## **Cryptography**



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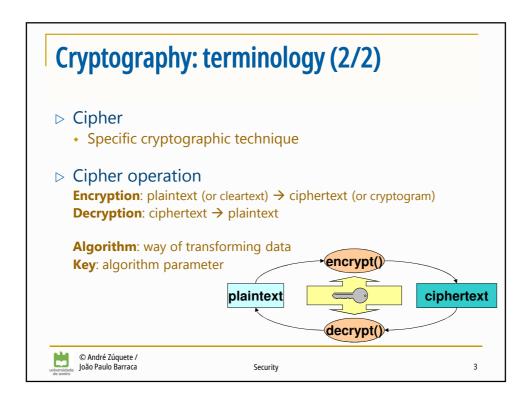
## **Cryptography: terminology (1/2)**

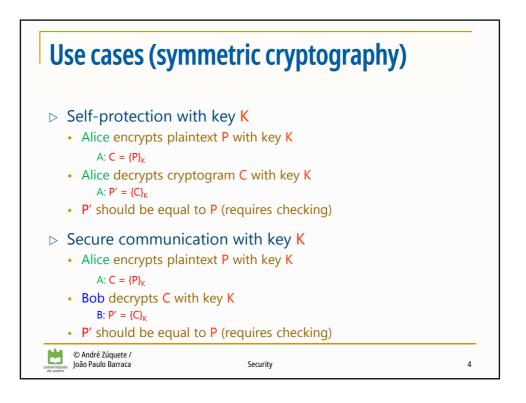
- Cryptography
  - · Art or science of hidden writing
    - from Gr. kryptós, hidden + graph, r. of graphein, to write
  - It was initially used to maintain the confidentiality of information
  - Steganography
    - from Gr. steganós, hidden + graph, r. of graphein, to write
- Cryptanalysis
  - Art or science of breaking cryptographic systems or encrypted information
- - Cryptography + cryptanalysis



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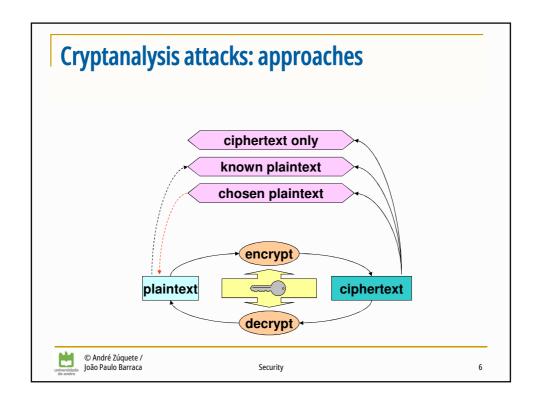


## **Cryptanalysis: goals**

- Discover original plaintext
  - · Which originated a given ciphertext
- Discover a cipher key
  - · Allows the decryption of ciphertexts created with the same key
- Discover the cipher algorithm
  - Or an equivalent algorithm
  - Usually algorithms are not secret, but there are exceptions
    - · Lorenz, A5 (GSM), RC4 (WEP), Crypto-1 (Mifare)
    - · Algorithms for DRM (Digital Rights Management)
  - · Reverse engineering



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### **Cryptanalysis attacks: approaches**

#### ▶ Brute force

- Exhaustive search along the key space until finding a suitable key
- Usually infeasible for a large key space
  - e.g. 2128 random keys (or keys with 128 bits)
  - · Randomness is fundamental!

#### 

 Reduce the search space to a smaller set of potential candidates



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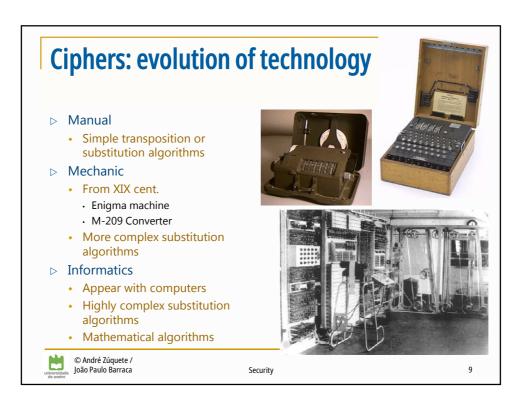
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#### **Size matters!**

- ≥ 2<sup>32</sup>
  - IPv4 address space
  - World population
  - · Years for the Sun to become a white dwarf
- > 2<sup>128</sup>
  - IPv6 address space
- > 2¹66
  - Earth atoms
- $\triangleright$  2<sup>265</sup>
  - · Hydrogen atoms in the known universe
- - Only cryptography uses them



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#### > Transposition

- Original cleartext is scrambled
   Onexcl raatre ilriad gctsm ilesb
- Block permutations
   (13524) → boklc pruem ttoai ns

#### > Substitution

- Each original symbol is replaced by another
  - · Original symbols were letters, digits and punctuation
  - · Actually they are blocks of bits
- · Substitution strategies
  - Mono-alphabetic (one→one)
  - · Polyalphabetic (many one→one)
  - Homophonic (one→many)



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## Ciphers: basic types (2/3): Mono-alphabetic

- - With #α elements
- Examples
  - Additive (translation)
    - crypto-symbol = (symbol + key) mod #  $\alpha$
    - symbol = (crypto-symbol key) mod #  $\alpha$
    - Possible keys =  $\#\alpha$
    - · Caesar Cipher (ROT-x)
  - With sentence key ABCDEFGHIJKLMNOPQRSTUVWXYZ QRUVWXZSENTCKYABDFGHIJLMOP
    - Possible keys = #  $\alpha$ !  $\rightarrow$  26!  $\approx$  288
- ▶ Problems
  - Reproduce plaintext pattern
    - · Individual characters, digrams, trigrams, etc.
  - Statistical analysis facilitates cryptanalysis
    - · "The Gold Bug", Edgar Alan Poe

53##1305)|6\*;4826|4#.)
4#);806\*;48#860)|85;1#
(;:#\*8+83(88)5\*+;46(;8
8\*96\*;8)\*#(;485);5\*+2
:\*#(;4956\*2(5\*-4)88\*;4
069285);)6+8)4##;1(#9;
48081;8:8#1;48+85;4)48
5+528806\*81(#9;48;(88;4)(#34;48)4#;161;:188;
#?;

A good glass in the bishop's hostel in the devil's seat fifty-one degrees and thirteen minutes northeast and by north main branch seventh limb east side shoot from the left eye of the death's-head a bee line from the tree through the shot forty feet out



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## Ciphers: basic types (3/3): Polyalphabetic

