about the comparative behaviour of particular instances of a given problem of the NP classes.

2. **Computational Geometry**. In 1973, Victor Klee posed the following questions: How many guards are necessary and how many are sufficient to patrol the paintings and works of art in a gallery with n walls? Since publication of the first original result (*Chvátal Art Gallery Problem*), tremendous amount of research on illumination problems has been carried out by mathematicians and computer scientists. In 1981 the Minimum Vertex Guard Problem, i.e. the problem of finding the minimum number of guards needed to cover any polygon, was proved to be NP-hard. In the Handbook of Computational Geometry (Elsevier, 2000), Urrutia asserts that one approach that has been neglected in the study of Art Gallery Problems is the one of finding algorithms that obtain approximate solutions. The most well-know result on this subject is an algorithm that finds in $\mathcal{O}(n^5 \log n)$ a vertex guard set that is at most $\mathcal{O}(\log n)$ times the minimum number of vertex guards needed.

3. **Quantum Computation**.

   - **Generalized redundant number systems in quantum computation.** Last two decades have brought many and significant advances in both theoretical and practical understanding of computation in qubit systems. However, the development of a theoretical framework for quantum computation in qudit systems started only recently. Universal quantum gates for hybrid quantum systems and quantum codes are hot topics in the area.
   - **Design of quantum arithmetic circuitry.** Considerable effort is being devoted to the building of a scalable quantum processor, using a wide range of competing technologies with several promising results. The design and complexity analysis of quantum circuits for arithmetic operations is an important and open research area.

- **Quantum computer simulation.** Quantum computer simulators are imperative at present, since quantum hardware prototypes hardly exist. A variety of quantum simulators is either available or under development. However, the execution time and memory requirements of these simulators increase exponentially with the number of qubits which constrains the size of problem instances to be solved.

3.5 Tasks

3.5.1. Complexity Theory

- **Task duration - months**: 36
- **persons*month**: 3,6
  - **Research team**: Maria Rosália Dinis Rodrigues (10%).
- **Expected results**: The completion of a model, presently being developed, based on the concepts of Algorithmic (or Kolmogorov) Complexity, Information Theory and Maximum Entropy Principle, for the computational evaluation of the complexity of particular problem instances.
- **Task description**:
  - To refine the existing model for the computational evaluation of the complexity of problem instances.
  - To derive efficient Entropy estimators.
  - To establish a basis for a complete Classification of instances of NP-hard problems.
  - To search for a relationship between known classes of heuristics and the classes of instances derived from the model.

3.5.2. Computational Geometry

- **Task duration - months**: 36
- **persons*month**: 9
  - **Research team**: Antonio Leslie Bajuelos Dominguez (25%).
- **Expected results**:
  - The development of Approximation Algorithms and Heuristics for the Minimum Vertex Guard Problem.
  - Identification of structural properties of certain classes of polygons which might prove relevant for the building of heuristics, as well as the characterization and recognition of the Visibility Graph.
  - Development of auxiliary tools and random generators for geometric structures.

  - Application of parallel processing techniques to the solving of illumination problems.
  - Implementation and assessment of the developed methods.

- **Task description**: We intended to study and develop algorithms and heuristics for the visibility and illumination problems. Most of these problems belong to the NP classes. Two aspects must be considered. The first one is the identification of structural properties of certain classes of polygons and terrains which might prove relevant for the building of heuristics for the resolution of the Minimum Vertex Guard Problem, that is the problem of finding the minimum number of guards needed to cover any polygon. On the other hand, we intend to study some visibility and illumination problems that still remain open, e.g. Visibility Graph Recognition and Vertex π-Floodlights Problem.

3.5.3. Quantum Computation

- **Task duration - months**:
  - **Research team**: Maria Rosália Dinis Rodrigues (20%), António Ferreira Pereira (100%).
- **Expected results**:
  - **Generalized redundant number systems in quantum computation.** To investigate the implications of redundant integer encodings as qudit states in the complexity of search problems.
  - **Design of quantum arithmetic circuitry.** To design efficient quantum arithmetic circuits.
  - **Quantum computer simulation.** To develop a full working package in *Mathematica* for the symbolic simulation of quantum computation. To evaluate the results of this new method, when compared with known quantum simulators.
- **Task description**:
  - **Generalized redundant number systems in quantum computation.** Existing methods assume a non-redundant encoding of integer digits as states in the canonical basis of the d-dimensional Hilbert space associated with each qudit. Redundant numbers systems have widely been used in the development of very efficient classical solutions of many problems, e.g. in digital arithmetic algorithms, but only a few and localized applications of redundancy in quantum algorithms are known.

    The use of redundancy changes the space structure of search problems. Our goal is to analyze these changes, from a computational complexity viewpoint.

- **Design of quantum arithmetic circuitry.** A sound universal set of redundant quantum gates still needs to be established, as well as the subsequent design of efficient quantum circuits for redundant arithmetic.
- **Quantum computer simulation.** The development of specific simulators for quantum computation based on symbolic and algebraic manipulation of states and operators is a new topic of research with several important practical applications. Our task is to implement symbolic algebraic software for sub-exponential simulations of quantum algorithms for *medium* size problem instances.

