11 Distortions

"People do what they get paid to do; what they don't get paid to do, they don't do". [William Easterly].

Learning Goals:

- Identify real world examples of market and government-imposed distortions.
- Understand how taxes and subsidies can be manipulated to "get the prices right".
- Understand the intervention dilemmas in a second-best framework.

11.1 Introduction

In a well-functioning economy, each resource is valued in the market according to its contribution to social welfare. In the real life, however, different types of impediments drive wedges between market prices and social returns. These cases are referred to as distortions. Whether government-imposed (e.g., taxes) or inherent to certain markets (e.g., externalities, natural monopolies), distortions cause misallocation of resources keeping the economy below its attainable productivity frontier.

This chapter provides a systematic view of the distortions that interfere with capital accumulation and with the allocation of different inputs to production. For mathematical convenience, the underlying framework is the AK model. This means that efficiency losses caused by distortions will translate into lower *growth*. A similar analysis could be spelled out in terms of the neoclassical model, with level effects. Sticking with the AK model, however, we gain in mathematical simplicity[214].

In Section 11.2 we discuss the case of distortions that affect consumption-saving decisions. In Section 11.3 we address the case where the distortion affects the relative use of two inputs to production. Section 11.4 extends the analysis in the previous section to the case in which a policy to subsidize one input comes at the cost of a tax in other input. Section 11.5 summarizes.

11.2 Distortion in the consumption-investment decisions

This section gives an example, whereby a distortion affecting the relative price of capital leads to a suboptimal rate of capital accumulation. The simpler manner to model a market distortion is to think it as an unjustified tax that introduces a wedge between private returns and social returns. In a broader interpretation, you can interpret this "tax" as capturing the effect of any policy or institution that reduces artificially the return on investment. This includes bureaucratic barriers to investment, licensing, tariffs on imported equipment, financial market frictions, corruption fees on entrepreneurs, etc.

Assume that the aggregate production function takes the AK form:

$$Y_t = AK_t \ , \ \mathrm{A} > 0 \qquad (11.1)$$

Where K denotes for private (excludable, rival) inputs, only[215].

It is also assumed that households are able to smooth consumption, according to:

[214] The chapter draws on Easterly (1993, 2005). Easterly, W. 1993. "How much do distortions affect growth?". Journal of Monetary Economics 32, 187-212. Easterly, W. 2005. National policies and economic growth: a reappraisal. In Aghion, P., and Durlauf, S. (eds), Handbook of Economic Growth, North Holland, Amsterdam.

[215] You may interpret K as including both human and physical capital. As long as the tax rate is uniform across capital inputs, there is no gain in modelling the different types of capital separately. Later in this chapter we will address specifically the case with non-uniform taxation.

$$\gamma = r - \rho \qquad (11.2)$$

Because in this model *K* is a purely private good and consumption-saving decisions are optimal there are no market failures. Thus, there is *no role* for government intervention. The market mechanism delivers the first best outcome and any government attempt to interfere in the price mechanism will be welfare worsening.

Assume now that the government imposes a tax, τ_K, on capital incomes, which proceeds are used for unproductive consumption. Profit maximization by an individual firm i takes the following form:

$$\pi_{it} = AK_{it} - (r_t + \delta)(1 + \tau_K)K_{it} \qquad (11.3)$$

The first order condition of profit maximization is:

$$\frac{\partial \pi_{it}}{\partial K_{it}} = A - (r_t + \delta)(1 + \tau_K) = 0$$

With all firms equal, the economy' interest rate will be:

$$r = \frac{A}{(1 + \tau_K)} - \delta \qquad (11.4)$$

From (11.4), the households' disposable income (Y_d) will be equal to:

$$Y_d = (r + \delta)K = Y/(1 + \tau_K) \qquad (11.5)$$

To see how much the tax rate affects growth, let's first solve the model as if the saving rate was *exogenous*. Because the saving rate applies to Y_d, the equality between savings and gross investment becomes:

$$\frac{sY}{1 + \tau_K} = \dot{K} + \delta K \qquad (11.6)$$

Dividing both members of (11.6) by *K*, rearranging and subtracting *n* on both sides, one obtains the growth rate of capital per worker (and of per capita income), for each level of the saving rate:

$$\gamma = \frac{\dot{K}}{K} - n = \frac{sA}{1 + \tau_K} - (n + \delta) \qquad (11.7)$$

This equation shows that an increase in the tax rate, by reducing the households' disposable income, impacts negatively on capital accumulation and by then, on growth, *for each* level of the saving rate. When the saving rate is endogenous, it will depend on the reward of capital. Because the distortion impacts negatively on capital returns (equation 11.4) consumers will optimally decide to save less.

An interesting implication of (11.7) and (11.8) is that the lower the A, the lower the derivative of growth with respect to the tax rate, τ_K. This means that the distortion is less pervasive when TFP is low. In the words of William Easterly, *bad policies are more likely to be tolerated in low-A countries than in high-A countries*. This, in turn, may help perpetuate the bad policies! This observation has important implications for convergence: if rich countries adopt better policies, then convergence forces like technological diffusion and imitation of best practices might not be enough to induce cross-country economic convergence. If that was the case, the *initial conditions* that shaped the differences in A at the very beginning - natural causes like the quality of arable land and climate should have a key role in explaining the cross-country differences in per capita incomes today.

11.2.1 The case with endogenous savings

Replacing (11.4) in the optimal consumption rule (11.2), one obtains the growth rate of per capita income in the economy with endogenous savings:

$$\gamma = \frac{A}{1+\tau_K} - \rho - \delta \qquad (11.8)$$

Comparing to (11.7), se see that the impact of taxation is much larger in this version of the model. The reason is that, when savings are endogenous, an increase in the tax rate not only decreases the productivity of capital but also the saving rate. To disentangle the two effects, let's equal (11.8) and (11.7) and solve for the (endogenous) saving rate:

$$s = 1 - \frac{\rho - n}{A}\left(1+\tau_K\right) \qquad (11.9)$$

This equation shows that an increase in the tax rate leads households to save less out of their disposable income. Thus, equation (11.8) accounts for two effects: on one hand, by reducing the households' disposable income, τ_K reduces the total amount of savings - and by then investment – for a given saving rate; on the other hand, by reducing the return on investment, τ_K induces a lower saving rate.

It is important to distinguish the circumstances in which, when assessing the growth prospects of a given country, one shall refer to equation (11.7) or instead to equation (11.8). The difference is that (11.8) presumes that the financial system is well developed, so that households are able to transfer income across time through borrowing and lending. This is less likely to be the case wherever financial systems are underdeveloped and households face borrowing constraints. This conjecture reinforces the previous conclusion: bad economic policies (as reflected in lower parameter A) are likely to impact more severely in countries with developed financial systems (with endogenous savings) than in countries with underdeveloped financial systems (where the saving rate is not necessarily responsive to the interest rate).