- - Periodical ciphers, with period N
- - · Vigenère cipher
- ▶ Problems
  - Once known the period, are as easy to cryptanalyze as N monoalphabetic ones
    - · The period can be discovered using statistics
    - Kasiski method
      - · Factoring of distances between equal ciphertext blocks
    - Coincidence index
      - Factoring of self-correlation offsets that yield higher coincidences



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# 

 a
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 X
 Y
 Z
 A

▷ Example of encryption of character M with key S, yielding cryptogram E
• Decryption is the opposite, E and S yield M



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## **Cryptanalysis of a Vigenère cryptogram: Example (1/2)**

Plaintext:

Eles não sabem que o sonho é uma constante da vida tão concreta e definida como outra coisa qualquer, como esta pedra cinzenta em que me sento e descanso, como este ribeiro manso, em serenos sobressaltos como estes pinheiros altos

▷ Cipher with the Vigenère square and key "poema"

- Kasiski test
  - · With text above:
  - With the complete poem:

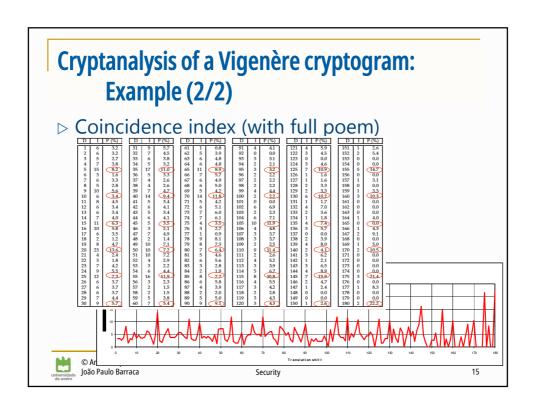
mpa	$20 = 2 \times$	$2 \times 5$
tp	$20=2\times$	$2\times 5$

$175 = 5 \times 5 \times 7$	1
$105 = 3 \times 5 \times 7$	3
$35 = 5 \times 7$	1
$20 = 2 \times 2 \times 5$	4



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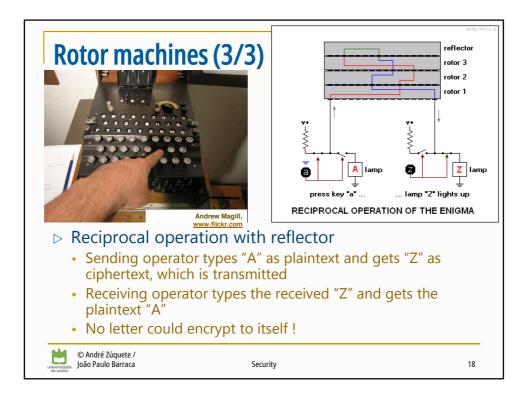
#### Rotor machines (2/3)

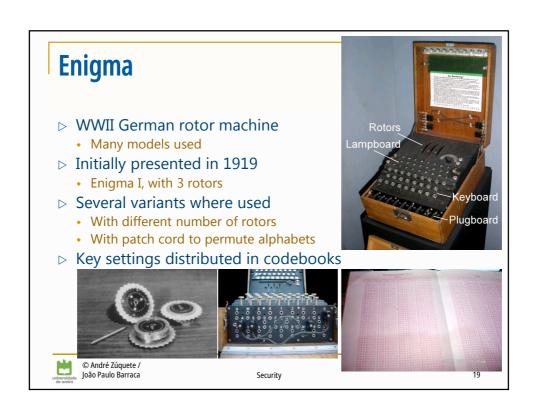
- ▶ Rotor machines implement complex polyalphabetic ciphers
  - Each rotor contains a permutation
    - · Same as a set of substitutions
  - The position of a rotor implements a substitution alphabet
  - Spinning of a rotor implements a polyalphabetic cipher
  - Stacking several rotors and spinning them at different times adds complexity to the cipher
- - The set of rotors used
  - The relative order of the rotors
  - · The position of the spinning ring
  - The original position of all the rotors
- Symmetrical (two-way) rotors allow decryption by "double encryption"
  - Using a reflection disk (half-rotor)

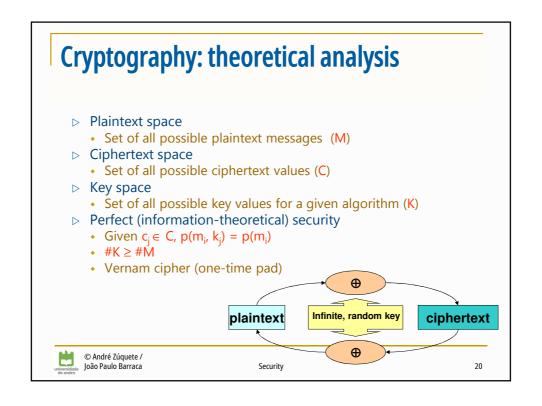


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#### **Cryptography: practical approaches (1/4)**

- > Theoretical security vs. practical security
  - Expected use ≠ practical exploitation
  - · Defective practices can introduce vulnerabilities
    - Example: reuse of keys

#### Computational security

- Security is measured by the computational complexity of break-in attacks
  - · Using brute force
- · Security bounds:
  - Cost of cryptanalysis
  - · Availability of cryptanalysis infra-structure
  - · Lifetime of ciphertext



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#### Cryptography: practical approaches (2/4)

- - · The amount of offered secrecy
    - · e.g. key length
  - Complexity of key selection
    - · e.g. key generation, detection of weak keys
  - · Implementation simplicity
  - Error propagation
    - · Relevant in error-prone environments
    - · e.g. noisy communication channels
  - · Dimension of ciphertexts
    - · Regarding the related plaintexts



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#### Cryptography: practical approaches (3/4)

#### 

- Complex relationship between the key, plaintext and the ciphertext
  - Output bits (ciphertext) should depend on the input bits (plaintext + key) in a very complex way

#### ▶ Diffusion

- Plaintext statistics are dissipated in the ciphertext
  - If one plaintext bit toggles, then the ciphertext changes substantially, in an unpredictable or pseudorandom manner
- Avalanche effect



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#### **Cryptography: practical approaches (4/4)**

- - · Cryptanalysts knows the algorithm
    - · Security lies in the key
  - Cryptanalysts know/have many ciphertext samples produced with the same algorithm & key
    - · Ciphertext is not secret!
  - Cryptanalysts partially know original plaintexts
    - · As they have some idea of what they are looking for
    - · Know-plaintext attacks
    - · Chosen-plaintext attacks



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#### **Cryptographic robustness**

- > The robustness of algorithms is their resistance to attacks
  - No one can evaluate it precisely
    - · Only speculate or demonstrate using some other robustness assumptions
  - · They are robust until someone breaks them
  - There are public guidelines with what should/must not be used
    - · Sometimes antecipating future problems
- Public algorithms without known attacks are likely to be more robust
  - More people looking for weaknesses
- > Algorithms with longer keys are likely to be more robust
  - And usually slower ...



