COMMENTED BIBLIOGRAPHY

ALGEBRAIC NUMBER THEORY

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The books and papers I relied upon when preparing the course:

BOOKS

- Kenneth Ireland and Michael Rosen, A CLASSICAL INTRODUCTION TO MODERN NUMBER THEORY, Springer 1990

  https://link.springer.com/book/10.1007/978-1-4757-2103-4

  A nice introduction to many topics in Number Theory, including reciprocity laws (quadratic, cubic, biquadratic, and Eisenstein's), the basics of Algebraic Number Theory (chapters 6, 12 and 13), Elliptic Curves (chapters 18 and 19), and a proof of Fermat's Last Theorem for the exponent 3, and for the first case for regular exponents (in chapter 17). Beautiful, crystal clear exposition!

- Paul Pollack, A CONVERSATIONAL INTRODUCTION TO ALGEBRAIC NUMBER THEORY, American Mathematical Society, 2017

  A delighful introduction to Algebraic Number Theory, starting with quadratic fiels before taking up general number fields. Very well written. A joy to read!

- Harry Pollard and Harold G. Diamond, THE THEORY OF ALGEBRAIC NUMBERS, The Mathematical Association of America (The Carus Mathematical Monographs), 2nd edition, 1975.

  An extraordinarily concise but detailed introduction to Algebraic Number Theory. A small and short book that will give you all the basics of the arithemtic of number fields.

- J.S. Milne, ALGEBRAIC NUMBER THEORY, online book:

  https://www.jmilne.org/math/CourseNotes/ant.html

  Very complete introduction to all classical topics in Algebraic Number Theory. Very well writen, clear, and it is completely free!

- Pierre Samuel, THÉORIE ALGÉBRIQUE DES NOMBRES , Hermann 1967.

  (English translation: Pierre Samuel, ALGEBRAIC THEORY OF NUMBERS, Dover, 2008)

  https://maa.org/press/maa-reviews/algebraic-theory-of-numbers

  https://web.math.ucsb.edu/~agboola/teaching/2021/fall/225A/samuel.pdf

  A classic: a short, clear introduction to the basics of Algebraic Number Theory.

- Borevich and Shafarevich, NUMBER THEORY, Academic Press, 1966

  https://www.maths.ed.ac.uk/~v1ranick/papers/borevich.pdf

  Another classic! Deeper and covering more material than Samuel's book.

- J.-P. Serre, LOCAL FIELDS, Springer, 1979

  https://link.springer.com/book/10.1007/978-1-4757-5673-9

  A very serious, concise, elegant book! It contains a neat, condensed introduction to Algebraic Number Theory in the first chapter. The main goal of this book is to expose Class Field Theory from a cohomological point of view.

PAPERS

Each of the following papers is a masterpiece of exposition.

- B. F. Wyman, *What is a Reciprocity Law?*, The American Mathematical Monthly, Vol. 79, No. 6 (June–July, 1972), pp. 571–586

- P. Stevenhagen and H. W. Lenstra, *Chebotarëv and his Density Theorem*, The Mathematical Intelligencer, Volume 18, pages 26–37 (1996)

  https://link.springer.com/article/10.1007/BF03027290

  or https://websites.math.leidenuniv.nl/algebra/chebotarev.pdf

  or https://www.math.leidenuniv.nl/~hwl/papers/cheb.pdf

- H. W. Lenstra and P. Stevenhagen, *Artin Reciprocity and Mersenne Primes*, Nieuw Arch. Wisk., Volume1, Issue 5, pp. 44–54

  https://websites.math.leidenuniv.nl/algebra/artin.pdf

- Jared Weinstein, *Reciprocity Laws and Galois Representations: Recent Breakthroughs*, Bulletin (New Series) of the American Mathematical Society, Volume 53, Number 1, January 2016, Pages 1–39

  http://dx.doi.org/10.1090/bull/1515