This discussion adds to the general point that questions like "what happens to per capita income (or to economic growth) when some parameter increases" do not have a unique answer: it depends on specific circumstances.

Box 11.1 Tax on production versus tax on capital

In the model (11.1)-(11-8) we use a tax on capital returns, τ_K. Conclusions do not change if we set a tax on production, τ, instead. In that case, the interest rate will be

$$r = sA(1-\tau)-\delta \quad , \quad (11.4a)$$

And the growth rate in the case with endogenous savings becomes:

$$\gamma = A(1-\tau)-\rho-\delta \qquad (11.8a)$$

As you may easily check, the tax on production equivalent to a given tax on capital returns is:

$$\tau = 1-\frac{1}{1+\tau_K} \qquad (11.10)$$

11.2.2 Example: Financial intermediation and economic growth

When we described the financial system in the flow income chart, we ignored the frictions that underlie the transfer of funds as between savers and borrowers. In the real world, however, financial transactions involve transaction costs, most of which related to information failures. These include costs related to searching and matching lenders and borrowers (brokerage costs), gathering information on the borrowers to evaluate their risk characteristics (evaluation costs), negotiating and designing contracts (negotiation costs), monitoring the implementation of projects (agency costs) and assuring the enforcement of the contract' obligations (enforcement costs). All these costs imply that the gross return paid by the borrower exceeds the net return received by the lender. The same applies to some types of government intervention in the banking system, such as the imposition of high reserve requirements, or to imperfect competition in the banking system.

In term of our model, the costs related to financial intermediation can be captured by τ_K. That is: for each euro saved, only $1/(1+\tau_k)$ translates into acquisition of new capital; the remaining $\tau_k/(1+\tau_k)$ is retained in the financial system. In that interpretation, a less efficient transformation of savings into investment will translate into a lower τ_K. Wherever financial institutions are fragile, poorly regulated, and contract enforcement is low, one will expects the intermediation spreads to be higher, implying more losses in the process of transformation of savings into investment.

Financial development makes investment more attractive for borrowers and savings more attractive for lenders. By allowing risk to be spread across different assets and reallocated from risk averse agents to risk takers, by pulling together small and short term savings of many households and channelling them to large long term loans to finance big projects, by scrutinizing risks, by collecting and releasing relevant information, the financial system helps deliver more favourable risk-return-maturity combinations for savers while creating incentives for entrepreneurs to engage in long-term and riskier projects, which otherwise would not be under consideration[216]. Empirically, there is an extensive evidence showing that financial development is a key determinant of economic performance.

Box 11.2: the research of King and Levine

In a serious of research papers, Robert King and Ross Levine analysed the relationship between financial development and economic performance. In one of these papers[217] the authors assessed whether the level of financial development affects real per capita GDP growth, (2) capital accumulation and (3) productivity growth. This study involved 80 countries over the period 1960-1989.

Table 11.1 reproduces some of the author's findings. In the table, dependent variables are in columns and independent variables are in rows. The later group includes the log of initial per capita GDP (capturing conditional convergence), the log of secondary school enrolment, government consumption divided by GDP, inflation, a measure of trade openness and a measure of financial depth, consisting on the amount of a country liquid liabilities (currency plus demand and interest bearing liabilities of financial intermediaries) divided by GDP. As shown in the table, the three regressions suggest that financial development is a good predictor of growth, capital accumulation and TFP.

Table 11.1: Growth and initial financial depth, 1960-1989

[216] The link between finance and economic growth also runs through the efficiency parameter A: for instance, the function of evaluating and selecting the most profitable projects and of monitoring their implementation tends to enhance the average productivity of capital. Sometimes, governments try to influence banks' decisions. Whenever this translates into the deviation of credit to socially inefficient projects or to loss-making companies, there will be a negative impact on efficiency and thereby on the rate of economic growth.

[217] King, R., Levine, R., 1993. Finance and growth: Schumpeter might be right. Quarterly Journal of Economics, 108 (3), 717-37

	Per capita GDP Growth, 1960-1989	Per Capita Capital Growth, 1960-1989	Per capita Productivity Growth, 1960-1989
Constant	0,035***	0,002	0,034***
	[0,001]	[0,682]	[0,001]
Log(real GDP per person in 1960)	-0,016***	-0,004*	-0,015**
	[0,001]	[0,068]	[0,001]
Log(secondary school enrollment in 1960)	0,013***	0,007***	0,011***
	[0,001]	[0,001]	[0,001]
Government Consumption/GDP in 1960	0,07*	0,049*	0,056*
	[0,051]	[0,064]	[0,076]
Inflation in 1960	0,037	0,02	0,029
	[0,239]	[0,0238]	[0,292]
(Imports plus exports)/GDP in 1960	-0,003	-0,001	-0,003
	[0,604]	[0,767]	[0,603]
DEPTH (Liquid Liabilities in 1960)	0,028***	0,019***	0,022***
	[0,001]	[0,001]	[0,001]
R^2	0,61	0,63	0,58

*significant at the 0,10 level, **signficant at the 0,05 level, ***significant at the 0,01 level.
[p-values in brackets]
Observations = 57
Source: King and Levine (1993a)

11.3 Discriminatory taxation

11.3.1 The efficient allocation

The model in Section 11.2 describes the growth effects of policies that impact in the consumption-saving decisions. By pooling together all forms of capital, however, the analysis misses an important category of distortions: those that affect the relative user cost of different inputs. This section and the section that follows address specifically this case.

Assume that there are two types of private capital: human capital (H) and physical capital (K), More generally, you can interpret K and H as simply referring to different types of capital, such as machinery versus vehicles, imported equipment versus domestically produced, etc. The aggregate production function takes the following form:

$$Y = AK^{\beta}H^{1-\beta} \qquad (11.11)$$

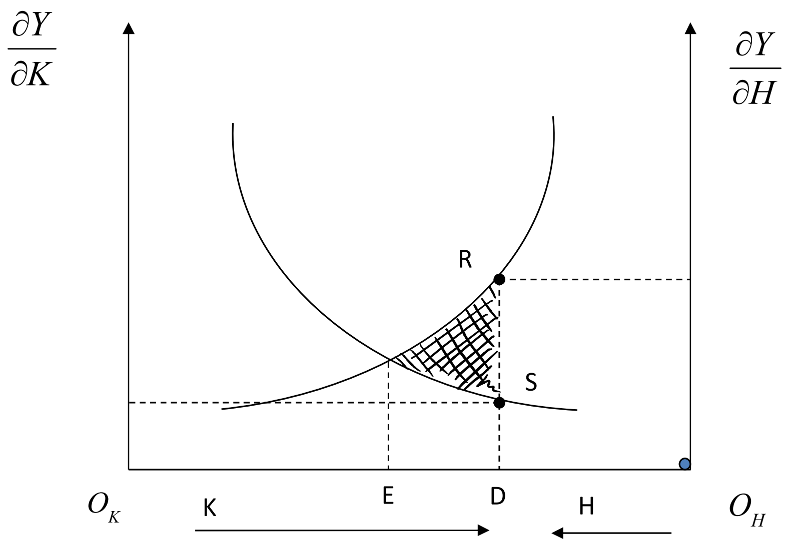
Also assume that one unit of output can be converted at no cost into either one unit of physical capital or into one unit of human capital: that is, the opportunity costs of investing on unit of output in human capital and in physical capital are the same. It is also assumed that the depreciation rates for the two types of capital are the same. These assumptions are not necessarily realistic, but allow us to abstract from other effects.