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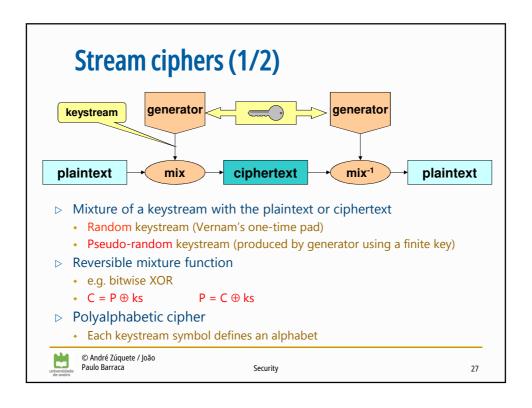
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#### **Cryptographic guidelines**

- □ Guideline for Using Cryptographic Standards in the Federal Government: Cryptographic Mechanisms, NIST Special Publication 800-175B, August 2016
- Cryptographic Storage Cheat Sheet, OWASP Cheat Sheets (last revision: 06/18/2018)
- Guidelines on cryptographic algorithms usage and key management, European Payments Council, EPC342-08 Version 7.0, 4 November, 2017
- △ Algorithms, Key Size and Protocols Report, ECRYPT Coordination
   & Support Action, Deliverable D5.4, H2020-ICT-2014 Project 645421,
   28 February, 2018



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#### Stream ciphers (2/2)

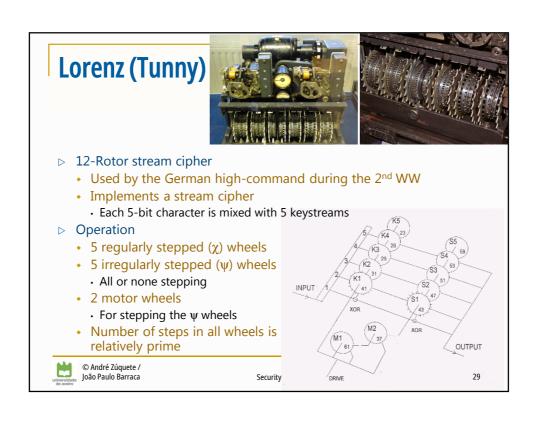
- Keystream may be infinite but with a finite period
  - The period depends on the generator
- Practical security issues
  - Each keystream should be used only once!
    - · Otherwise, the sum of cryptograms yields the sum of plaintexts

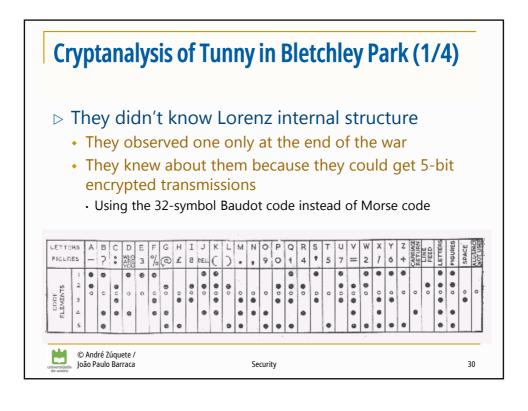
$$C1 = P1 \oplus Ks$$
,  $C2 = P2 \oplus Ks \rightarrow C1 \oplus C2 = P1 \oplus P2$ 

- Plaintext length should be smaller than the keystream period
  - Total keystream exposure under know/chosen plaintext attacks
  - · Keystream cycles help the cryptanalysts knowing plaintext samples
- Integrity control is mandatory
  - · No diffusion! (only confusion)
  - · Ciphertexts can easily be changed deterministically



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#### Cryptanalysis of Tunny in Bletchley Park (2/4)

- - A German operator had a long message (~4,000) to send
    - He set up his Lorenz and sent a 12 letter indicator (wheel setup) to the receiver
    - After ~4,000 characters had been keyed, by hand, the receiver said "send it again"
  - The operator resets the machine to the same initial setup
    - · Same keystream! Absolutely forbidden!
  - The sender began to key in the message again (by hand)
    - But he typed a slightly different message!
    - . C = M ⊕ Ks
    - $\cdot$  C' = M'  $\oplus$  Ks  $\rightarrow$  M' = C  $\oplus$  C'  $\oplus$  M  $\rightarrow$  text variations
    - If you know part of the initial text, you can find the variations



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#### Cryptanalysis of Tunny in Bletchley Park (3/4)

- Breakthrough
  - Messages began with a well known SPRUCHNUMMER "msg number"
    - The first time the operator keyed in SPRUCHNUMMER
    - The second time he keyed in SPRUCHNR
    - Thus, immediately following the N the two texts were different!
  - Both messages were sent to John Tiltman at Bletchley Park, which was able to fully decrypt them using an additive combination of the messages (called *Depths*)
    - $\cdot$  The 2nd message was ~500 characters shorter than the first one
    - Tiltman managed to discover the correct message for the 1st ciphertext
  - They got for the 1st time a long stretch of the Lorenz keystream
    - · They did not know how the machine did it, ...
    - · ... but they knew that this was what it was generating!



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## Cryptanalysis of Tunny in Bletchley Park (4/4): Colossus

- - But deciphering it required knowing the initial position of rotors
- □ Germans started using numbers for the initial wheels' state
  - · Bill Tutte invented the double-delta method for finding that state
  - The Colossus was built to apply the double-delta method
- - · Design started in March 1943
  - The 1,500 valve Colossus Mark 1 was operational in January 1944
  - Colossus reduced the time to break Lorenz from weeks to hours



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#### **Modern ciphers: types**

- > Concerning operation
  - Block ciphers (mono-alphabetic)
  - Stream ciphers (polyalphabetic)
- - Symmetric ciphers (secret key or shared key ciphers)
  - Asymmetric ciphers (or public key ciphers)
- > Arrangements

	Block ciphers	Stream ciphers
Symmetric ciphers		
Asymmetric ciphers		



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#### **Symmetric ciphers**

- Secret key
  - · Shared by 2 or more peers
- - Confidentiality among the key holders
  - · Limited authentication of messages
    - · When block ciphers are used
- Advantages
  - Performance (usually very efficient)
- Disadvantages
  - N interacting peers, pairwise secrecy ⇒ N x (N-1)/2 keys
- ▶ Problems
  - Key distribution



