Aveiro Summer School

Local Fields

Exercise Sheet 1

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1. Let p be a prime number. Show that the absolute value $|\,|_p$ is non-Archimedean.

2. Let $(K, |\,|)$ be a valued field.

   (a) Show that if $x \in K$ is a root of unity, then $|x| = 1$.

   (b) Show that
   $$\left|\frac{x}{y}\right| = \frac{|x|}{|y|},$$
   for all $x, y \in K$ with $y \neq 0$.

   (c) If $K = \mathbb{F}_{p^n}$ (a finite field) then any absolute value on K is the trivial one. Deduce the same result for $\overline{\mathbb{F}_p} = \cup_n \mathbb{F}_{p^n}$.

   (d) Show that the sequence $1, 11, 111, 1111, 1111, \ldots$ converges to $-\frac{1}{9}$ for $|\,|_5$ and $|\,|_2$, but is not Cauchy for all other absolute values of $\mathbb{Q}$.