In a competitive equilibrium with no market failures, profit maximization ensures that human and physical capital are employed so that their marginal products are equal. That is:

$$\frac{H}{K} = \frac{1-\beta}{\beta} \qquad (11.12)$$

Figure 11.1 illustrates this. In the figure, the horizontal axes measures the *total* amount of capital available in the economy at a given moment in time, with the endowment of physical capital being measured from left to right and the endowment of human capital being measured from right to left. The vertical axes measures the marginal products of the two types of capital. The equilibrium described by allocation E corresponds to efficient allocation: if both types of capital cost the same and if their depreciation rates are equal, then profit maximization and absence of arbitrage opportunities will ensure that their marginal products are the same.

Figure 11.1 Marginal products of physical and human capital



In figure 11.1, an allocation like D is inefficient. In allocation D, more physical capital and less human capital are used in comparison to E. Due to diminishing returns, the marginal product of physical capital in D (DS) is lower than the marginal product of human capital (RD). This means that more output could be produced if less physical capital and more human capital were employed in production. The shaded area is a *deadweight loss*.

The implication for economic growth is straightforward. Since in the AK model efficiency has growth effects, allocation E is the one that delivers the maximum possible rate of economic growth[218]. In a context with diminishing returns, a similar conclusion would hold, but in levels.

11.3.2 Distortionary taxes

We now examine how factors affecting the user costs of the two types of capital cause firms to deviate from the socially optimal allocation. We illustrate this assuming that the government taxes differently the returns on physical capital and on human capital. Consider an economy composed by many identical firms, with production functions of the form:

$$Y_i = AK_i^{\beta} H_i^{1-\beta} . \qquad (11.11a)$$

Let τ_K be the tax rate on physical capital incomes and τ_H the tax rate on human capital incomes. These taxes create a wedge between the user cost of capital, paid by firms, and the net worth to households. The individual firm' profits become:

$$\pi_i = Y - (1+\tau_K)(r+\delta)K_i - (1+\tau_H)(r+\delta)H_i \quad . \qquad (11.13)$$

Profit maximization in that case leads to:

$$\frac{\partial Y_i}{\partial K_i} = \beta \frac{Y_i}{K_i} = (r+\delta)(1+\tau_K) \ ; \qquad (11.14)$$

$$\frac{\partial Y_i}{\partial H_i} = (1-\beta)\frac{Y_i}{H_i} = (r+\delta)(1+\tau_H) . \qquad (11.15)$$

[218] To see this, let $\breve{k} = K/H$ denote for the ratio of physical to human capital. The question is to find out the value of $\breve{k}$ that delivers faster growth. To solve this problem, rewrite the production function, as $Y = \left[\breve{k}^{\beta}/(1+\breve{k})\right]\overline{K}$, where $\overline{K} = K + H$ denotes for the total capital in the economy each moment in time. Assuming for a moment an exogenous saving rate, the change in total capital will be given by $\gamma = \dot{\overline{K}}/\overline{K} = sY/\overline{K} - \delta = s\left[\breve{k}^{\beta}/(1+\breve{k})\right] - \delta$. Maximizing this in respect to $\breve{k}$, you will obtain $\breve{k} = \beta/(1-\beta)$.

Due to arbitrage, the *net* rental prices of human and physical capital ought to be equal, implying:

$$\frac{H_i}{K_i} = \frac{H}{K} = \frac{1-\beta}{\beta}\frac{1+\tau_K}{1+\tau_H} \tag{11.16}$$

Equation (11.16) illustrates how discriminatory taxation, affecting the relative use of the two types of capital, impacts on the marginal rate of technical substitution. If both tax rates were zero, the economy would achieve the most efficient allocation, E.

Summing (11.11a) across firms and substituting (11.16), one obtains the aggregate production function as a function of the tax rates:

$$Y_t = A\left(\frac{1+\tau_K}{1+\tau_H}\right)^{1-\beta}\left(\frac{1-\beta}{\beta}\right)^{1-\beta} K_t, \tag{11.17}$$

The novelty in this version of the AK model is that the efficiency component (the average productivity of capital) now depends on two policy parameters, τ_K and τ_H. Solving (11.14) for the interest rate, substituting (11.17) and using the optimal consumption rule (11.2), one obtains the growth rate of per capita income in an economy with endogenous savings:

$$\gamma = \frac{B}{P_I} - \delta - \rho \tag{11.18}$$

where

$$B = A\beta^{\beta}(1-\beta)^{1-\beta} \tag{11.19}$$

$$P_I = (1+\tau_K)^{\beta}(1+\tau_H)^{1-\beta} \ . \tag{11.20}$$

The term P_I is the average user cost of capital.

To interpret equation (11.18), consider first the case in which $\tau_K = 0$ and $\tau_H = 0$. In this case, P_I=1, the marginal products of the two types of capital are equal and the maximum possible growth rate is achieved. Now assume that $\tau_K = \tau_H > 0$, e.g., both taxes are positive and equal. In this case, equation (11.16) becomes equal to (11.12): there is a penalty on investment, but there is no distortion on the *relative use* of the two types of capital. The economy will grow less because of the overall tax on investment (like in 11.8), but there is no efficiency loss in the use of the two types of capital. The economy is still operating in point

E. Finally, when taxation is not uniform (say, $\tau_H > 0$ and $\tau_K = 0$), there will be distortions affecting both the relative price of capital P_I, and the relative price of the two types of capital. In terms of Figure 11.1, the economy will be operating in an allocation different from E.

Box 11.3: Other Distortions

In the real world, different factors may alter the relative user cost of different inputs to firms, preventing an economy from operating in a point like E. Among others, taxes, tariffs, import quotas, price controls, interest rate controls, dual exchange rates, employment protection rules, nominal rigidities, corruption fees and imperfect competition. The following sections provide some examples.

- Import protection: In many developing countries, import tariffs are used to raise government revenues. In terms of the model outlined above, one may examine the effects of import protection interpreting τ_K and τ_H as tariffs, instead as of taxes. Suppose that *H* refers to imported equipment while *K* refers to locally produced equipment. If a tariff is levied on the imported capital, this will lead to a distortion in the relative use of the two types of capital. The same applies to non-tariff import protection mechanisms, such as import quotas, licenses, and technical requirements. All these instruments will cause the relative price of inputs to depart from the competitive equilibrium, causing the economy to move away from E.
- High inflation: Inflation, by affecting nominal variables but not real variables, impacts differently in the user costs of different inputs. In terms of our model, let *K* denote for physical capital, and *H* for firms' working capital. In most activities, w*orking capital* is an essential input to production, consisting in cash, short-term financial instruments, and credit to customers. High inflation, , drives a wedge between the costs of holding working capital

(except inventories) relative to, for instance, equipment, which secondary market price goes along with inflation. In response, firm will tend to operate with less working capital than in the efficient allocation[219].

