# DGS 2013 - International Conference and Advanced School Planet Earth, Dynamics, Games and Science 

Mini symposium on graphs and combinatorics September 3-4, Calouste Gulbenkian Foundation, Lisbon.<br>Organizer: Domingos M. Cardoso<br>CIDMA, DMat, Univ. de Aveiro

1. Graphs and Combinatorics - I
1.1 Title: The maximum k-regular induced subgraph problem.

- Speaker: Sofia J. Pinheiro (spinheiro@ua.pt), CIDMA, DMat, Univ. de Aveiro.
- Abstract: Many optimization problems on graphs are reduced to the determination of a subset of vertices of maximum cardinality which induces a $k$-regular subgraph. We denote this maximum cardinality by $\alpha_{k}(G)$. Since the determination of the order of a $k$-regular induced subgraph is in general a NP-hard problem, it is crucial to find upper bounds determined in polynomial time as good approximations. In this presentation some upper bounds on $\alpha_{k}(G)$ are proposed and an extension to arbitrary graphs of the convex quadratic upper bound introduced in Cardoso, Kamiński, and Lozin [J. Comb. Optim., 14, 455-463, 2007] for regular graphs is presented. Based on this approach, new spectral upper bounds on the order of maximum size k-regular induced subgraphs are deduced. Finally a few computational experiments are presented.
- Joint work with: Domingos M. Cardoso.
1.2 Title: $(0,2)$-regular sets and applications.
- Speaker: Maria de Fátima Pacheco (pacheco@ipb.pt), Polytechnic Inst. of Bragança, CIDMA, Univ. de Aveiro.
- Abstract: A $(\kappa, \tau)$-regular set in a graph is a subset of vertices inducing a $\kappa$-regular subgraph and such that each vertex not in the set has exactly $\tau$ neighbors in it. We will present a new algorithm for the determination of $(0,2)$-regular sets in line graphs as well as its application to the determination of maximum matchings in arbitrary graphs.
- Joint work with: Domingos M. Cardoso and Carlos J. Luz.
1.3 Title: The construction of the poset of regular exceptional graphs using equitable partitions.
- Speaker: Inês Barbedo (inesb@ipb.pt), Polytechnic Inst. of Bragança, CIDMA, Univ. de Aveiro.
- Abstract: An exceptional graph is a connected graph with least eigenvalue greater than or equal to -2 which is not a generalized line graph. In [3] it is shown that the set of regular exceptional graphs is partitioned in three layers. A $(\kappa, \tau)$-regular set is a subset of the vertices of a graph, inducing a $\kappa$-regular subgraph such that every vertex not in the subset has $\tau$ neighbors in it [2]. In [1] a new recursive construction of regular exceptional graphs is proposed, where each regular exceptional graph of the first and the second layer is constructed by a $(0,2)$-regular set extension. In this talk we present an algorithm based on this recursive construction and show that this technique induces a partial order relation on the set of regular exceptional graphs. The process of extending a graph is reduced to the construction of the incidence matrix of a combinatorial 1-design, considering several rules to prevent the production of isomorphic graphs, and we show that each regular exceptional graph has an equitable partition which, by this construction technique, is extended with a new element, the set of the additional vertices. The recursive construction is generalized to the construction of arbitrary families of regular graphs, by extending a regular graph $G$ with another regular graph $H$ such that $V(H)$ is a $(\kappa, \tau)$-regular set of the regular graph produced. This technique is used to construct the exceptional regular graphs of the third layer. The Hasse diagrams of the posets of the three layers are presented.
- Joint work with: Domingos M. Cardoso and Paula Rama.


## References

[1] D.M. Cardoso and D. Cvetković. Graphs with least eigenvalues -2 attaining a convex quadratic upper bound for the stability number., Bull. Acad. Serbe Sci. Arts, Cl. Sci. Math. Natur., Sci. Math., CXXXIII (31):41-55, 2006.
[2] D. M. Cardoso and P. Rama, Spectral results on regular graphs with $(\kappa, \tau)$ regular sets, Discrete Math. (307) 1306-1316, 2007.
[3] D. Cvetković, P. Rowlinson and S. Simić. Spectral Generalizations of line graphs: on graphs with least eigenvalue -2, Cambridge University Press, Cambridge, 2004.

