6. The real exchange rate in the long run

Index:

6.1 Introduction

A key relative price in open economies is the real exchange rate. The real exchange rate is an index that compares the cost of buying a given basket of goods in the home economy to that of buying the same basket of goods abroad. A real exchange rate depreciation means that foreign goods are becoming more expensive relative to domestic goods. In the short run, the real exchange rate may drift up or down, for instance because of price stickiness. In the long run, the real exchange rate is expected to approach some *equilibrium level*.

In this handout we discuss the determinants of the equilibrium real exchange rate. In doing so, we focus on the long run, restricting the analysis to the case where the economy is in external balance. We start out with the theory of Purchasing Power Parity (PPP), that basically assumes that the equilibrium real exchange rate is constant over time. Although the PPP hypothesis is useful to describe the behaviour of the real exchange rate in many circumstances, it is incomplete as a theory in that it does not explain why costs of living differ across countries. It also does not account for the possibility of relative costs of living not being constant. To address these limitations, we introduce the Tradable-Non-Tradable goods (TNT) model[1]. The TNT model steps into the real-world fact that not all sectors within an economy are equally exposed to international competition. In light of the TNT model, cross-country differences in the cost of living can be explained by cross-country differences in productivity.

This note is organized as follows: in Section 2, we discuss the simpler theory of the real exchange rate, the purchasing power parity. In Section 3, we introduce the TNT model, to show the relationship between the real exchange rate and productivity. In section 4, we

[1] The model born out of the pioneer ideas of Meade (1956), Salter (1959) and Swan (1960) [Meade, J. 1956. The price mechanism and the Australian balance of payments. Economic Record 32, 239-56. Salter, W. 1959. "Internal and External Balance: The Role of Price and Expenditure Effects". Economic Record 35: 226-38. Swan, T. 1960. "Economic Control in a Dependent Economy." Economic Record 36: 51-66].

augment our basic TNT model with the demand side, to analyse how the pattern of production changes with the composition of aggregate demand, and with terms of trade. In Section 5, we depart from the assumption of constant returns to assume that the production possibilities frontier is concave, In that case, shifts in aggregate demand impact on the real exchange rate. Section 6 summarises the main ideas.

6.2 The purchasing power theory

6.2.1 The real exchange rate

The real exchange rate is an index that compares the price of a given basket of goods abroad and at home, with both being expressed in the same currency. A simple definition is as follows:

$$\theta = \frac{eP^*}{P} \qquad (1)$$

where θ is the real exchange rate, P stands for the consumer price index (CPI) at home, P^* stands for the consumer price index abroad, and e is the nominal exchange rate (i.e, the price of foreign currency in units of domestic currency - a higher e means "depreciation"). When θ increases, this means that foreign goods are becoming relatively more expensive. In this case, we say that the (home) real exchange rate is depreciating.

Because the real exchange rate is a *real variable*, its long run level should be independent of the units in which nominal magnitudes are measured. In the long run - that is, after all nominal prices adjust to their equilibrium levels - the real exchange rate is expected to be determined by real factors only, such as technology and preferences. In the short run, price stickiness may cause the actual real exchange rate to depart from the one that is expected to hold in the long run.

6.2.2 Actual versus equilibrium

As any other relative price, the real exchange rate reacts to economic shocks and may depart, in the short run, from the level that is implied by economic fundamentals. These departures are called *real exchange rate gaps*.

To define a real exchange rate gap, one needs to have a reference for what the "equilibrium" should be. In what follows we distinguish two concepts of real exchange rate:

- The *actual* real exchange rate, θ: the one that holds each moment in time;
- The *fundamental equilibrium* real exchange rate, $\tilde{\theta}$: the one that should hold in the long run.

Exchange rate gaps refer to the difference between the actual exchange rate and the fundamental equilibrium real exchange rate, $\theta/\tilde{\theta}$. Deviations between the actual real exchange rate and the equilibrium real exchange rate measure the extent to which the currency is overvalued or undervalued relative to its long run level.