4. Control theory group

4.1 Researcher team

- **Coordinator**: Delfim F. M. Torres
- **Research Team**:
  1. Delfim F. M. Torres (PhD, 30%)
  2. Eugénio M. Rocha (PhD, 30%)
  3. Manuel Guerra (PhD, 30%)
  4. Paulo D. F. Gouveia (MSc, PhD student, 40%)
- **Scientific cooperation**:
  1. Boris Miller, Russian Academy of Sciences, Russia
  2. Emmanuel Trélat, Université Paris-Sud XI, Orsay, France
  3. Hans Munthe-Kaas, University of Bergen, Norway
  4. Matthias Kawski, Arizona State University, USA
  5. Ugo Boscain, SISSA, Italy
  6. Valida Sesadze, Georgian Technical University, Georgia
  7. Yacine Chitour, Université Paris-Sud XI, Orsay, France
- **Post-Docs**:
  1. Olena Mul, Ternopil Academy of National Economy, Ukraine

4.2 Project's Summary

The purpose of mathematical control theory is to analyze the properties of controlled systems, i.e., dynamic systems in which one can act using a command or control. Controls can be introduced in most types of systems, like systems of differential or difference equations. Control systems appear naturally in many areas, such as mechanics, biology and economics. The goal can be, for example, to steer the system from an initial state to a prescribed target, according to certain constraints on the state and/or the control, or to design trajectories that minimize some given cost (optimal control). Since the 1950's, with the discovery of Pontryagin's Maximum Principle, control theory had a great development and became a recognized field of mathematical research. Optimal control can be viewed as a generalization of the calculus of variations. The aim of "cotg"is to make advances in the following areas:

1. Nonlinear Control Theory and Optimal Control.

2. Calculus of Variations.

3. Motivated Applications and Computational Mathematics.

4.3 Project's Description Objectives

1. To obtain new versions of Pontryagin's maximum principle covering classes of generalized controls; to give geometric characterizations of generalized extremals and accessible sets; to investigate singular optimal control problems; to obtain explicit formulas for the Chen logarithm and develop a formal control theory with applications to stabilizability and noncommutative symmetric functions; to obtain conditions for boundedness of optimal controls; to investigate the role of symmetry and conservation laws.

2. Extension of Noether's theorem of the calculus of variations to the wider class of Lipschitz functions, and obtain a corresponding formulation for higher-order variational problems; to obtain conserved quantities along all minimizers of the problems; to identify new and more general classes of well and bad behaved problems in the calculus of variations.

3. To study systems of rolling bodies by designing time-variant feedback stabilizers with possible application to robotic manipulators; to develop a computer algebra package for determining symmetries and conservation laws in the calculus of variations and optimal control; to prove convergence of digital arithmetic algorithms by using results on feedback stabilization; develop numerical and asymptotical methods to study nonlinear dynamical control systems with distributed and discrete parameters, decreasing the effect of vibrations.

4.4 Relationship with the state of the art

1. Singular linear-quadratic problems with finite lower bound always admit a generalized optimal control of an appropriate class. Generalized solutions for these problems admit a simple geometric characterization using "generalized Hamiltonian flows"(Guerra'00'01). Different methods exist to construct classes of generalized controls for other types of problems (Miller&Rubinovich'01,Sarychev'91). In general, existence of generalized Hamiltonian flows seems to be related to existence of local feedback linearizations, described in (Jakubczyk'98'01).

Formulas for the Chen logarithm of a nonlinear control system are quite useful as its alternative, coordinates of second kind (Sussmann'86), in resolving problems of controllability (Sussmann'87, Murray et al'94), stabilizability (Rocha&Sarychev'02), motion planning (Hermes'89), or averaging of mechanical systems (Bullo'02). Several results exists in control theory (Kawski'00) or related with the continuous Baker-Campbell-Haudorff formula in quantum mechanics (Vinokurov'69), but none presented explicit formulas in a Hall basis, turning them useless for general applications.

The fundamental ideas of regularity in optimal control, for linear time-invariant control systems, are due to Clarke&Vinter'90. Results for the control-affine case were obtained by Sarychev&Torres'00. We intend to generalize such results to nonlinear problems.

In the optimal control setting, the relation between invariance of a problem and the existence of expressions which are constant along any of its extremals, has been obtained in the publications by van-der-Schaft'82 and Sussmann'95. Recent extensions were obtained by Torres'01'02. We will attempt now to enlarge the range of application of the previous results by extending the very concept of invariance.

2. It is well-known that problems in the calculus of variations may present absolutely continuous solutions which are not Lipschitz, and which fail to satisfy the Euler-Lagrange equations (Ball&Mizel'85, Sarychev&Torres'00). Different but related pathologies are also possible (Sarychev'97). We want to investigate a new perspective on the results concerning the Lipschitz regularity of the minimizing trajectories in the calculus of variations, by using appropriate conservation laws (Torres'02). For that it is important to obtain a proper extension of the classical Noether's symmetry theorem (Heinricher&Mizel'88).

3. The problem of controlling rolling bodies inspired much attention in the literature due to its importance in robotic applications. An intrinsic geometric method for two solid bodies rolling over each other, without slipping and twisting, was proposed by Agrachev&Sachkov'99.

Many powerful and versatile Computer Algebra Systems are available nowadays, putting at our disposal sophisticated environments of mathematical computing. Symbolic methods for control is now a very active research area. In particular, we are interested in developing a package to find symmetries and conservation laws in in the calculus of variations and optimal control.

Digital Arithmetic has been an active field of research over the past two decades. A large number of algorithms have been published for hardware implementation. However, a solid mathematical framework is still missing.

Analysis and control of oscillations in dynamical systems with distributed and discrete parameters were investigated by Kravchenko&Mul&Shut'99. Such systems are widespread in heavy, extractive and manufacturing industry, and also in space-system engineering.

4.5 Tasks

4.5.1 Nonlinear Control Theory and Optimal Control

- **Task duration - months**: 36
- **persons*month**: 19,8
  - **Research team**: Delfim F. M. Torres (10%), Eugénio M. Rocha (15%), Manuel Guerra (30%).