- Monopoly: a common source of efficiency-loss is imperfect competition: whenever one side of the market is characterized by a small number of players, these may have the ability to influence prices. In terms of our model, suppose that that the supply of human capital was unionized and that the only concern of the union was to maximize its monopsony rents. In that case, the cost of using human capital would rise relative to that of physical capital, inducing a departure from point[220].

- Segmented labour markets: In many economies, there are barriers to labour mobility that prevent otherwise identical workers get paid equally. Sources of labour market segmentation include differences in the institutional nature of the employer (public vs. private), restrictive labour laws (high severance payments, restrictive conditions for dismissals), dual productive structures (modern vs. traditional, formal vs. informal), barriers to geographical mobility (housing rents, high licensing costs) or the rational for employees to pay wages above the competitive level (efficiency wages). All these factors may prevent real wages to be equalized across workers with similar qualifications. In terms of hour model, these frictions imply different user costs in using different inputs (in this case, two types of human capital), causing again a departure from allocation E.

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[219] On the negative relationship between inflation and growth, see Barro (1995, 1997), Sarrel (1996), Gylfason (1998). Bruno and Easterly (1998).

[220] Comparing to (11.16), we see that a monopolized market for input H is equivalent to a subsidy to physical capital accumulation. This suggests that the government can use taxes and subsidies to remove the distortion.

11.3.3 Self-financed subsidies

In light of (11.20), while taxes on capital are harmful to growth, subsidies to capital lead to faster growth. A question might be raised then: should the government give large subsidies to capital accumulation, so as to induce faster growth? To answer this question, one needs to take into account the fact that subsidies must be financed. Since lump-sum taxes are not, in general, available, the positive effect on growth due to a higher subsidy on a type of capital must be balanced against the negative effects of distortionary taxation anywhere else.

To analyse this problem in the context of our model, suppose that the government launches a subsidy to human capital accumulation financed with the proceeds of a tax on physical capital. For a moment, assume that there are no government services (G=0). The government budget constraint is:

$$(r+\delta)(\tau_H H+\tau_K K)=0 \qquad (11.21)$$

The main point here is that, when the subsidy to human capital is financed with the proceeds of taxation on physical capital, P_I rises unambiguously. The reason is the policy alters the relative price of the two inputs, according to (11.16), giving rise to substitution effects that shrink the tax base and enlarge the subsidy base. Thus, when the subsidy rate increases, this requires a more than one-for-one increase in the tax rate in order to keep the budget balance unchanged. Thus, the price of capital rises unambiguously, and the rate of economic growth is lower[221] [222].

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[221] Mathematically, replacing (11.21) in (11.20) and using (11.16), we get $P_I=(1+\tau_H)\left[\beta/(\beta+\tau_H)\right]^{\beta}$. This price is minimum when $\tau_H=0$.

[222] Even if lump-sum taxation was available, a question would arise as to why should the government try to maximize the growth rate of the economy: if there were no distortions and private agents optimally decide their consumption paths, there would be no point in trying to modify the competitive growth rate. Policies that artificially increase the growth rate of the economy are in this context detrimental to welfare and shall be compared to immisering growth (Easterly, 2005).

Box 11.4: Examples of Tax-cum-Subsidy schemes

In the real world, there are many mechanisms of implicit transfers that act as tax-cum subsidy schemes, that are detrimental to economic growth. This box provides some examples:

- P*rice controls:* in many countries, governments set maximum price ceilings in some goods, to promote their consumption. Such policy calls for budgetary transfers so as to compensate firms from the implied losses. If taxes on other goods are levied to finance the policy, this is equivalent to a tax-cum-subsidy scheme that impacts negatively on efficiency and therefore on growth.
- Ceiling in banks' lending rates: sometimes, central banks impose maximum interest rates on bank loans, giving rise to excess demand in credit markets. In response, parallel markets of finance tend to develop, at very high interest rates. In the official market, the scarce credit is often allocated administratively, favouring "priority sectors" or state-owned enterprises. also induces. All in all, the segmentation of the credit market acts as a tax-cum subsidy scheme: some types of borrowers are implicitly subsidized at the cost of other borrowers.
- *Dual exchange rates*. In many developing countries, central banks set an official exchange rate, but fail to enforce it in the market, buying or selling foreign exchange. In these cases, the currency is said to be unconvertible. Inconvertible currencies are typically characterized by a dual exchange rate system, with an official exchange rate available to public companies or other priority bodies, and a black market exchange rate for the rest of the economy. Those who are allowed to import goods at the official exchange rate receive an implicit subsidy, while those forced to pay the black market rate pay an implicit tax. In terms of the model, this is equivalent to a tax-cum subsidy scheme.

11.3.4 Tax evasion

A major problem in government finance is tax evasion. When a significant share of economic activity is informal and doesn't pay taxes, the burden of financing government activities falls on a narrower base, giving rise to unfair competition, wrong incentives, and an implicit transfer from those who pay taxes to those who benefit with the public good without paying taxes. This phenomenon is particularly pervasive in developing countries.

To analyse this, assume that a non-excludable input G is essential to production:

$$A = \left(\frac{G}{Y}\right)^{\eta} \qquad (11.22)$$

Substituting (11.22) in (11.11s), aggregating up and rearranging, one obtains the economy' production function in terms of the three inputs:

$$Y = G^{\frac{\eta}{1+\eta}} K^{\frac{\beta}{1+\eta}} H^{\frac{1-\beta}{1+\eta}} \qquad (11.23)$$

This equation reveals that the *actual* contributions of physical capital and human capital to production are less than what individual firms *perceive* to be, in (11.11a).

The question now is what combination of τ_K and τ_H shall the government use to finance the provision of government services? With public provision, the government budget constraint becomes:

$$(r+\delta)(\tau_H H + \tau_K K) = G \qquad (11.24)$$

The rest of the model is equal to that in section 11.3.2: profit maximization at the firm level leads to (11.14) and (11.15) and the growth rate of the economy is just as described by (11.18). The only difference is that now the parameter A (inside B) is related to public provision, according to (11.22). Using (11.14) and (11.15), the government balanced budget constraints simplifies to:

$$\frac{G}{Y} = \frac{\beta\tau_K}{1+\tau_K} + \frac{(1-\beta)\tau_H}{1+\tau_H} \qquad (11.25)$$

Substituting this in (11.22), one obtains the expression for A in terms of the two tax rates:

$$A = \left(\frac{\beta\tau_K}{1+\tau_K} + \frac{(1-\beta)\tau_H}{1+\tau_H}\right)^{\eta} \qquad (11.26)$$

The growth rate of this economy is given by (11.18), with the only difference that A shall now be replaced by (11.26).