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#### **Symmetric block ciphers**

- Usual approaches
  - Large bit blocks
    - · 64, 128, 256, etc.
  - Diffusion & confusion
    - · Permutation, substitution, expansion, compression
    - · Feistel Networks
    - $L_i = R_{i-1}$   $R_i = L_{i-1} \oplus f(R_{i-1}, K_i)$
    - Iterations

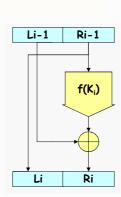
#### Most common algorithms

- DES (Data Enc. Stand.), D=64; K=56
- IDEA (Int. Data Enc. Alg.), D=64; K=128
- AES (Adv. Enc. Stand., aka Rijndael), D=128, K=128, 192, 256
- Other (Blowfish, CAST, RC5, etc.)



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#### **DES (Data Encryption Standard) (1/4)**

- > 1970: the need of a standard cipher for civilians was identified
- ▶ 1972: NBS opens a contest for a new cipher, requiring:
  - The cryptographic algorithm must be secure to a high degree
  - Algorithm details described in an easy-to-understand language
  - The details of the algorithm must be publicly available
    - $\boldsymbol{\cdot}$  So that anyone could implement it in software or hardware
  - The security of the algorithm must depend on the key
    - Not on keeping the method itself (or part of it) secret
  - The method must be adaptable for use in many applications
  - · Hardware implementations of the algorithm must be practical
    - · i.e. not prohibitively expensive or extremely slow
  - · The method must be efficient
  - · Test and validation under real-life conditions
  - · The algorithm should be exportable



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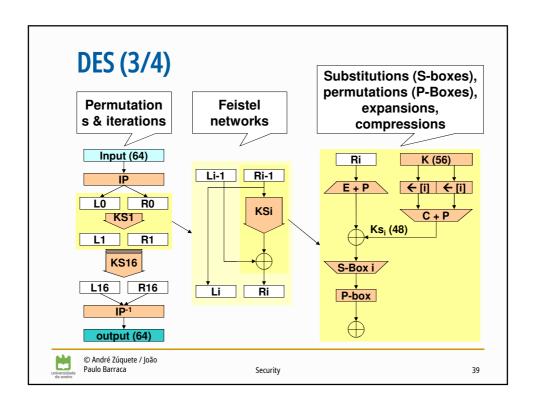
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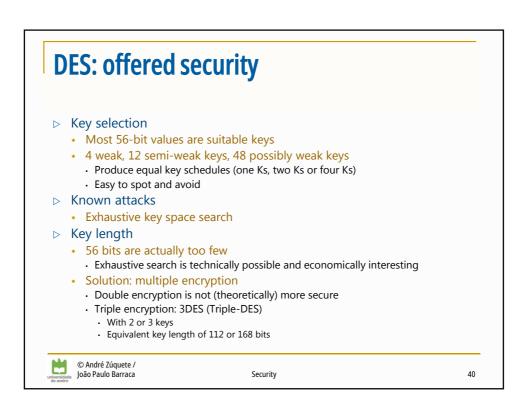
#### **DES (2/4)**

- ▶ 1974: new contest
  - Proposal based on Lucifer from IBM
  - 64-bit blocks
  - 56-bit keys
    - · 48-bit subkeys (key schedules)
  - Diffusion & confusion
    - · Feistel networks
    - · Permutations, substitutions, expansions, compressions
    - 16 iterations
  - Several modes of operation
    - ECB (Electronic Code Book), CBC (Cypher Block Chaining)
    - **OFB** (Output Feedback), **CFB** (Cypher Feedback)
- ▶ 1976: adopted at US as a federal standard



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#### (Symmetric) stream ciphers

- Approaches
  - Cryptographically secure pseudo-random generators (PRNG)
    - · Using linear feedback shift registers (LFSR)
    - · Using block ciphers
    - · Other (families of functions, etc.)
  - · Usually not self-synchronized
  - · Usually without uniform random access
    - · No immediate setup of generator's state for a given plaintext/cryptogram offset
- - A5/1 (US, Europe), A5/2 (GSM)
  - RC4 (802.11 WEP/TKIP, etc.)
  - E0 (Bluetooth BR/EDR)
  - SEAL (w/ uniform random access)



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#### **Uniform random access**

- > Same time to reach and process any piece of information regardeless of its position
- ▶ Uniform
  - Memory
  - Disks (magnetic, optical)
- ▶ Non-uniform
  - Tapes (audio, video, computer)

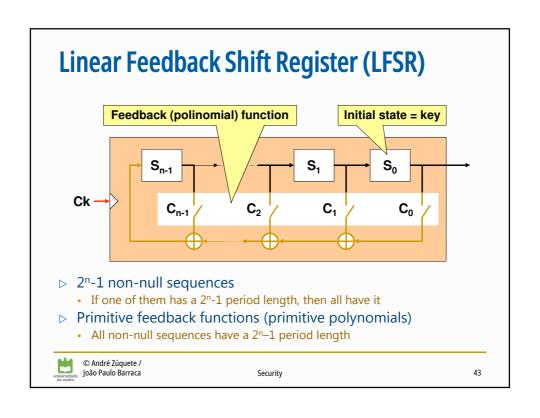


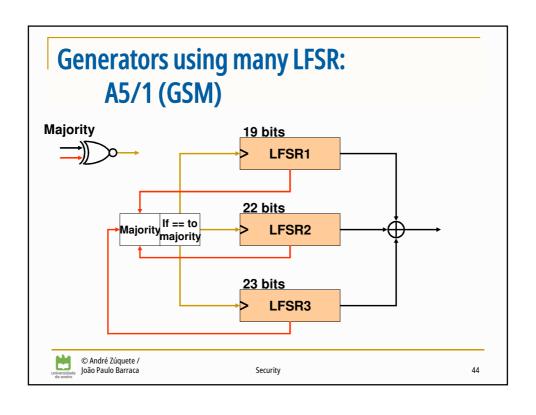




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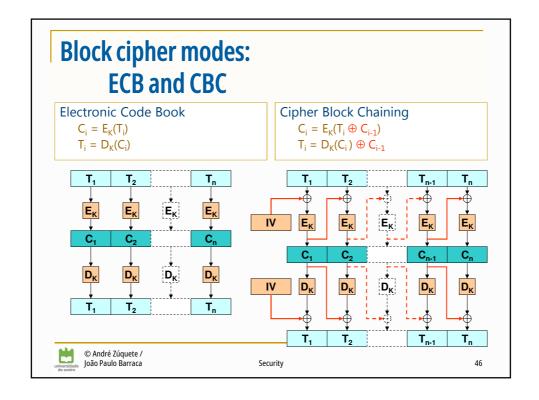