## 2. Graphs and Combinatorics - II

2.1 Title: Matchings and orthogonality of symmetrized tensors.

- Speaker: Maria Manuel Torres (mmanuel@ptmat.fc.ul.pt), CELC, DMat, Univ. de Lisboa.
- Abstract: Many scheduling problems are stated in terms of edge colorings on bipartite graphs, that is, decompositions by disjoint matchings covering the entire edge set. We will show how the orthogonality of certain symmetrized tensors can be described by a scheduling condition.
- Joint work with: José Perdigão Dias da Silva.
2.2 Title: Graph Decomposition: A blend of Turán numbers and Ramsey numbers
- Speaker: Teresa Maria Sousa (tmjs@fct.unl.pt), DMat, FCT-Univ. Nova de Lisboa.
- Abstract: Let $H=\left(H_{1}, \ldots, H_{k}\right)$ be a fixed $k$-tuple of graphs and let $G$ be a graph on $n$ vertices whose edges are colored with $k$ colors. A monochromatic $H$-decomposition of $G$ is a partition of its edge set such that each part is either a single edge or a copy of $H_{i}$ monochromatic in color $i$. The aim is to find the smallest number, denoted by $f(n, H, k)$, such that, any $k$-edge colored graph with $n$ vertices admits a monochromatic $H$-decomposition with at most $f(n, H, k)$ parts. We will consider the problem when $H$ is a fixed $k$-tuple of cliques (a clique is a complete graph) for all values of $k$. The results presented will involve both the Turán numbers and the Ramsey numbers.
- Joint work with: Henry Liu.


## 3. Graphs and Combinatorics - III

3.1 Title: Topological Indices of graphs.

- Speaker: Paula Carvalho (paula.carvalho@ua.pt), CIDMA, DMat, Univ. de Aveiro.
- Abstract: A topological index of a graph is a numerical parameter that is mathematically derived from the structure of the graph. Many topological indices emerged in Chemical Graph Theory in the development of new relationships between the physico-chemical properties of a molecule and its chemical structure, represented by a graph. In this talk we focus on the modified Schultz index: $S^{*}=\sum_{\{u, v\}} d_{G}(u) d_{G}(v) \operatorname{dist}_{G}(u, v)$. We consider some graph operations, namely cartesian product, complete product, composition and subdivision, and we obtain explicit formulae for the modified Schultz index of a graph in terms of the number of vertices and edges as well as some other topological invariants such as the Wiener index, the Schultz index and the first and second Zagreb indices.
- Joint work with: Paula Rama.
3.2 Title: Min-max regret robust shortest path problem in a finite multi-scenario model.
- Speaker: M. Pascoal (marta@mat.uc.pt), INESC-Coimbra, DMat, Univ. de Coimbra.
- Abstract: The min-max regret robust shortest path problem is a classical problem based on the determination of a path with the minimal maximum deviation from the shortest path between a pair of nodes in a given network. The research on this subject has been mainly focused on interval data models. Nevertheless, for discrete models with a bounded number of scenarios the problem has received little attention. In this talk the min-max regret robust shortest path problem is defined considering a finite number of scenarios, and some properties of the optimal solutions are presented. Two types of methods are then proposed, the first is a labelling approach and the second is based on the ranking of paths by order of cost. Two variants of the latter method are described. One consists of ranking paths in every scenario and of using a stopping condition dependent on cost upper bounds given by the paths that
are computed, the other improves this version by including further reduction techniques. The performances of the three algorithms are compared based on computational experiments performed on randomly generated networks.
- Joint work with: Marisa Resende.
3.3 Title: A Generalization of the Krein parameters and some admissibility conditions for strongly regular graphs.
- Speaker: Vasco Moço Mano (vascomocomano@gmail.com), DMat, Univ. do Porto.
- Abstract: In this work, we describe a Jordan-algebraic approach to the theory of strongly regular graphs. In particular, we associate a three dimensional Euclidean Jordan algebra to the adjacency matrix of a strongly regular graph. With this approach, we generalize the Krein parameters and derive the generalized Krein admissibility conditions for such graphs. In light of these generalizations, we were able to extract further information over the classical Krein parameters and obtain some tight upper bounds for some of the generalized Krein parameters.
- Joint work with: Enide A. Martins and Luis A. Vieira.