A benevolent planner in this economy would choose τ_K and τ_H so as to maximize the growth rate of per capita income, as given by (11.18), taking into account that the parameter *A* (inside B) is determined according to (11.26). As usual, this problem is solved setting the partial derivatives $\partial\gamma/\partial\tau_K$ and $\partial\gamma/\partial\tau_H$ equal to zero. The algebra of the exercise is tedious, but the solution should be, according to our previous findings, intuitive: from (11.14), the share of capital returns (net of taxes) in income is $K(r+\delta)/Y = \beta/(1+\tau_K)$. Thus, the tax rate that makes this share equal to the contribution of capital to production in (11.23) is $\tau_K = \eta$. Using the same reasoning for human capital, the first best solution will be:

$$\tau_K = \tau_H = \eta \qquad (11.27)$$

With no surprise, the first best policy is the one that sets a *uniform tax rate* (this, in turn, is equivalent to a tax on production, as specified in Chapter 10). Substituting the optimal taxation rule (11.27) in (11.25), one obtains the corresponding (optimal) level of public provision:

$$\frac{G}{Y} = \frac{\eta}{1+\eta} \qquad (11.28)$$

Note that this corresponds to the contribution of government services to production, as captured by equation (11.23).

To analyse the implications of tax evasion in this model, let *K* denote for the *formal* sector (the one that pays taxes) and *H* denote for the *informal* sector (the one that does not pay taxes). To find the benevolent planner solution in this case, we must impose $\tau_H = 0$ in (11.18) and (11.26) and maximize the growth rate of the economy in respect to τ_K, only. This leads to:

$$\tau_K = \frac{\eta}{\beta} > \eta \qquad (11.29)$$

Comparing to (11.27), we see that the tax rate on physical capital is now higher than in the first best: since the government cannot tax human capital, it sets a higher tax rate on physical capital. Also note that, the lower the β, that is, the smaller the size of the formal sector, the higher the tax rate must be.

Substituting (11.29) in (11.25), we get an interesting result:

$$\frac{G}{Y}=\frac{\eta\beta}{\beta+\eta}<\frac{\eta}{1+\eta} \qquad (11.30)$$

This equation states that the optimal provision of government services under tax evasion is lower than the contribution of government services to production, as stated in (11.23). The reason is intuitive: because rising revenues forces the government to impose a distortion in the factor markets, a *benevolent planner will balance the benefits of providing one extra unit of public input against the extra cost resulting from a further move away from E*. Equation (11.30) shows that the optimal balance between these two effects translates into a lower provision of government services than in the case where uniform taxation is available. The public provision deviates from the first best, and the more, the lower the contribution of the formal sector to production (β)[223].

All in all, under tax evasion, the second best solution will imply *those in the formal sector paying very high tax rates and the economy as whole enjoying a very lower level of government services*. No wonder why economies with large informal sectors find it difficult to attract foreign direct investment!

Box 11.5. Second-best decision-making

The case with optimal provision of the public input under tax evasion illustrates a problem of second-best decision-making. The Theory of Second-Best concerns with the optimal choice by the planner when the range of available instruments is somehow restricted. In a second-best framework, *eliminating one distortion does not necessarily leads to higher efficiency*. The reason is that the alleviation of one distortion may impact negatively on other

[223] The Cobb-Douglas production function implicitly postulates a unit elasticity of substitution between *H* and *K*. But you could assume instead a CES production function, with a very high elasticity of substitution between the two inputs. In that case, the effects just described would be amplified: the size of the formal sector would shrink even more, the tax rates would be set even higher and the level of public provision would be even lower.

distortions that cannot be removed. Thus, the optimal policy shall involve a balance between the benefits of alleviating one distortion against the costs of aggravating other distortions.

In the example of tax evasion, providing more of the public input implies taxing further the formal sector, exacerbating the distortion in the factor markets. If informality cannot be eliminated (that would be the first best), the second best policy (as summarized in 11.30) involves a balance between the benefits of providing the economy with more of the public input with the cost of further distorting the factor markets.

Box 11.6. Growth diagnostics

The theory of second-best tells us that a policy reform that looks efficiency-enhancing when considered in isolation may end up being counterproductive, due to adverse interaction effects with other distortions. Since in the real world, policymakers do not have the complete knowledge of all prevailing distortions in an economy nor a precise quantification of all possible adverse second-best interactions, the strategy of simply tackling any distortion may reveal unsuccessful. Under this reasoning, Ricardo Hausmann, Dani Rodrik and Andres Velasco proposed a minimalistic approach to economic reform, that they labelled as "growth diagnostics"[224]. In a complex world, with many distortions and complex interaction effects between them, finding out the appropriate sequence of reforms is very difficult. The authors argue that, instead of trying to tackle all distortions at the same time ("laundry list approach to economic reform"), policymakers should carefully identify the one or two most important "binding constraints" in a given economy and tackle these constraints.

To help identify the relevant binding constraints, the authors propose a diagnostic analysis, using a conceptual model that can be interpreted in light of the simple AK model. According to that framework, economic activity in a given developing country may be constrained by: (a) Low A (low return on private investment) (b) High τ_K (high cost of

[224] "Getting the Diagnosis Right". Hausmann, R., Rodrik, D., Velasco, A., 2006. Finance and Development 43(1), March 2006.

finance). When the main problem is a low A (in which case the interest rate is expected to be low and capital is expected to be flowing out), this should be related to low social returns (low human capital, poor infrastructure, bad geography) or to a large gap between social and private returns (distortionary taxation, government failures). When the main problem is instead a high cost of finance, then the country should exhibit high interest rates, and the distortions should be related to inefficiencies in the financial market, such as low enforcement of loan contracts, restrictions to capital inflows, or imperfect competition on domestic banking. Using this conceptual model, policymakers should identify the one or two most binding constraints to economic growth and adjust the policy to tackle this small number of constraints, only.

11.3.5 Externalities

In the examples above, discriminatory taxation was inefficient because the equilibrium without taxes was efficient. In this section, we argue that, in the presence of market failures, discriminatory taxation may not only be desirable, it may also be the optimal solution.

Assume that there is a positive externality associated to Human Capital:

$$A = C\left(\frac{H}{Y}\right)^{\varepsilon} \qquad (11.31)$$

Equation (11.31) implicitly assumes that the externality is subject to congestion: as the economy gets larger, human capital must increase proportionally to generate the same external effect. Summing the production function (11.11a) across firms and substituting (11.31), one obtains the aggregate production function for this economy:

$$Y_t = BK_t^{1-\alpha} H_t^{\alpha} \qquad (11.32)$$

where $B = C^{\frac{1}{1+\varepsilon}}$ and $1-\alpha = \dfrac{\beta}{1+\varepsilon}$ (11.33).