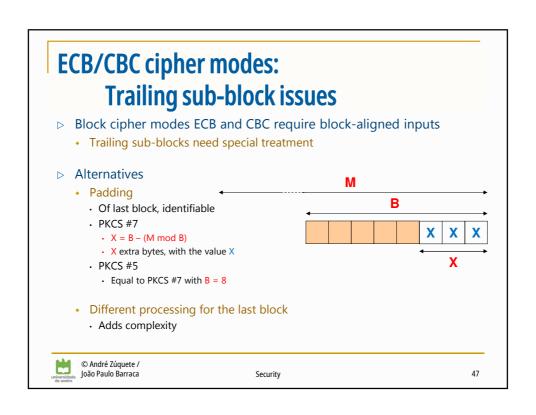
## Deployment of (symmetric) block ciphers: Cipher modes

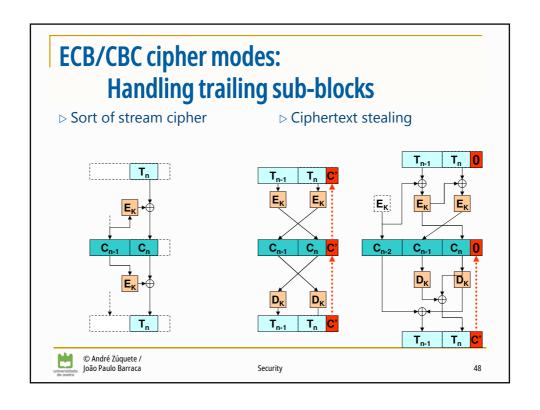
- - ECB (Electronic Code Book)
  - CBC (Cipher Block Chaining)
  - OFB (Output Feeback)
  - CFB (Cipher Feedback)
- - In principle ...
- - CTR (Counter Mode)
  - GCM (Galois/Counter Mode)

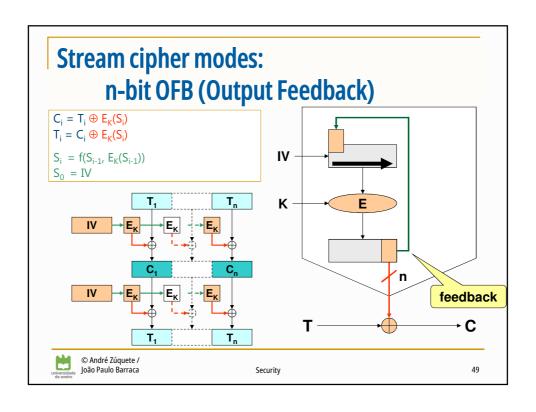


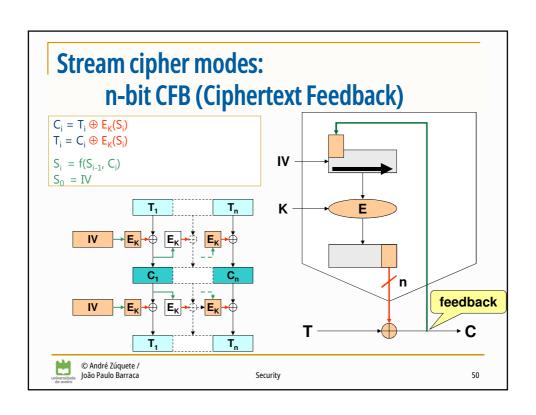
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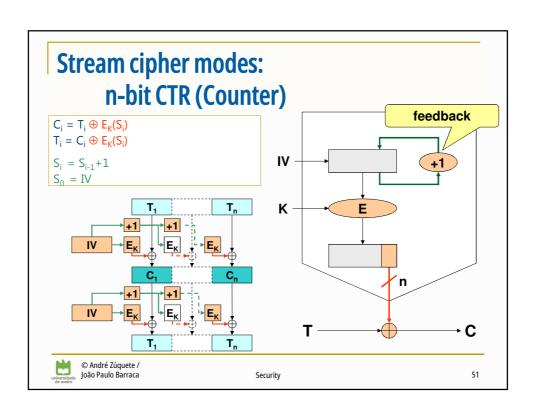