- **Expected results**: New versions of Pontryagin's maximum principle or other "extremality conditions", covering various classes of generalized controls; geometric description of generalized extremals and accessible sets; explicit formulas for the Chen logarithm, and a formal control theory; new regularity conditions for boundedness of optimal controls; Lipschitzian regularity of optimal trajectories; extensions of Noether's first and second theorems, in the direction which enlarges the scope of its application, and corresponding discrete-time analogous; useful necessary and sufficient conditions of invariance.

- **Task description**: We plan to generalize previous results (Guerra'00'01) to affine and non-autonomous systems. We consider commutative and non-commutative cases. We expect to construct generalized solutions having good properties and obtain extremality conditions using the appropriate generalized Hamiltonian flow. Short segments of generalized extremals have a special structure in linearized local coordinates. We expect to use this relationship to provide global geometric descriptions of generalized extremals, even when global linearization does not exist. We plan to obtain expressions for the logarithm of the Chen-Fliess series of nonlinear control systems, as a development of high-order averaging. The idea is to propose a calculus to deal with formal non-autonomous ordinary differential equations evolving on an algebra of formal series. We will address the question: under what conditions one can assure optimal controls to be bounded? This question is related to the one of Lipschitzian regularity of optimal trajectories, and the answer to it is crucial for closing the gap between the conditions arising in the existence theory and necessary optimality conditions. We will look for conditions beyond standard hypotheses of the existence theory. We will explore the relation between Noether's theorem, symmetry, Pontryagin's maximum principle, invariance under a family of transformations, and the existence of preserved quantities.

4.5.2 Calculus of Variations

- **Task duration - months**: 36

- **persons*month**: 10,8

  - **Research team**: Delfim F. M. Torres (10%), Paulo D. F. Gouveia (20%).

- **Expected results**: A proper extension of Noether's theorem of the calculus of variations to the wider class of Lipschitz functions; new invariants for problems of the calculus of variations with higher-order derivatives; conserved quantities along all the minimizers of the fundamental problem of the calculus of variations, even for those absolutely continuous minimizers which do not satisfy the classical Euler-Lagrange equations; new Lipschitzian regularity conditions for the problems of the calculus of variations; more general classes of ill behaved problems.

- **Task description**: A difficulty which appears in the calculus of variations (and optimal control) is to prove that the minimizers predicted by the existence theory satisfy the corresponding Euler-Lagrange equations. This is due to the fact that existence of minimizers is given in the class of absolutely continuous functions, while Euler-Lagrange equations require more regularity from the minimizers in order to be obtained. We propose a new perspective on the results concerning the regularity of the minimizing trajectories. Our claim is that such results are fruit of the existence of certain symmetry properties, and follow from appropriate conservation laws. For nonsmooth extremals, Noether's conservation laws cease to be valid. We expect to obtain new results showing that Noether's theorem is still valid in the wider class of Lipschitz functions, as long as some appropriate restriction of the Euler-Lagrange extremals is taken in account. We intend to obtain a proper extension, in contrast with the recent developments of Noether's symmetry theorems to the optimal control setting, which give rise to non-proper extensions when specified for the problems of the calculus of variations.

4.5.3 Motivated Applications & Computational Mathematics

- **Task duration - months**: 36
- **persons*month**: 16,2
  - **Research team**: Delfim F. M. Torres (10%), Eugénio M. Rocha (15%), Paulo D. F. Gouveia (20%).
- **Expected results**: Time-variant feedback stabilizers for systems of rolling bodies without twisting or slipping; results which turn the origin of rolling bodies control systems into locally asymptotically stable equilibrium points; a computer algebra package for determining symmetries and conservation laws automatically, from the data of the problems of the calculus of variations or optimal control; convergence results of digital arithmetic algorithms; numerical and asymptotical methods which decrease the effect of vibrations for nonlinear dynamical control systems with distributed and discrete parameters.
- **Task description**: We will study the stability and asymptotic stability of time-variant differential control equations, trying to design time-variant stabilizers for nonlinear control systems, in particular for nonholonomic systems of two bodies rolling one over the other without twisting or slipping. Our approach is to use Chen logarithm expressions, the monodromy map of a flow, and Lyapunov functions. Some computational tools will be developed. The equations given by necessary optimality conditions are, generally speaking, nonlinear, and very hard to solve. One way to address the problem is to obtain conservation laws to lower the order of the equations. The crucial point is to derive the invariance-variational-symmetries.

We will develop symbolic computational facilities, based on necessary and sufficient invariance-conditions, to identify symmetries and conservation laws. We intend to model redundant arithmetic algorithms as discrete-time time-variant control systems, with the strings of digits being treated as integer-valued controls, and obtain convergence of digital arithmetic algorithms by using results on feedback stabilization. We will study the effect of vibration on stability of mechanical systems. We will be particularly interested in nonlinear control systems with distributed and discrete parameters. Both numerical and asymptotical methods will be developed.