Comparing (11.11a) and (11.32) we see that there is again a divergence between the *perceived* contributions (betas) and the *actual* contributions (alfas) of physical and human capital to production. Since *1−α<β*, this means that the *actual* contribution of physical capital to output is lower than that *perceived* by firms. In turn, the elasticity of human capital, α, is larger than that perceived by firms, $1-\beta$.

From (11.32), we know that the *first best* resource allocation (point E) is:

$$\frac{H}{K}=\frac{\alpha}{1-\alpha} \tag{11.34}$$

From (11.16), we know that a careful choice of taxes and subsidies can be used to push the economy towards (11.34). Setting (11.16) equal to (11.34), the following rule is obtained:

$$\frac{1+\tau_K}{1+\tau_H}=1+\frac{\varepsilon}{1-\beta} \tag{11.35}$$

If the two tax rates are set according to (11.35), this means that private firms, following their maximization problem – that is, deciding according to (11.16) – will be induced to choose a relative employment of H and K satisfying the efficiency condition (11.34). For the policy to be self-financed, it must satisfy condition (11.21). Using (11.21), (11.34) and (11.35), one obtains the first best solution $\tau_K^*=\varepsilon$ and $\tau_H^*=-\varepsilon\beta/(1-\beta+\varepsilon)$. This means that a now unique combination of the *two* instruments satisfies the *two* targets: the correction of the externality and a balanced budget.

11.4 Policies and growth

This chapter reviewed different types of market or government-imposed distortions that impact negatively on economic performance. The chapter also illustrates how government policies can be used to "get the prices right". It is suggested that a careful intervention using taxes and subsidies has the potential to improve economic efficiency and drive the economy to a faster growth path (or, in alternative, to a higher level of per capita income, assuming diminishing returns to capital).

The view that policy reforms should address market failures and deadweight triangles in the first place has been at the core of the policy prescriptions adopted by the main international institutions, since the 1990s. In practice, however, the experience has revealed that similar policies may lead to different results, depending on other factors, including the cultural, institutional, social and political environment (Box 11.7).

With no question, tackling economic efficiency and getting the prices right is a key ingredient to economic success. However, one should not overstate this principle. Saying that a country is poor because economic policies are wrong, and that the solution is simply too implement market oriented reforms may be too short-seeing. At a deeper level, one shall ask why policymakers in developing countries do not implement better economic policies. In the economic debate, it has been recognized that, although good policies are critical for economic performance, implementing the right policies may not be within reach for many developing countries, because they lack the necessary institutions.

Institutions set the framework in which policy makers implement their policies. If institutions are poor and allow policymakers to deviate from the public interest, most probably, policies will deviate from the public interest. The claim that the quality of institutions determined the quality of economic policies has been supported by extensive evidence in cross-country growth regressions showing that variables measuring the quality of polices (such as trade openness, inflation, etc) tend to lose significance when the quality of institutions is accounted for (an example in Box 5.3). This is not to say that policies are not important: simply, because good institutions tend to generate good policies, in cross-country regressions variables capturing the quality of institutions also capture the quality of economic policies in general.

Box 11.7 The Washington Consensus

The view that policy reforms should address market failures and deadweight triangles in the first place was coined "The Washington Consensus" by John Williamson, in a conference held at the Institute for International Economics, in 1990. In that conference, the author listed 10 policy reforms that synthetized the view: (1) fiscal discipline; (2) reorientation of public expenditures from non-merit subsidies to basic health care, education and infrastructure; (3) tax reform, so as to enlarge the tax base and reduce marginal tax rates; (4) liberalization of interest rates; (5) unified and competitive exchange rates; (6) trade liberalization; (7) openness to inward foreign direct investment; (8) privatisation; (9) deregulation, to ease barriers to entry and exit; (10) secure property rights.

The Washington consensus constituted a departure from the thinking in the 1950s and the 1960s, according to which economic development was a complex process of economic, social, political and historical transformation, requiring a specific diagnostics for each particular country. Along the 1990s, the policy prescription of the Washington Consensus was implemented in many developing countries, namely in Latin America, in Sub-Saharan Africa, and in former socialist republics. By the turn of the century, many developing countries had achieved more open economies, sounder public finance, lower inflation, fewer restrictions on private business, and more efficient financial sectors.

In practice, the experience with the implementation of the Washington consensus revealed different results. In Latin America, for instance, growth rates in the 1990s were on average lower than along 1960-1980, a period, characterized by extensive state intervention and import substitution policies. At the same time, countries like Thailand, Malaysia, China, Vietnam and India were experimenting fast growth, despite their insistence on industrial policies. China and India, in particular, made significant market-based reforms, but maintained high levels of state intervention, import restrictions and targeted industrial policies. All in all, some countries that followed the Washington Consensus didn't achieve faster growth, while some countries following less orthodox approaches achieved fast improvements in their living standards. Moreover, similar reforms delivered different growth performances in different countries.

In 2005 the World Bank launched a study, *Economic Growth in the 1990s: Learning from a Decade of Reform,* where it is explicitly recognised that no one single prescription shall be viewed as applying to all countries at the same time. The document argues that, while good policies are in general important for growth, their effects may be offset or reinforced by other factors, including the cultural, institutional, social and political environment[225]. The report concludes that a correct assessment of each country particular circumstances is essential, in order to define the optimal sequencing of reforms.

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[225] The recognition that sound policies work better if embedded in strong institutions had been already materialized in 2001, with a new list of priority reforms that became known as the Monterrey Consensus. This list complemented the original (the so-called *first generation reforms*) with a number of "action points" addressing issues such as governance, corruption and human rights (*second generation reforms*).

Box 11.8. Easterly: Policies can destroy growth!

To illustrate the role of policies in economic growth, William Easterly (2005) estimated a number of cross-country growth regressions, using a panel of 5-year averages along 1960-2000. The author considered 6 variables capturing distinct dimensions of policy (details in Table 11.3): the inflation rate (INFL), the government deficit (BB), a measure of real exchange rate overvaluation (LREALOVR), the black market premium on foreign exchange (LBMP), a measure of financial development (M2/GDP) and a measure of trade openness (TRADE). The author then regressed the growth rate of GDP per capita (five years average, 1960-2000) in all these six variables.

Table 11.3 describes Easterly' findings. Column (1) presents the estimated coefficients of a simple regression where the 6 dependent variables are included. All signs are in accordance to the theory and most coefficients are significant[226].

At the first sight, the results in Column (1) suggest that improving the quality of economic policies may have a significant impact on growth. To quantify this, Easterly computed the implied effects on growth resulting from a one standard deviation improvement in each of the policy variables. These effects are displayed in Column (2); the one standard deviation changes in the policy variables are displayed in Column (4). For instance, a reduction in the inflation rate by one standard deviation (that is, by 32 p.p, as showed in Column 4) would augment per capita income growth by 0.6 p.p (Column 2). According to this table, if all variables were improved at the same time, the overall impact on per capita GDP would be as much as 3p.p. This finding suggests that getting the policies right is good for growth.