6.2.3 The Law of one price

In a frictionless economy, similar goods must be priced the same everywhere. If the same good was quoted in different locations at different prices, arbitrage opportunities would arise: profits could be made buying the good where the price was lower and selling it where the price was higher, until prices were equal everywhere.

The tendency for prices of similar goods to evolve together is known as the *Law of One Price*. Considering a particular good i, the Law of one Price can be expressed as follows:

$$P_i = eP_i^* \qquad (2)$$

where P_i and P_i^* denote, respectively, the domestic price of good i in units of domestic currency, and the foreign price of good i in units of foreign currency.

In the real world the Law of one Price may not hold instantaneously. For instance, following a nominal exchange rate depreciation, a period of time may take place, during which prices in the two countries are different (departure from 2). During this period, entrepreneurs engage in a profitable trade. Buying and selling in different locations involves searching costs, the establishment of contacts with local retail traders, shipment, and other factors that slow down the arbitrage movement. This means that temporary deviations from the Law of One Price are very likely.

In what follows, temporary deviations from the law of one price will be captured by a a parameter, χ:

$$P_i = eP_i^*(1+\chi) \qquad (3)$$

As time goes by, the parameter χ is expected to approach the value of zero[2].

6.2.4 Permanent deviations for the LOOP

The parameter χ captures short-term frictions that prevent the law of one price to hold each moment in time. These frictions are likely to vanish overtime. A different question relates to structural factors that prevent the law of one price from holding, even in the long run. For instance, transport costs, tariffs, market segmentation and cultural factors are permanent in nature. Since these impediments do not disappear in the long run, they give rise to permanent deviations from the Law of one Price.

To model this, assume that *ad valoren* tariffs are imposed domestically and abroad to a given product i. Denoting for τ and τ^* the tariffs set by the domestic government and the foreign government, respectively, absence of arbitrage opportunities in this case would materialize as:

$$P_i \leq (1+\tau)eP_i^* \qquad (4)$$

$$P_i(1+\tau^*) \geq eP_i^* \qquad (5)$$

The first condition applies to imports: whenever the foreign price plus the import tariff exceeds the domestic price, importers are priced out in the domestic market; the second condition applies to exports: whenever the domestic price plus the foreign tariff exceeds the foreign price, domestic exporters are priced out in the foreign market.

Using (4) and (5), we see that, with the tariffs, the law of one price holds only within a band:

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[2] Note that short-term deviations from the Law of one Price tend to be more problematic for the home economy in case $\chi > 0$, than when $\chi < 0$. The reasons is that downward adjustments in prices are more difficult to achieve than upward movements.

$$\frac{1}{1+\tau} \leq \frac{eP_i^*}{P_i} \leq 1+\tau^* \qquad (6)$$

Whenever relative prices depart from this band, there will be profit opportunities: then, import and export movements will press the relative price back to inside the band. Within the band, prices in both countries are free to drift up and down, without facing the threat of competing imports. You may interpret parameters τ and τ^* as capturing other barriers to international trade, such as quantitative restrictions, red tape and transportation costs.

In sum, due to different types of transaction costs, the law of one price cannot hold exactly for most goods, even in the long run. Still, as long as these costs are not prohibitive, there will be a long-run constraint in the form of a band, preventing prices in different locations from departing *too much* from each other. Only in the limiting case with no transaction costs the law of one price will hold exactly.

6.2.5 Absolute PPP

The macroeconomic counterpart of the Law of One Price is the theory of Purchasing Power Parity (PPP). The difference between the LOOP and PPP is that the former applies to singular goods, while the later applies to economy-wide price indexes.

The theory of Purchasing Power Parity, in its *absolute version*, states that costs of living should be the same in different locations. That is:

$$P = eP^*, \qquad (7)$$

where P stands for the consumer price index (CPI) in the home country and P^* stands for the consumer price index abroad. In light of this theory, the equilibrium real exchange rate should be equal to one:

$$\tilde{\theta} = 1 \quad . \qquad (8)$$

To interpret this, suppose that the consumer basket was the same everywhere – that is, households in different countries were consuming the same goods exactly in the same proportions. In the absence of transport costs, information failures, and other barriers to trade, international competition should ensure the verification of the law of one price for each good that basket. In that case, one would expect consumer *baskets* to cost the same everywhere.