Scientific report 2003

1. Optimization, graph theory and combinatorics

1.1 Activities during 2003

During 2003, part of a Portuguese book on discrete mathematics organized in four parts: I - General concepts and general results; II - Combinatorics; III - Algebraic combinatorics; IV - Graph theory and algorithms, was written by Domingos M. Cardoso, Jerzy Szymanski (from Adam Mickiewicz University, Poznan, Poland) and Mohammad Rostami (from Universidade da Beira Interior). Also a MSc course was prepared in cooperation with Adam Mickiewicz University (namely, Michal Karonski and Jerzy Szymanski) and Universitat Politecnica de Catalunya (namely, Marc Noy and Oriol Serra). Several results about Laplacian and adjacency eigenvalues and eigenvectors and their relations with (k, τ)-regular sets were obtained, namely in the framework of strongly regular graphs (this research was done in cooperation with Charles Delorme from University of Paris-Sud, Orsay, and the results were submitted for publication). Agostinho Agra (supervised by Miguel Constantino from Universidade de Lisboa), Cristina Requejo (supervised by Luis Gouveia from Universidade de Lisboa) and Paula Carvalho (supervised by António Guedes de Oliveira from Universidade do Porto) finishing their PhD dissertations. Some results relating the convex quadratic upper bound for the stability number with the Lovasz theta number were obtained and submitted for publication. Some relations between the optimal parameter of a self-concordant barrier over a symmetric cone, the Carathédory number of the cone and the rank of every underlying Euclidean Jordan algebra were also obtained and submitted for publication (this research was done in cooperation with Luis A. Vieira from Universidade do Porto which are finishing his PhD dissertation under the supervision of Domingos M. Cardoso). A multi-attribute ranking solution procedure was developed and applied to confirm the results obtained by a multi-attribute ranking decision methodology on a tender for the supply of buses to the Porto Public Transport Operator (STCP) (this research was done in cooperation with Jorge F. Sousa from Universidade do Porto and submitted for publication). An algorithm of dynamic step size adaptation was constructed that allows to increase the step size when the algorithm is relatively far from the solution, in the 'deterministic' phase, and

to decrease it in the 'stochastic' phase, when the algorithm is near the solution. The algorithm allows to increase efficiency significantly on the 'transient' stage. Convergence almost surely of the algorithm is proved. We have demonstrated the second order sufficient optimality conditions for nonlinear programming problems with mixed constraints that in the case of abnormal extrema guarantee isolatedness of extremal point in the admissible set.
Essentially foundational studies were made with attempts at approximate minimization via embeddings in hyperfinite-dimensional normed spaces as well as from hyperfinite families of curves hereby including versions of min-max theorems for finite dimensional spaces, in collaboration with Maria João Borges of the IST, in Lisbon. Preparatory studies about generalized manifolds were made by the beginning PhD student Ricardo Almeida and were presented at the weekly task seminar.

1.2 List of publications

- **Books**

  1. A. Batel Anjo, Ricardo Fernandes e A. Simões Carvalho, "Curso de MatLab", Principia - Publicações Universitárias e Científicas, 2003.

- **Articles in International Journals (including book chapters)**

  1. Domingos M. Cardoso, "On graphs with stability number equal to the optimal value of a convex quadratic program", Matemática Contemporânea, 25 (2003): 9-25.
  2. Domingos M. Cardoso and Jorge F. Sousa, "Numerical tools for multiattribute ranking problems", Networks, 41, 4 (2003): 229-234.
  3. Jürg Hüster, Pedro Cruz, Andreia Hall, Carlos M. Fonseca, "On optimization and extreme value theory. Methodology and computing in applied probability, 5 (2003): 183-195.
  4. Rosa A. Martins and João F. Queiró, "2-widths of Hölder unit balls", Linear Algebra and its Applications, 361 (2003): 245-255.

- **Articles in National Journals (including book chapters)**

  1. Carlos J. Luz, "The graph bisection minimization problem", Investigação Operacional", 23 (2003): 85-101.

1.3 List of talks

- **Talks at International Conferences**

  1. Agostinho Agra and Miguel Constantino, "MIP cuts based on knapsaks with 2 integer variables", 18th International Symposium on Mathematical Programming, August 18-22, 2003, Copenhagen, Denmark.

2. Domingos M. Cardoso, "Graphs with convex-QP stability number", 18th International Symposium on Mathematical Programming, August 18-22, 2003, Copenhagen, Denmark.

3. Domingos M. Cardoso, Charles Delorme and Paula Rama, "On Laplacian eigenvectors and eigenvalues an almost equitable partitions", Combinatorics in Oporto, September 12-17, 2003, Porto, Portugal.

4. Domingos M. Cardoso e Maria H. Silva, "Abordagens analíticas de um problema combinatório de descodificação de imagens", Congresso Galego de Estatística e Investigación de Operacións, November 5-7, 2003, Vigo, Spain.

5. Luis Gouveia, Thomas Magnanti and Cristina Requejo, "An intersecting tree model for odd-diameter-constrained minimum spanning and steiner trees", Proceedings of the International network optimization conference - INOC 2003, October 27-29, Evry/Paris, France.

6. Luís Gouveia, Thomas Magnanti and Cristina Requejo, "MA 2-paths approach for odd-diameter-constrained minimum spanning and steiner trees", 18th International Symposium on Mathematical Programming, August 18-22, 2003, Copenhagen, Denmark.

7. Natália Martins and Vítor Neves, "Nonstandard Discrete Derivatives and Existence Theorems for ODES", Second European Junior Meeting on Control Theory and Stabilization, December 2003, University of Torino, Italy.

8. Rommel Barbosa and Domingos M. Cardoso, "On a subclass of well-covered graphs". Thirty-fourth Southestearn Conference on Combinatorics, Graph Theory, and Computing, March 3-7, 2003, Boca Raton, Florida, USA.

9. Vítor Neves, "Nonstandard Calculus of Variations", Variational Analysis and Applications, (June/July 2003), Erice, Sicily.

10. Vítor Neves, "Nonstandard Calculus of Variations", 12th St.Petersburg Summer Meeting in Mathematical Analysis, August 2003.

- **Talks at National Conferences**

1. Domingos M. Cardoso, "Uma abordagem algébrica de grafos fortemente regulares", Sessão de Homenagem ao Professor Mário da Silva Rosa, October 15, 2003, Coimbra, Portugal.