Easterly pointed out, however, that such an exercise has enormous limitations. First, he observed that a one standard-deviation change in any of these six policy variables (Column 4) is outside the experience of most countries: reducing the inflation rate by 32 p.p,

[226] M2 and TRADE perform less well than the other variables, but the author shows that they become significant once any one of the other five variables is dropped out of the regression equation.

improving the budget balance by 5 p.p, improving the M2/GDP ratio in 25 pp, etc, are certainly not easy to achieve for most countries. The reason, he argues, is that these large standard deviations are related to the presence of very *extreme* observations (extreme inflation, extremely high deficits and extremely high black market premiums "on the bad side", extremely high monetization ratios and extreme degree of openness "in the good side"). These extreme observations influence critically the statistical significance of the policy variables in growth regressions.

To abstract from the presence of these outliers, the author run the regression again, restricting the sample to observations where all six policies variables lie in the range of "moderate" policies, that is, removing the extreme values form the sample (this reduces the sample to roughly one half). Column 3 of Table 11.3 displays the corresponding results. Excluding observations where *any* of the six policy variables are extreme, all policy variables become insignificant!

Table 11.3 – Easterly regressions on polices and growth

Regressions of per capita growth on basic set of 6 policy variables.
Dependent variable : LGDPG (log per capita growth, five year averages, 1960-2000)

	(1)	(2)	(3)	(4)
	Coefficient in growth regression	Change in growth from one standard deviation change in policy (%)	Sample: Moderate Policies, only	Memo: Improvement of one standard deviation in policy variable
INFL	-0.018	0.6	-0.064	-0.325
	(2.61)**		-1.23	
BB	0.092	0.5	0.018	0.054
	(2.81)**		0.22	
M2	0.01	0.3	-0.004	0.253
	1.37		0.27	
LREALOVR	-0.014	0.5	0.001	-0.387
	(2.97)**		0.06	
LBMP	-0.012	0.7	-0.038	-0.558
	(2.33)*		-0.95	
TRADE	0.01	0.5	0.01	0.454
	1.92		1.09	
Constant	0.016		0.027	
	(3.62)**		(2.52)*	
Observations	422		193	
R-squared	0.18		0.03	

Notes: Robust *t* statistics in parentheses. *Significant at 5%; **Significant at 1%.

(3) Restrictions under moderate policies: INFL between -0.05 and 0.3, BB between -0.12 and 0.02, M2<1.0, LREALOVR between -0.5 and 0.5, trade < 1.20, LBMP between -0.05 and 0.3

Variables used: LGGDP: Log per capita growth rate; INFL: Log (1+ inflation rate); BB: Government budget balance/GDP; M2: M2/GDP; LREALOVR: Log (overvaluation index/100), above zero indicates overvaluation; LBMP: log(1+black market premium on foreign exchange; TRADE: (exports+imports)/GDP.

Source: Easterly (2005)

This finding is quite suggestive. It suggests that the results in column (1) are mainly driven by extreme values. In other words, the results in column (1) may be reflecting mainly the potential for destruction of bad policies. Thus, countries with moderate values of these variables are not expected to obtain large gains by achieving moderate improvements in their policies. Based on this, Easterly concluded that: "Although extremely bad policy can probably destroy any chance of growth, it does not follow that good macroeconomic or trade policy alone can create the conditions for high steady state growth" (pp. 1017).

Key ideas of chapter 11

- A market distortion can be thought as a tax that drives a wedge between private returns and social returns, reducing the level of market activity.
- A first type of distortions analysed in the chapter are those that influence the consumption-savings decision. By affecting the marginal product of capital, these distortions impact on the reward of investment and thereby on saving rates, causing capital accumulation to slow down.
- An example of such type of distortion relates to financial markets. Financial market frictions give rise to transaction costs that arise in the form of an intermediation margin, creating a gap between the marginal product of capital paid by firms and the net return actually received by borrower. The larger this intermediation margin, the lower the return on savings. Financial development, by reducing the costs of intermediation, impacts positively on capital accumulation and economic performance.
- A second type of distortions analysed in this chapter are those that alter the relative price of two inputs, inducing firms to change the proportions in which they are used. Examples of this type of distortion occur with import protection, segmented labour markets, high inflation, monopoly and externalities.
- Some policies act as a subsidy to particular sectors that come as a cost to other sectors, with a negative overall impact on growth. Examples of these "tax-cum-subsidy" schemes include interest rate ceilings, directed banking credits, and dual exchange rates.
- In the presence of tax evasion, there is an incentive for firms to use more of the type of capital that escapes taxation. When this is so, the optimal intervention will involve a lower provision of the public input. This case provides an illustration of second best decision-making.
- The view that reform agendas should focus on market failures and deadweight losses was well reflected in the Washington Consensus. The mixed experience with these market oriented reforms lead economies to call for more attention on the quality of institutions.

Problems and Exercises

Key concepts

- *Distortion. Tax cum subsidy schemes. Second best decision-making. The Washington consensus*

Exercises

11.1. (**Tax on production**) Consider an economy, where the production function is given by Y=0,2K, the population grows at 1% per year and physical capital depreciates at 4%. (a) Assume for the moment that the saving rate is constant and equal to 30%. Describe the main equations of this model and find out the growth rate of per capita income in this economy. Discuss, with the help of a graph the dynamic properties of this model. (b) Consider now that the government imposes a tax on production, which proceedings are used to finance unproductive government consumption. Describe the main income identities of this economy and place them in a flow income chart. Find out the growth rate of per capita income in this economy when: τ=0%; and τ=20%. Explain. (c) Now assume that the saving rate is endogenous, so as to satisfy the following inter-temporal consumption rule: $\gamma_t = r_t - 0{,}15$ (c1) Explain this equation. (c2) Departing from the identity $\left[\dot{K} + \delta K\right] = s(1-\tau)Y$, compute the endogenous saving rate in this economy, as a function of the tax rate. Explain. (c3) Compute again the growth rates of per capita income when τ=0%; and τ=20%.

11.2. (**Tax on Capital income**) Consider an economy where consumers do not face borrowing constraints and where the production function is given by Y=0,1K. In this economy, the government imposes a tax on capital incomes, τ_K, which proceedings are used for government consumption. Assume also that δ=2.5%. The population in this economy is constant. For the moment consider that the saving rate is exogenous and equal to 25%. (a) Describe the main income identities of this economy and place then in a flow income chart. (b) Find out the growth rate of per capita income in this economy when: τ_K τ_K =0% and τ_K =25%. Explain. *(Answer: γ_0 = 0; γ_1 = -0.005) (c)* Consider now that households are able to optimize their intertemporal consumption, implying $\gamma_t = r_t - \rho$, where ρ = 7.5%. Find out the growth rate of per capita income when τ_K =0% and τ_K =25%. Explain. *(Answer: γ_0 = 0; γ_1 = -0.02) (d)* Solve for the endogenous saving rate.