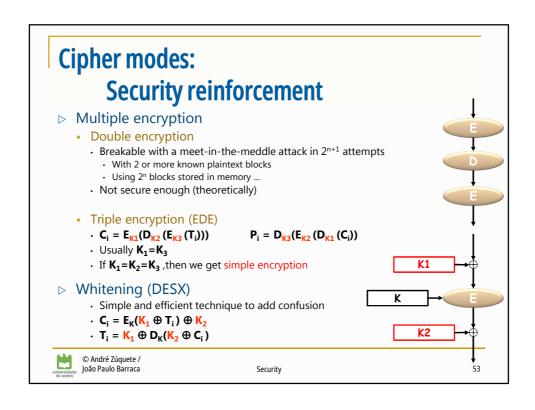


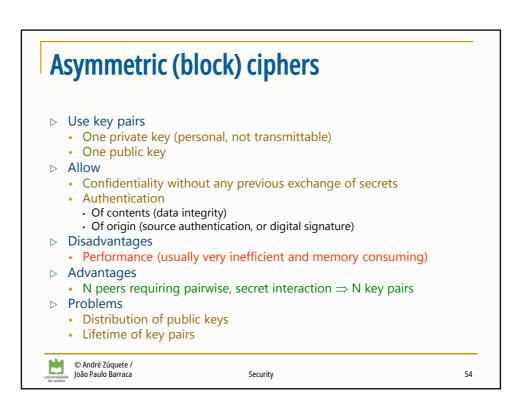


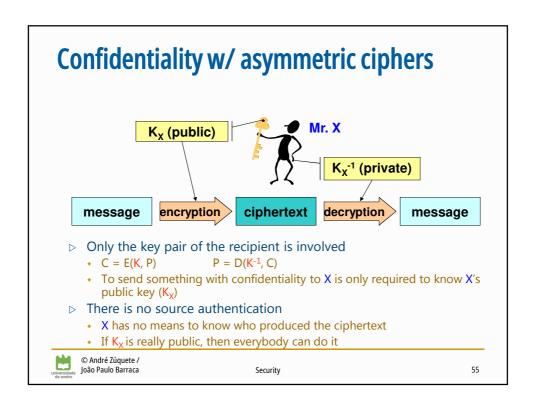


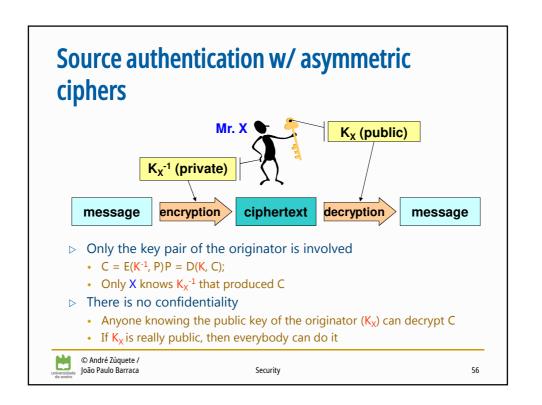


Cipher modes: Pros and					
i i oo ana	Block		Stream		
	E <i>C</i> B	СВС	OFB	CFB	CTR
Input pattern hiding		✓	✓	✓	✓
Confusion on the cipher input		<b>√</b>		<b>√</b>	Secret counter
Same key for different messages	<b>*</b>	<b>~</b>	other IV	other IV	other IV
Tampering difficulty	✓	<b>√</b> ()		✓	
Pre-processing			✓		✓
Parallel processing	1	Decryption Only	w/ pre- processing	Decryption only	<b>✓</b>
Uniform random access					
Error propagation	Same block	Same block Next block		Some bits afterwards	
Capacity to recover from losses	Block Losses	Block Losses		<b>√</b>	



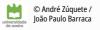




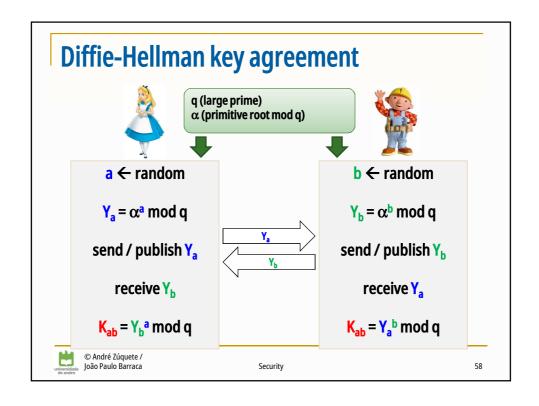


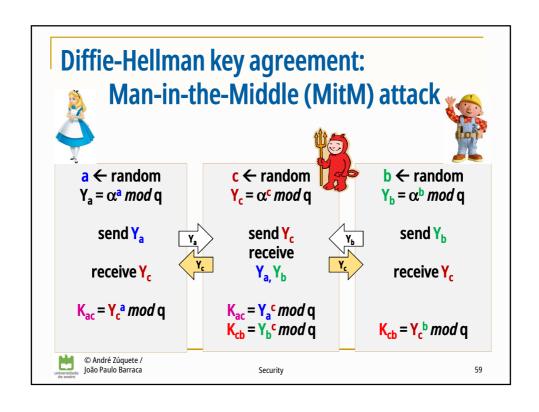
#### Asymmetric (block) ciphers

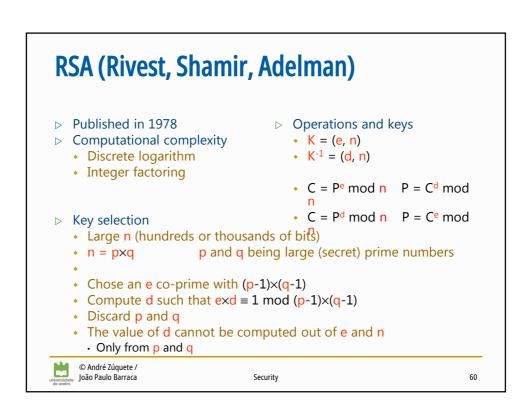
- > Approaches: complex mathematic problems
  - Discrete logarithms of large numbers
  - Integer factorization of large numbers
  - Knapsack problems
- > Most common algorithms
  - RSA
  - ElGamal
  - Elliptic curves (ECC)
- > Other techniques with asymmetric key pairs
  - Diffie-Hellman (key agreement)