2. Carlos J. Luz e M. Odete Pereira "Análise da Interferência da Variáveis Demográfica e Organizacionais nas Atitudes face à Melhoria da Qualidade", I Jornadas de Gestão e Empreendedorismo, Janeiro 23, Universidade Internacional da Figueira da Foz.

1.4 List of reports (including proceedings)

- 1. Agostinho Agra and Miguel Constantino, "Description of 2-integer continuous knapsack polyhedra", Centro de Investigação Operacional, Working Paper n. 3 (2003): 12 p.
  2. Agostinho Agra and Miguel Constantino, "Lifting 2-integer knapsack inequalities", Optimization Online, Integer programming submissions, August (2003): 33 p.
  3. Alexander Plakhov and Pedro Cruz, "A stochastic approximation algorithm with step size adaptation", Cadernos de Matemática, Universidade de Aveiro, CM03/I-04, 2003.
  4. Carlos J. Luz, "Relating the Lovász theta number with some convex quadratic bounds on the stability number of a graph", Cadernos de Matemática, Universidade de Aveiro, CM03/I-22, 2003).
  5. Domingos M. Cardoso e Luís A. Vieira, "Conceitos e resultados sobre álgebras de Jordan", Cadernos de Matemática, Universidade de Aveiro, CM03/I-20 (2003): 44 p.
  6. Domingos M. Cardoso e Luís A. Vieira, "Representação quadrática de uma álgebra de Jordan", Cadernos de Matemática, Universidade de Aveiro, CM03/I-23 (2003): 31 p.
  7. Domingos M. Cardoso e Luís A. Vieira, "On the optimal parameter of a self concordant barrier over a symmetric cone", Cadernos de Matemática, Universidade de Aveiro, CM03/I-32 (2003): 13 p.
  8. Luís Gouveia, Thomas Magnanti and Cristina Requejo, "A 2-path approache for odd-diameter-constrained minimum spanned trees", Centro de Investigação Operacional, Working Paper n. 2 (2003): 25 p.
  9. Luis Gouveia, Thomas Magnanti and Cristina Requejo, "An intersecting tree model for odd-diameter-constrained minimum spanning and steiner trees", Proceedings of the International network optimization conference - INOC 2003, pag. 254-260.

1.5 List of organized seminars and conferences

- **Organized seminars**
  1. Natália Martins, "Uma demonstração não standard do teorema de existência de Caratheodory", Seminários do CEOC, November 28, 2003, Universidade de Aveiro.
  2. Paula Rama, "Valores e vectores próprios de laplacianos e partições quase equilibradas de grafos", Seminários do CEOC, October 10, 2003, Universidade de Aveiro.
  3. Jayme L. Szwarcfiter, On self-clique graphs", Seminários do CEOC, September 18, 2003, Universidade de Aveiro.

4. Miguel Constantino, Planeamento integrado da produção e sequenciamento de lotes na indústria de tintas: programação inteira mista, decomposição e programação por restrições", Seminários do CEOC, June 20, 2003, Universidade de Aveiro.
5. Tim Hultberg, "Formulation of linear optimization problems in C++", Seminários do CEOC, June 13, 2003, Universidade de Aveiro.
6. Jerzy Szymanski, "Propriedades de "dags"aleatórios", Seminários do CEOC, May 16, 2003, Universidade de Aveiro.
7. A. Leal Duarte, "Valores próprios múltiplos de matrizes cujo grafo é uma árvore", Seminários do CEOC, April 4, 2003, Universidade de Aveiro.
8. Filipa D. Carvalho, "Contribuições da optimização combinatória na protecção de dados estatísticos", Seminários do CEOC, March 28, 2003, Universidade de Aveiro.
9. Vitor Noeves, "O cálculo de variações estocástico não standard - II", Seminários do CEOC, March 21, 2003, Universidade de Aveiro.
10. Carlos J. Luz, "Relating the Lovász varthteta number with some convex quadratic bounds on the stability number of a graph", Seminários do CEOC, March 7, 2003, Universidade de Aveiro.

1.6 List of PhD and MSc dissertations

- **MSc dissertations**

1. Ana Helena Tavares, "Aspectos Matemáticos da Entropia", Universidade de Aveiro, 2003 (supervisor: António Batel Anjo).
2. António M. V. Rodrigues, "As não conformidades e a inovação", Universidade Aberta, 2003 (supervisor: Carlos J. Luz).
3. Maria H. Silva, "Abordagens analíticas de um problema combinatório de descodificação de imagens", Universidade de Aveiro, 2003 (supervisor: Domingos M. Cardoso).
4. Sandra Pinho, "Optimização Matemática e Entropia", 2003 (supervisor: António Batel Anjo).

2. Problems of minimal resistance and problems of mass transfer

2.1 Activities during 2003

Newton's problem of minimal resistance is formulated as follows. A body is moving through a rarefied medium. It is required to find shape of the body minimizing resistance of the medium to the motion of body. Newton solved this problem in the class of convex axisymmetric bodies. In the early 1990's the interest in the problem revived; interesting minimization results have been obtained in the classes of non-convex and/or non-axisymmetric bodies (see works of Buttazzo, Kawohl, Lachand-Robert and others). All these results were obtained under the assumption that every particle collides with the body at most once. In 2003, we considered the problem in the case where multiple collisions are allowed. It is proved that resistance of an arbitrary connected body can be made less than any positive number, by arbitrarily small deformation of its boundary. Furthermore, the problem of minimal resistance for non-convex non-axisymmetric bodies with fixed height and fixed horizontal circular cross section was considered. It is proved that if double and triple collisions are allowed, the infimum of resistance is zero. A two-dimensional version of the problem was considered.