11.3. (**Tax cum Subsidy Scheme**): Consider an economy composed by a large number of identical firms, with production functions of the form: $Y_i = 0.25K_i^{1/2}H_i^{1/2}$. In this economy, households are able to optimize their intertemporal consumption, implying $\gamma_t = r_t - \rho$, where ρ = 5%. Population is constant and the depreciation of both physical and human capital is 5%. (a) Find out the growth rate of per capita income in this economy, assuming perfect competition. (b) Now, assume that the government launched a subsidy to the returns of human capital amounting to $\tau_{H=-0.25}$, to be financed with a tax on capital returns, to $\tau_{K.}$ (b1) From the firm' optimization problem, find out the implications for the use of the two types of capital. Explain, with the help of a graph. (b2) Find out the balanced-budget tax on capital incomes. (b3) Quantify the impact of the policy on the growth rate of the economy. (c) Beyond the scope of this exercise, shall we conclude that, in general, the optimal policy involved uniform taxation? Why not?

11.4. (**Tax evasion**) Consider an economy composed by a large number of identical firms, with production functions of the form: $Y_i = AK_i^{1/3}H_i^{2/3}$. In this economy, the government has the ability to coerce citizens to pay taxes out of their factor incomes. We also know that the saving rate is 12,5%, population is constant and the depreciation of both physical and human capital is 4%. Consider the following optimal consumption rule γ = r – 0,05. (a) (**distortionary taxation**) Solve the individual firm problem and find out the implied factor income shares. Describe, with the help of a graph, the effects of taxation on the relative use of the two types of capital. (b) Find out the growth rate of per capita income in this economy, depending on the tax rates. (c) Now assume that $A = (G/Y)^{1/2}$, where G are public inputs. Compute the aggregate production function of this economy. Explain the market failure. (d) Obtain an expression for A in terms of the two tax rates. (e) Compute the benevolent planner solution. Graph the equilibrium and explain. (f) Now assume that the government could not tax Human Capital. Find out the optimal tax on physical capital and the corresponding provision of public inputs. Compare with the first best outcome and explain.

11.5. In Unevenland, the aggregate production function can be described as $Y = EK$, where E denotes for aggregate efficiency and K denotes for physical capital. The later depreciates at the rate δ=0,05. Capital markets are perfect and there is no uncertainty, so households can smooth consumption inter-temporally, according to $\gamma = r - 0{,}1$. (a) (**AK**) Find out the growth rate of per capita income in this economy in terms of *E*. Describe the dynamic properties of the model. (b) (**Cost of capital**) Elaborating a bit more, suppose that the efficiency term is better described as a ratio of two terms, $E = 0.5A/P_I$, where A is constant and P_I denotes for the relative price of capital goods. Which policies or country circumstances may be captured by P_I ? Assume that initially $A = 1/2$. Compute the growth rates of per capita income in this economy when $P_I = (1/2)^{1/2}$ and when $P_I = (3/2)$. With the help of a graph, compare with the impact of a similar change in the context of the Solow model. (c) (**Public good**) The economy of Unevenland is actually more complex than at the first sight. In particular, each individual firm *i* faces a production function of the form: $Y_i = AK_i^{1/2}H_i^{1/2}$, where *H* is human capital and $A = (G/Y)^{1/2}$, where G denotes for a non-excludable good. The government budget constraint is given by $G = (1/2)[\tau_K/(1+\tau_K) + \tau_H/(1+\tau_H)]Y$, where τ_K is the tax rate on physical capital incomes and τ_H the tax rate on human capital incomes. P_I becomes $P_I = [(1+\tau_K)(1+\tau_H)]^{1/2}$. Compute the aggregate production function and explain why there is a market failure. (d) (**Tax evasion**) The benevolent planner of Unevenland chooses the intervention level G/Y so as to maximize the growth rate of per capita income or - which is the same - the efficiency term, *E*. For the moment, however, assume that the government can only levy taxes on physical capital τ_K (that is, $\tau_H = 0$). Find out the optimal level of τ_K and the corresponding level of government intervention.(e) **Optimal taxation**) Now assume that the government is able to impose a uniform tax rate (that is, $\tau_H = \tau_k$). Find out the optimal level of τ_K and the corresponding level of government intervention. Explain the effect on P_I and compare this case to the one in question c. Compare the solutions of d and e and discuss.

11.6. (**Externality**). Consider an economy composed by a large number of identical firms, with production functions of the form: $Y_i = AK_i^{1/3}H_i^{2/3}$. We also know that the saving rate is 12,5%, population is constant and the depreciation of both physical and human capital is 4%. Assume that there was a positive externality associated to the use of Physical capital, according to: $A = \left(K/Y\right)^{\varepsilon}$. Compute the aggregate production function of this economy. Explain the market failure. Suppose that the government want to solve the market failure imposing a tax on human capital, with a balanced budget, so it would use the tax proceeds to finance a subsidy on human capital. What would be the optimal intervention?

11.7. (**Monopoly**) Consider an economy composed by many identical firms, with production functions in the final good sector of the form: $Y_i = 0.5K_i^{1/3}H_i^{2/3}$, where K and H are physical and human capital. We also know that one unit of output can be transformed in one unit of physical capital or in one unit of human capital and that the depreciation rate for both types of capital is 3%. Consider that each firm *i* in the final good sector is a price taker in factor markets. Let P_K be the price of physical capital and P_H be the price of human capital. (a) From the profit maximization problem of firms in the final good sector, find out the optimal relation between H and K as a function of factor prices. (b) Now consider the case in which both factor markets are competitive, that is, each firm producing physical and human capital is price taker. Describe the profit maximization problem of a representative firm producing (human or physical) capital, find out the optimal price and the corresponding profits. (c) Now assume that the supply of physical capital is undertaken by a monopolist. Solve its profit maximization problem, finding out its optimal price and the corresponding profits. (d) Compare the relative employment of physical and human capital in the case of perfect competition and in the case of a monopoly. Compare it with the efficient allocation. € Using the optimal consumption rule γ = r – p, compare the growth rates of this economy in the two cases. (f) How could the government remove this distortion?

11.8. (**Financial market imperfections**) Consider an economy where aggregate output is produced using two types of capital, according to $Y = AK_1^{1/2}K_2^{1/2}$. The total capital available each moment in time evolves according to $sY = \left[\dot{K} + \delta K\right](1 + \tau_K)$, where τ_K refers to financial market imperfections. (a) Interpret the equation describing capital accumulation (b) Draw the income flow chart of this economy. (c) Show that, as long as the ratio of the two types of capital is constant, the production function as an AK representation. (d) Find out the ratio K_1/K_2 that maximizes the efficiency level in this economy. Interpret. (e) Suppose that investment decisions for each type of capital were undertaken by two independent firms, which take the other' firm decision as given. Show that, as long as both firms have access to credit at the same interest rate, the maximum efficiency will be achieved. (f) Describe the growth rate of this economy as a function of the exogenous parameters. Interpret.