

AVEIRO SUMMER SCHOOL

Local Fields

Exercise Sheet 2

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1. Show that $\mathbb{Z}_p/p\mathbb{Z}_p \simeq \mathbb{Z}/p\mathbb{Z}$.
2. Show that $\mathbb{Q}_p = \text{Frac}(\mathbb{Z}_p) = \cup_n p^{-n}\mathbb{Z}_p$.
3. Show that $\mathbb{Z} \subset \mathbb{Z}_p$ are dense, and that \mathbb{Z}_p is the completion of \mathbb{Z} with respect to $||_p$.
4. Show that the extension of $||_p$ to \mathbb{Q}_p is a well-defined absolute value on \mathbb{Q}_p .