## 4. Graphs and Combinatorics - IV

4.1 Title: Some results on the modified Schultz index.

- Speaker: Paula Rama (prama@ua.pt), CIDMA, DMat, Univ. de Aveiro.
- Abstract: The topological indices are graph invariants much studied in both mathematical and chemical literature. The modified Schultz index, also known as the Gutman index, is defined as $S^{*}(G)=\sum_{\{u, v\} \subset V(G)} d(u) d(v) \operatorname{dist}(u, v)$, where $d(u)$ is the degree of vertex $u$ and $\operatorname{dist}(u, v)$ is the distance between vertices $u$ and $v$. In this talk, we present several results on the modified Schultz index, in particular, we determine a lower bound of the modified Schultz index of tricyclic graphs with three cycles and $n$ vertices, that is, connected graphs of order $n$ with exactly three cycles, and we characterize all extremal graphs which attain the lower bound.
- Joint work with: Paula Carvalho.
4.2 Title: Singleton free set partitions avoiding a 3-element set.
- Speaker: Ricardo Mamede (mamede@mat.uc.pt), CMUC, DMat, Univ. de Coimbra.
- Abstract: The definition and study of pattern avoidance for set partitions, which is an analogue of pattern avoidance for permutations, begun with Klazar. Sagan continued his work by considering set partitions which avoids a single partition of three elements, and Goyt generalized these results by considering partitions which avoids any family of partitions of a 3-element set. We continue this work with the enumeration and description of set partitions, even set partitions and odd set partitions without singletons which avoids any family of partitions of a 3-element set. The characterization of these families allows us to conclude that the corresponding sequences are $P$-recursive. We
also construct Gray codes for the sets of singletons free partitions that avoids a single partition of three elements.
4.3 Title: Maps and hypermaps with large symmetry group.
- Speaker: Rui Duarte (rduarte@ua.pt), CIDMA, DMat, Univ. de Aveiro.
- Abstract: Maps and hypermaps are cellular embeddings of graphs and hypergraphs into closed surfaces. An automorphism or symmetry of a hypermap $\mathcal{H}$ is an incidence-preserving permutation of each of the sets of vertices, edges and faces. In this talk we present some families of hypermaps with large symmetry group. We also present some constructions of hypermaps and their properties.


## 5. Graphs and Combinatorics - V

- Title: Shannon switching games and directed variants.
- Speaker: Ilda Perez da Silva (ipsilva@fc.ul.pt), CELC, DMat, Univ. de Lisboa.
- Abstract: Shannon switching game is a combinatorial game modeling break down repair of a network. The game was completely solved by Lehman in 1963. In the late 1980's Y.O. Hamidoune and M. Las Vergnas introduced and studied directed versions of the game. Despite the results obtained so far, Hamidoune-Las Vergnas directed switching games are still far from being completely understood. We will do a brief account of results and open questions on the subject.
- Joint work with: Luís Sequeira.
- Title: Growth diagrams, crystal operators and Cauchy kernel expansions.
- Speaker: Olga Azenhas (oazenhas@mat.uc.pt), CMUC, DMat, Univ. de Coimbra.
- Abstract: We formulate the reverse RSK and an analogue of RSK for pairs of certain combinatorial objects, in the framework of Fomin's growth diagrams. We recall crystal operators as operators on words, introduced by Lascoux and Schützenberger in the study of the plactic monoid, and later rediscovered by Kashiwara and Nakashima in the theory of crystal bases to define crystal graph. These tools are then used to regain expansions of non symmetric Cauchy kernels over Ferrers shapes in type A.
- Joint work with: A. Emami