2.2 List of publications

- **Articles in International Journals (including book chapters)**

  1. A.Yu. Plakhov, "Newton's problem of the body of minimal aerodynamic resistance", Doklady of the Russian Academy of Sciences 390, 3 (2003): 1-4.
  2. A. Yu. Plakhov, Newton's problem of the body of minimal resistance with a bounded number of collisions, Russ. Math. Surv. 58, 1 (2003): 191-192.

2.3 List of talks

- **Talks at International Conferences**

  1. A.Yu. Plakhov, "Newton's problem of minimal averaged resistance in the class of bodies of fixed volume", Int. Conf. "Kolmogorov and contemporary mathematics", June 16-21, 2003, Moscow, Russia.
  2. Delfim F. M. Torres and Alexander Plakhov, "Optimal control of Newton-type problems of minimal resistance", Second Junior European Meeting, Control Theory and Stabilization, December 2003, Torino, Italy,

2.4 List of reports (including proceedings)

- 1. A.Yu. Plakhov, "Newton's problem of the body of minimal aerodynamic resistance", Cadernos de Matemática, Universidade de Aveiro, CM03/I-05, (2003): 6 p.
  2. A.Yu. Plakhov, "Newton's problem of the body of least resistance: the case of few impacts", Cadernos de Matemática", Universidade de Aveiro, CM03/I-07, (2003): 7 p.
  3. A.Yu. Plakhov, "Newton's problem of minimal resistance for bodies containing a half-space", Cadernos de Matemática, Universidade de Aveiro, CM03/I-21, Universidade de Aveiro, (2003): 4 p.
  4. A.Yu. Plakhov, "Newton's problem of the body of minimal averaged resistance", Cadernos de Matemática, Universidade de Aveiro, CM03/I-34, (2003): 24 p.
  5. A.Yu. Plakhov, "Exact solutions of the one-dimensional Monge-Kantorovich problem, Cadernos de Matemática, Universidade de Aveiro, CM03/I-36, (2003): 18 p.

2.5 List of organized seminars and conferences

- **Organized seminars**
  1. Alexander Plakhov, "O problema de Newton de resistência mínima e o problema de Monge de transporte de massa: dois problemas antigos revisitados", Seminários do CEOC, October 3, 2003, Universidade de Aveiro.
  2. Alexander Plakhov, "O problema de Newton do corpo cuja resitência média é mínima", Seminários do CEOC, May 9, 2003, Universidade de Aveiro.

3. Computability and Algorithms

3.1 Activities during 2003

1. **Complexity Theory**.

   Previous work on two-dimensional compaction has been published in 2003 in an international journal with referee.

   Also during the year of 2003 a PhD dissertation was concluded and submitted (approved May 2004 on the classification of instances of NP-hard Problems, in terms of their computational cost.

2. **Computational Geometry**.

   Our research interest in 2003 was the study of art gallery and illumination problems in order to build approximation algorithms. We developed an anytime algorithm to compute successively better approximations of the optimum to minimum vertex guard. An experimental evaluation of the algorithm has been done, in which standard computational geometry algorithms and constraint programming techniques are used in an hybrid way. We also started research work on methods for generating random polygons. Besides its manifest theoretical interest, the generation of random geometric objects has applications that include the testing and verification of time complexity for computational geometry algorithms. We developed two different methods for generating random orthogonal polygons with a given number of vertices.

   During that year, 3 MSc dissertations were supervised, a local seminar and 2 communications in international conferences were presented. Also a book chapter and a paper in a conference proceedings, both international and with referees, were published.

3. **Quantum Computation**.

   Subsequent to previous work on Computer Arithmetic, along 2002 and 2003, joint research was developed together with members of the control theory group from CEOC. This has resulted on the publication in 2004 of two papers in an international journal with referee.

   Also during 2003 a PhD project was prepared (and started October 2003) in the area of quantum computation. Preliminary reading resulted in the publication of a report and a local seminar on Shor's Algorithm.

   Research work on quantum arithmetic started with the design of a $\mathcal{O}(1)$ adder, which explores the use of redundant number systems under the quantum paradigm.

3.2 List of publications

- **Articles in International Journals (including book chapters)**

1. A.M. Almeida and R. Rodrigues, "Trees, Slices and Wheels: On the Floorplan Area Minimization Problem", NETWORKS, vol 41(4), (2003), 235-244.
2. A. P. Tomás, A. L. Bajuelos and F. Marques, "Approximation Algorithms to Minimum Vertex Cover Problems on Polygons and Terrains", LNCS 2657, Springer-Verlag,(2003), 869-878.

- **Articles in National Journals (including book chapters)**

1. A. F. Pereira e M. R. D. Rodrigues, "O Problema das Torres de Hanoi: a lenda, algoritmos e generalizações", Gazeta de Matemática, n. 144, Janeiro de 2003, 8-16.

3.3 List of talks

- **Talks at International Conferences**

1. Antonio Leslie Bajuelos, "Approximation algorithms to minimum vertex cover problems on polygons and terrains", ICCS 2003, June 2-4, St. Petersburg, Russia.
2. Antonio Leslie Bajuelos, "Approximation algorithms to minimum vertex cover problems on polygons", X EGC, Jne 16-17, 2003, Seville, Spain.