In reality, however, there are many reasons to believe that condition (8) does not, in general, hold. First, because consumer patterns differ across countries: people in different locations spend different shares of their income in different goods and therefore the different good' prices enter with different weights in the corresponding price indexes. Thus, even if all prices were the same in different countries, consumer price indexes would not be the same[3]. Second, tariffs, transportation costs and other impediments prevent the Law of One Price from holding exactly for each good, even in the long run. In practice, the "absolute" version of PPP is rejected by the real data.

Box 1: PPP exchange rates and international comparisons of income

In the real World, costs of living differ significantly across countries. To measure cross-country differences in the cost of leaving, statistical entities compute the so-called "PPP exchange rates".

PPP exchange rates are computed as follows. First, at the product level, PPP exchange rates are obtained by dividing the price of a product at home in domestic currency units by the price of the same product abroad in foreign currency units. For instance, if a litre of milk

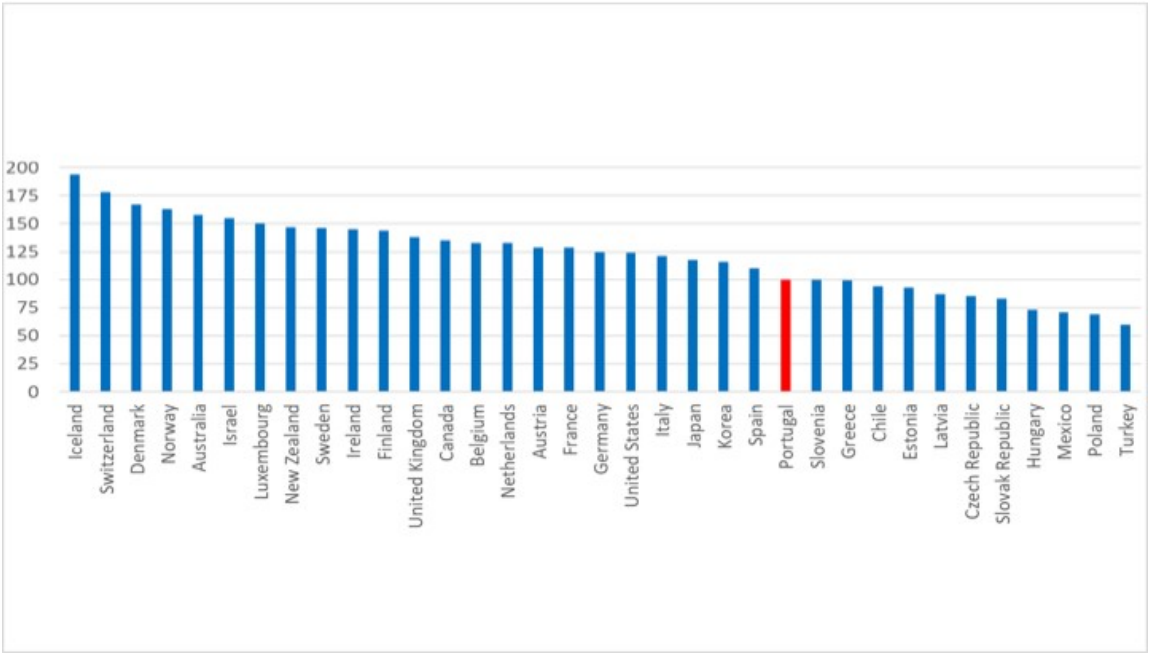
[3] See exercise 1 for a numerical example.

costs 3 euros in Portugal and CHF6 in Switzerland, then the PPP of milk between Portugal and the Switzerland is 3/6, or 0.5 euros per Swiss franc. PPP exchange rates are then obtained aggregating product-level PPPs along comparable consumer baskets. For instance, one may find that a basket of goods that costs 100 euros in Portugal would cost 220 CHF if purchased in Switzerland. In that case, the PPP exchange rate would be PPP=100/225=0.45 euros per CHF.