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#### RSA: example

```
(small primes)
\triangleright p = 5 q = 11
   • n = p \times q = 55
   • (p-1) \times (q-1) = 40
\triangleright e = 3
   • Co-prime with 40
> d = 27
   • e \times d \equiv 1 \mod 40
P = 26
                                  (note that P, C\in [0, n-1])
   • C = P^e \mod n = 26^3 \mod 55 = 31
   P = C^d \mod n = 31^{27} \mod 55 = 26
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```

#### **ElGamal**

- ▶ Published by El Gamal in 1984
- - · But using only the discrete logarithm complexity
- - DSA (Digital Signature Algorithm)
  - US Digital Signature Standard (DSS)
- ▷ Operations and keys (for signature handling)
  - $\beta = \alpha^x \mod p$  $K = (\beta, \alpha, p)$
  - $k \text{ random}, k \cdot k^{-1} \equiv 1 \mod (p-1)$
  - Signature of M:  $(\gamma, \delta)$   $\gamma = \alpha^k \mod p$   $\delta = k^{-1} (M x\gamma) \mod (p-1)$

 $K^{-1} = (x, \alpha, p)$ 

- Validation of signature over M:  $\beta^{\gamma} \gamma^{\delta} \equiv \alpha^{M} \pmod{p}$
- ▶ Problem
  - Knowing k reveals x out of  $\delta$
  - · k must be randomly generated and remain secret

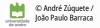


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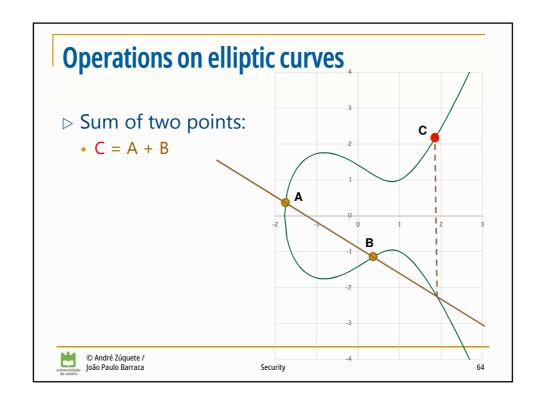
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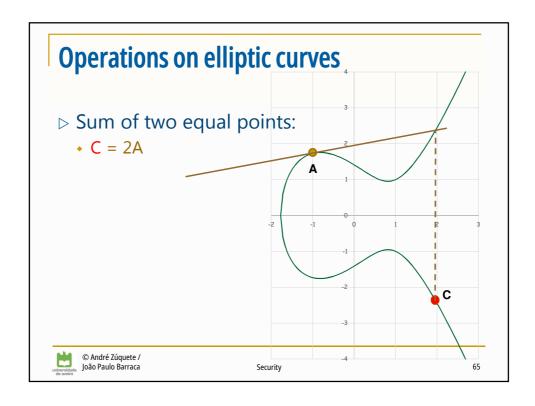
## **Elliptic curve**

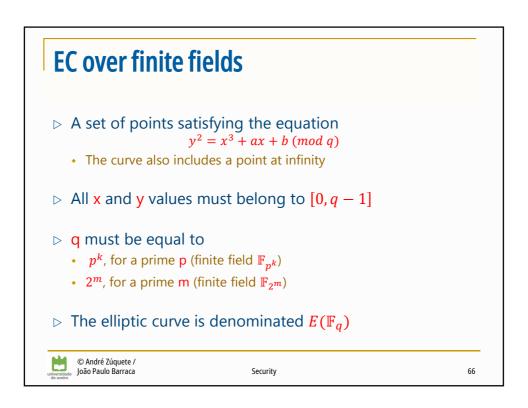
- $\triangleright$  A curve described by an equation  $y^2 + axy + by = x^3 + cx^2 + dx + e$
- Curves of this kind are symmetric to the X axis
  - And don't have solution for all x values

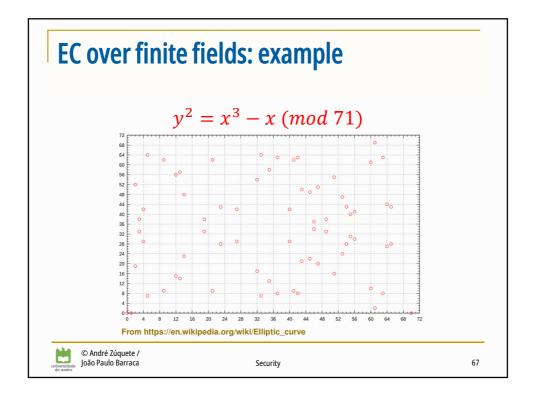


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### **EC discrete logarithm problem**

- ightharpoonup Given an elliptic curve  $E(\mathbb{F}_p)$ ,
  - a point G on that curve,
  - a point P which is an integer multiple of G,

find the integer x such that xG = P

For cryptographic operations, x will be the private key and P the public key



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## EC cryptography (ECC): curves' definition

- $\triangleright$  Prime p  $\rightarrow$  (p, a, b, G, n, h)
  - Constants a and b of the EC equation
  - A generator point (or base point) G
  - The order n of G
    - Normally prime
  - A (small) co-factor h
    - Given by  $\frac{1}{n} \# E(\mathbb{F}_p)$



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## EC Diffie-Hellman (ECDH)

- - (p, a, b, G, n, h)
- $\triangleright$  Alice chooses a random  $\alpha$ 
  - And publishes  $A = \alpha G$
- - And publishes  $B = \beta G$
- - $K = \alpha B$   $K = \beta A$



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#### **Recommended curves**

Length of n (bits)	p (bits)	m (bits)
161 - 223	192	163
224 - 255	224	233
256 - 383	256	283
384 - 511	384	409
≥ 512	521	571

#### NIST, 1999

• 5 P curves over prime fields  $\mathbb{F}_p$ 

$$\cdot y^2 = x^3 - 3x + b$$

• 5 B curves over binary fields  $\mathbb{F}_{2^m}$ 

$$y^2 + xy = x^3 + x^2 + b$$

- b randomly generated
  - SHA-1 hash of a seed
- 5 K (Koblitz) curves over binary fields  $\mathbb{F}_{2^m}$

$$y^2 + xy = x^3 + ax^2 + 1$$



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#### **Recommended curves**

#### □ IETF

• Daniel Bernstein's Curve25519

$$y^2 = x^3 + 486662 x^2 + x \pmod{q}$$

$$\cdot q = 2^{255} - 19$$

Curve448

$$y^2 = x^3 + 15632 x^2 + x \pmod{q}$$

$$\cdot q = 2^{448} - 2^{224} - 1$$