3.4 List of organized seminars and conferences

- **Organized seminars**

1. Ana Paula Tomás, "Geração de Polígonos Ortogonais", Seminários do CEOC, December 5, 2003, Universidade de Aveiro.
2. António Pereira, "Uma Introdução à Computação Quântica", Seminários do CEOC, October 24, 2003, Universidade de Aveiro.
3. Miguel Calejo, "Web Application Maker - a declarative web database tool", Seminários do CEOC, October 17, 2003, Universidade de Aveiro.

4. Control theory group

4.1 Activities during 2003

The application of symmetry and conservation laws in optimal control systems is considered in the book [1]. Particularly, variational methods, optimal control, and the Pontryagin maximum principle. The problem of synchronization of systems is considered as a problem of optimal control. Practical examples of using the obtained theoretical results are given.

In [2,8,11] we show that for nonsmooth Euler-Lagrange extremals, Noether's conservation laws cease to be valid. We prove that Emmy Noether's theorem of the calculus of variations is still valid in the wider class of Lipschitz functions, as long as one restricts the Euler-Lagrange extremals to those which satisfy the DuBois-Reymond necessary condition. In the smooth case all Euler-Lagrange extremals are DuBois-Reymond extremals, and the result gives a proper extension of the classical Noether's theorem.

We obtain a discrete time analog of E. Noether's theorem in Optimal Control, asserting that integrals of motion associated to the discrete time Pontryagin Maximum Principle can be computed from the quasi-invariance properties of the discrete time Lagrangian and discrete time control system. As corollaries, results for first-order and higher-order discrete problems of the calculus of variations are obtained [3,9,14].

Explicit formulas for the Chen logarithm are obtained in [4], together with a rule to generate the coefficients of any Lie bracket without using any information of previous brackets. Our formulation turns out to be quite economic and extends the results in Kawski'00. Although the domain set is still larger than a Hall basis, this formulation is powerful enough to be applied to resolving stabilizability issues of some classes of nonlinear control systems.

In [5,10] we address the issue of existence of generalized optimal solutions for nonautonomous singular linear quadratic problems. Both the constant, and the time-variant order of singularity cases are addressed.

We extend the second Noether theorem to optimal control problems which are invariant under symmetries depending upon k arbitrary functions of the independent variable and their derivatives up to some order m. As far as we consider a semi-invariance notion, and the transformation group may also depend on the control variables, the result is new even in the classical context of the calculus of variations [6].

We consider [7] the Lagrange problem of optimal control with unrestricted controls and address the question: under what conditions we can assure optimal controls are bounded? Rewriting the Lagrange problem in a parametric form, we obtain a relation between the applicability conditions of the Pontryagin maximum principle to the later problem and the Lipschitzian regularity conditions for the original problem. Under the standard hypotheses of coercivity of the existence theory, the conditions imply that the optimal controls are essentially bounded, assuring the applicability of the classical necessary optimality conditions like the Pontryagin maximum principle. The result extends previous

Lipschitzian regularity results to cover optimal control problems with general nonlinear dynamics.

We provide [16] a new, simpler, and more direct proof of the well known fact that for autonomous optimal control problems the Pontryagin extremals evolve on a level surface of the respective Pontryagin Hamiltonian.

We study Smarandache sequences of numbers, and related problems, via a Computer Algebra System. Solutions are discovered, and some conjectures presented [17].

4.2 List of publications

- **Books**

[1] A. Gugushvili, O. Khutsishvili, V. Sesadze, G. Dalakishvili, N. Mchedlishvili, T. Khutsishvili, V. Kekenadze, and Delfim F. M. Torres; Symmetries and Conservation Laws in Optimal Control Systems (book in Georgian), Georgian Technical University, Tbilisi, 2003. (ISBN 99940-14-53-6, 248 pages)

- **Articles in International Journals (including book chapters)**

[2] Delfim F. M. Torres, A Proper Extension of Noethers Symmetry Theorem for Nonsmooth Extremals of the Calculus of Variations. Lagrangian and Hamiltonian Methods for Nonlinear Control. Editors: Astolfi, Gordillo and van der Schaft, 2003, pp. 195-198.

[3] Delfim F. M. Torres, Integrals of Motion for Discrete-Time Optimal Control Problems. Control Applications of Optimisation. Editors: Bars and Gyurkovics, 2003, pp. 33-38.

[4] Eugénio M. Rocha, "On computation of the logarithm of the Chen-Fliess series for nonlinear systems", in Zinober, Alan (ed.) et al., Nonlinear and adaptive control. Lect. Notes Control Inf. Sci. 281, 317-326, 2003 [Zbl:01836488].

[5] Manuel Guerra, On nonautonomous singular L-Q problems, Lagrangian And Hamiltonian Methods In Nonlinear Control (Editors: A. Astolfi, F. Gordillo, A. van der Schaft) pp. 189-194, 2003.

[6] Delfim F. M. Torres, Gauge Symmetries and Noether Currents in Optimal Control, Applied Mathematics E-Notes, Vol. 3, 2003, pp. 49-57 [Zbl:01925174] [MR:1980565].

[7] Delfim F. M. Torres, Lipschitzian Regularity of the Minimizing Trajectories for Nonlinear Optimal Control Problems, Mathematics of Control, Signals, and Systems (MCSS), 16, 2003, pp. 158-174 [Zbl:01998222] [MR:2006825].

4.3 Talks at International Conferences

1) Delfim F. M. Torres, A Proper Extension of Noether's Symmetry Theorem for Nonsmooth Extremals of the Calculus of Variations, Invited Session on Optimal Control, 2nd IFAC Workshop on Lagrangian and Hamiltonian Methods in Nonlinear Control, Sevilla, Spain, April 2003.

2) E. Rocha, "Computation of coordinates of first kind for nonlinear control systems", SciCADE03, Norway, Jul/2003.