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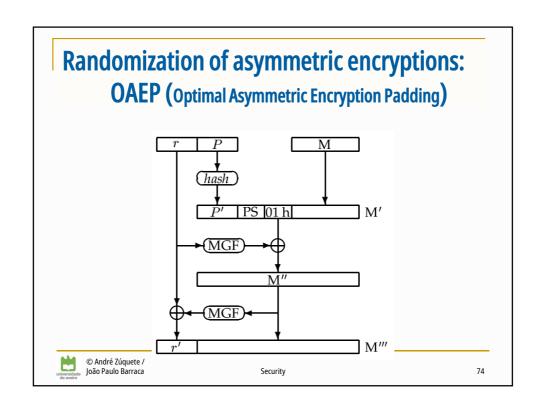
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#### **Randomization of asymmetric encryptions**

- Non-deterministic (unpredictable) result of asymmetric encryptions
  - N encryptions of the same value, with the same key, should yield N different results
  - Goal: prevent the trial & error discovery of encrypted values
- ▶ Technics
  - Concatenation of value to encrypt with two values
    - · A fixed one (for integrity control)
    - A random one (para randomization)

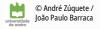


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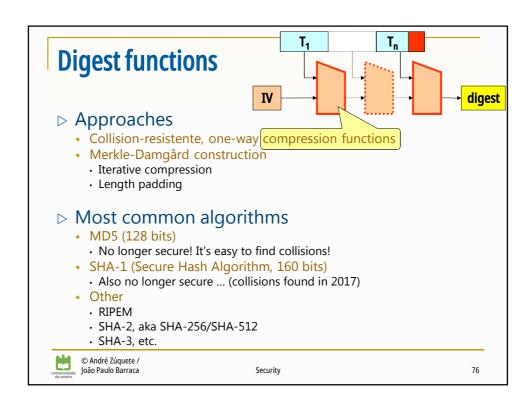


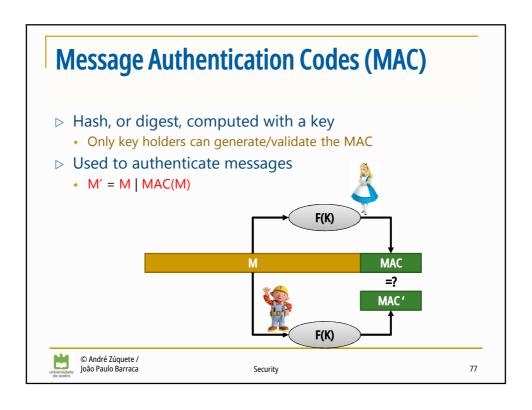
#### **Digest functions**

- - Sort of text "fingerprint"
- ▶ Produce very different values for similar texts
  - Cryptographic one-way hash functions
- Relevant properties:
  - · Preimage resistance
    - · Given a digest, it is infeasible to find an original text producing it
  - 2<sup>nd</sup>-preimage resistance
    - Given a text, it is infeasible to find another one with the same digest
  - Collision resistance
    - It is infeasible to find any two texts with the same digest
    - · Birthday paradox



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## Message Authentication Codes (MAC): Approaches

- - Using, for instance, a symmetric block cipher
- □ Using encryption with feedback & error propagation
  - ANSI X9.9 (or DES-MAC) with DES CBC (64 bits)
- Adding a key to the hashed data
  - Keyed-MD5 (128 bits)
    - MD5(K, keyfill, text, K, MD5fill)
  - HMAC (output length depends on the function H used)
    - H(K, opad, H(K, ipad, text))
    - ipad = 0x36 B times

opad = 0x5C B times

· HMAC-MD5, HMAC-SHA, etc.



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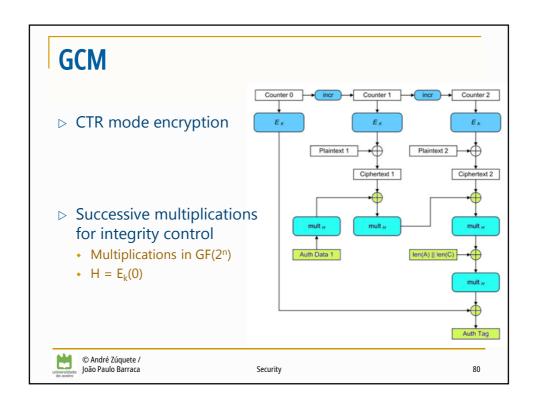
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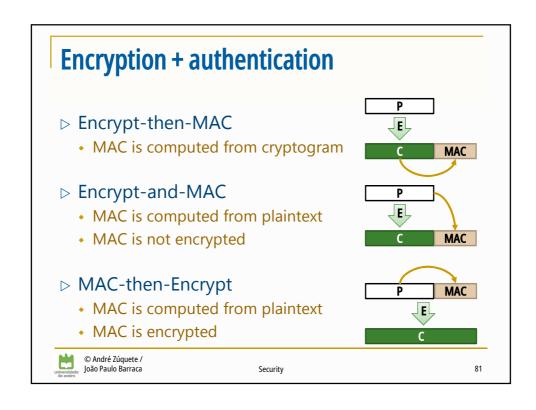
### **Authenticated encryption**

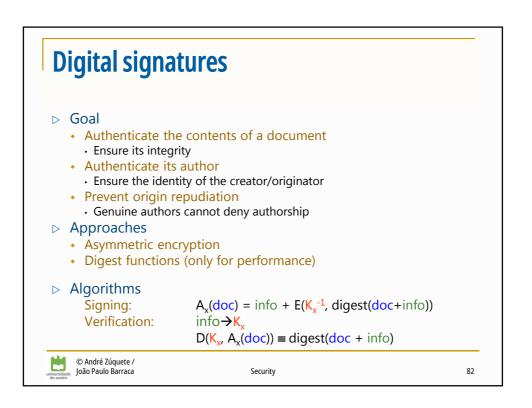
- ▷ Encryption mixed with integrity control
  - Error propagation
  - Authentication tags
- - GCM (Galois/Counter Mode)
  - CCM (Counter with CBC-MAC)

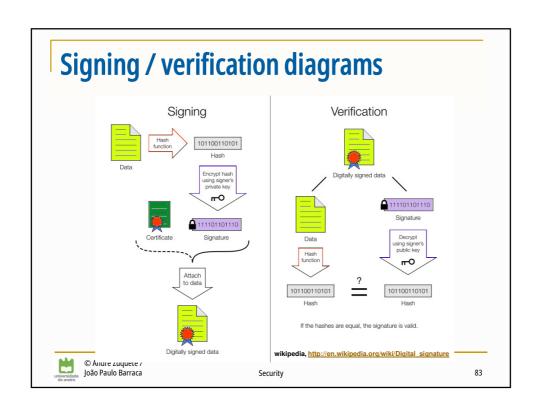


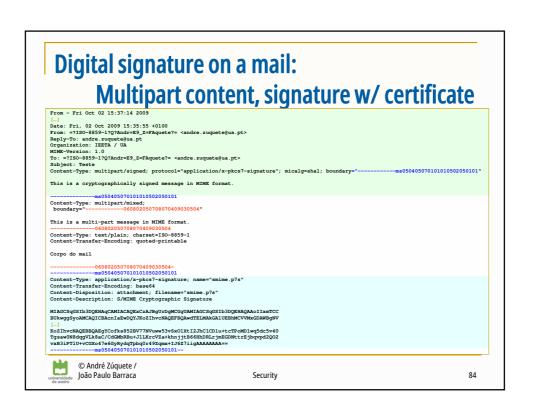
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## **Blind signatures**

- Signatures made by a "blinded" signer
  - Signer cannot observe the signed contents
  - Similar to a handwritten signature on an envelope containing a document and a carbon-copy sheet
- ► They are useful for ensuring anonymity of the signed information holder, while the signed information provides some extra functionality
  - Signer X knows who requires a signature (Y)
  - X signs T<sub>1</sub>, but Y afterwards transforms it into a signature over T<sub>2</sub>
    - Not any T<sub>2</sub>, a specific one linked to T<sub>1</sub>
  - Requester Y can present T<sub>2</sub> signed by X
    - But it cannot change T<sub>2</sub>
    - X cannot link T<sub>2</sub> to the T<sub>1</sub> that it observed when signing



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### **Chaum Blind Signatures**

- - Blinding
    - Random blinding factor K
    - $k \times k^{-1} \equiv 1 \pmod{N}$
    - $\cdot m' = k^e \times m \mod N$
  - Ordinary signature (encryption w/ private key)
    - $\cdot A_x (m') = (m')^d \mod N$
  - Unblinding
    - $\cdot A_x (m) = k^{-1} \times A_x (m') \mod$



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