3) M. Guerra, Discontinuous Hamiltonian flows for nonlinear control systems. Second Junior European Meeting "Control Theory and Stabilization", Politecnico de Torino, Torino, Italia, Dezembro de 2003.

4) Delfim F. M. Torres, Optimal Control of Newton-Type Problems of Minimal Resistance, Second Junior European Meeting, Control Theory and Stabilization, Dipartimento di Matematica, Politecnico di Torino, Torino, Italy, December 2003.

5) Delfim F. M. Torres, The role of symmetry in the regularity properties of optimal controls, Symmetry in Nonlinear Mathematical Physics, Kiev, Ukraine, June 2003.

6) Delfim F. M. Torres, Integrals of Motion for Discrete-Time Optimal Control Problems, session "Optimal Control"(co-chair), IFAC Workshop on Control Applications of Optimization, CAO 2003, Visegrád, Hungary, July 2003.

7) Delfim F. M. Torres, The Noether Principle of Optimal Control, Session on Optimal Control Theory, Eight Viennese Workshop on Optimal Control, Dynamic Games and Nonlinear Dynamics: Theory and Applications in Economics and OR/MS, Vienna, Austria, May 2003.

4.4 List of reports (including proceedings)

- **Proceedings with Referee**

  [8] Delfim F. M. Torres, A Proper Extension of Noethers Symmetry Theorem for Nonsmooth Extremals of the Calculus of Variations. Proceedings of the International Federation of Automatic Control, 2nd Workshop on Lagrangian and Hamiltonian Methods for Nonlinear Control (invited session on "Optimal Control"), Seville, Spain, April 3-5, 2003, pp. 225-228.

  [9] Delfim F. M. Torres, Integrals of Motion for Discrete-Time Optimal Control Problems. Proceedings of the International Federation of Automatic Control, Workshop on Control Applications of Optimization - CAO2003, Visegrád, Hungary, 30 June - 2 July 2003, pp. 44-49.

[10] Guerra, M. On nonautonomous singular L-Q problems. Second Workshop on Lagrangean and Hamiltonian methods for nonlinear control, April, 3-5 2003, Seville, Spain. Astolfi, A.; van der Schaft, A.J.; Gordillo, F. eds. Preprints of the IFAC, 2003 (CD-ROM), pp 219-224;

- **Research Reports**

[11] Delfim F. M. Torres, A Proper Extension of Noethers Symmetry Theorem for Nonsmooth Extremals of the Calculus of Variations. Cadernos de Matemática CM03/I-11, Dep. Matemática, Univ. Aveiro, February 2003.

[12] Delfim F. M. Torres, Autómatos Celulares, Cadernos de Matemática CM03/D-03, Dep. Matemática, Univ. Aveiro, June 2003. National Contest on Logic Programming, CeNPL03, University of Évora, Évora, May 9-11, 2003.

[13] Delfim F. M. Torres, Entropy Text Analyser, Cadernos de Matemática CM03/D01, Dep. Matemática, Univ. Aveiro, January 2003. MIUP2002, 2nd Portuguese ACM Programming Contest.

[14] Delfim F. M. Torres, Integrals of Motion for Discrete-Time Optimal Control Problems. Cadernos de Matemática CM03/I-01, Dep. Matemática, Univ. Aveiro, January 2003.

[15] Delfim F. M. Torres, Números Felizes e Sucessões de Smarandache: Digressões com o Maple, Cadernos de Matemática CM03/D-02, Dep. Matemática, Univ. Aveiro, June 2003.

[16] Delfim F. M. Torres, On the Constancy of the Pontryagin Hamiltonian for Autonomous Problems, Cadernos de Matemática CM03/I-14, Dep. Matemática, Univ. Aveiro, March 2003.

[17] Paulo D. F. Gouveia and Delfim F. M. Torres, Smarandache Sequences: Explorations and Discoveries with a Computer Algebra System. Cadernos de Matemática CM03/I-35, Dep. Matemática, Univ. Aveiro, Dezembro 2003.

4.5 List of organized seminars and conferences

- **Organized seminars**

1. Eugénio Rocha, "Contribuições da álgebra moderna para a teoria do controlo não linear", Seminários do CEOC, December 12, 2003, Universidade de Aveiro.
2. Luís N. Vicente, "A Técnica da aplicação de estado para problemas de controlo óptimo governados por EDPs"Seminários do CEOC, November 7, 2003, Universidade de Aveiro.
3. Boris Miller, "Dynamical systems with controlled singularities", Seminários do CEOC, September 26, 2003, Universidade de Aveiro.

4. Margarida Camarinha, "Polinómios cúbicos em espaços de curvatura constante", Seminários do CEOC, June 6, 2003, Universidade de Aveiro.
5. Rui C. Rodrigues, "Splines generalizados em espaços euclidianos", Seminários do CEOC, May 30, 2003, Universidade de Aveiro.
6. Manuel Guerra, "Fluxos hamiltonianos descontínuos e extremais generalizados para sistemas de controlo não linear", Seminários do CEOC, May 23, 2003, Universidade de Aveiro.
7. J. Clemente-Gallardo, "Quantum computation and spin systems", Seminários do CEOC, March 14, 2003, Universidade de Aveiro.
8. A. A. Davydov, "On generic singularities of parametric avarage optimization of control systems", Seminários do CEOC, February 21, 2003, Universidade de Aveiro.

- **Organized conferences**

8. Delfim F. M. Torres was member of the Organizing Committee of the Second Junior European Meeting on "Control Theory and Stabilization", Dipartimento di Matematica del Politecnico di Torino, Torino, Italy, 3-5 December 2003.

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