Differences in costs of living may then be assessed comparing the market exchange rate and the PPP exchange rate. For instance, if the market exchange rate was 0.8 euros per CHF, and the PPP exchange rate was 0.45, then the relative price level in comparable currency units would be $\theta = 0.8/0.45 = 0.8 * 225/100 = 1.76$. That would mean that the cost of living in Switzerland was 76% higher than in Portugal.

Figure 1 describes the differences in price levels calculated in March 2018, by the OECD. As shown in the figure, the most expensive country in this sample was Iceland, with the comparable basket costing twice the price of the same basket in Portugal. In contrast, the cheapest country in the sample was Turkey, with the comparable basket costing 40% less than in Portugal.

Figure 1 – Comparative price levels as of March 2018 (Portugal=100)



Source: OECD, http://stats.oecd.org/Index.aspx?DataSetCode=CPL

The fact that costs of living differ so much across countries complicates the cross country comparison of standards of living. For instance, in 2015, per capita GDP in the United States was around USD 56 thousand. In Uganda, the corresponding figure using the market exchange rate was USD 600. At the first sight, that would mean that the average citizen in the US was 93 times richer than the average citizen in Uganda. Such comparison fails, however, for not taking into account the fact that the cost of living is much lower in Uganda than it is in the United States. According to the IMF, a basket of goods that costs 100 dollars in the US costs 30 dollars in Uganda, only. This means that, in PPP units, per capita GDP in Uganda was equal to 600/0.3=2000. In other words, taking into account the differences in costs of living, the purchasing power of an average citizen in the United States was 28 times higher than that of an average citizen in Uganda. This is still a huge difference, but not as much as the one computed using market exchange rates.

6.2.6 Relative PPP

A less stringent version of the Purchasing Power Parity Theory is its relative version. The *relative PPP theory* only requires cross-country differences in costs of living to remain constant over time.

Suppose, for instance, that the price of a given basket of goods has been 30% more expensive in the United Kingdom than in Portugal. In light of the "Relative PPP theory" one would contend that the equilibrium real exchange rate of UK vis-à-vis Portugal should be $\tilde{\theta} = 1.3$. Hence, if the price level in the UK increased today, for instance, by 10% in pounds, then the return to parity would require a 10% increase in the price level in Portugal, or a 10% appreciation of the euro vis-à-vis the British pound.

Taking differences in (1), the relative PPP hypothesis implies that:

$$\hat{e} + \pi^* - \pi = 0, \qquad (9)$$

where $\hat{e} = \Delta e/e$, $\pi^* = \Delta P^*/P^*$, and $\pi = \Delta P/P$ denote, respectively, the percentage change in the nominal exchange rate, the percentage change in the foreign CPI (foreign inflation), and the percentage change in the domestic CPI (domestic inflation).

Clearly, the relative PPP assumption is more realistic than its absolute counterpart: it accounts for the fact that some countries are systematically more expensive than others. However, the relative PPP theory fails by postulating that differences in costs of living shall

remain constant in the long run. In practice, we observe that some real exchange rates are not trendless, even in the long run (the case of Japan, discussed in Box 2, illustrates this). This means that the relative PPP assumption does not provide a satisfactory theory for the real exchange rate.

A more general theory should explain, (a) why some countries are more expensive than others, and (b) why differences in costs of living sometimes change in a permanent manner. These questions are addressed in the following sections. For the moment, just hold with a key idea: in general, the relative PPP provides a reasonable benchmark for forecasting the real exchange rate when disturbances are nominal in nature: since the real exchange rate is a real variable, in the long run it should not be affected by monetary shocks. Thus, changes in prices should be transmitted to nominal exchange rates and vice-versa. Yet when shocks affecting the economy are real - such as tariffs, productivity or in preferences - the long run real exchange rate is expected to be affected. In that case, the PPP theory fails, even in its relative formulation.

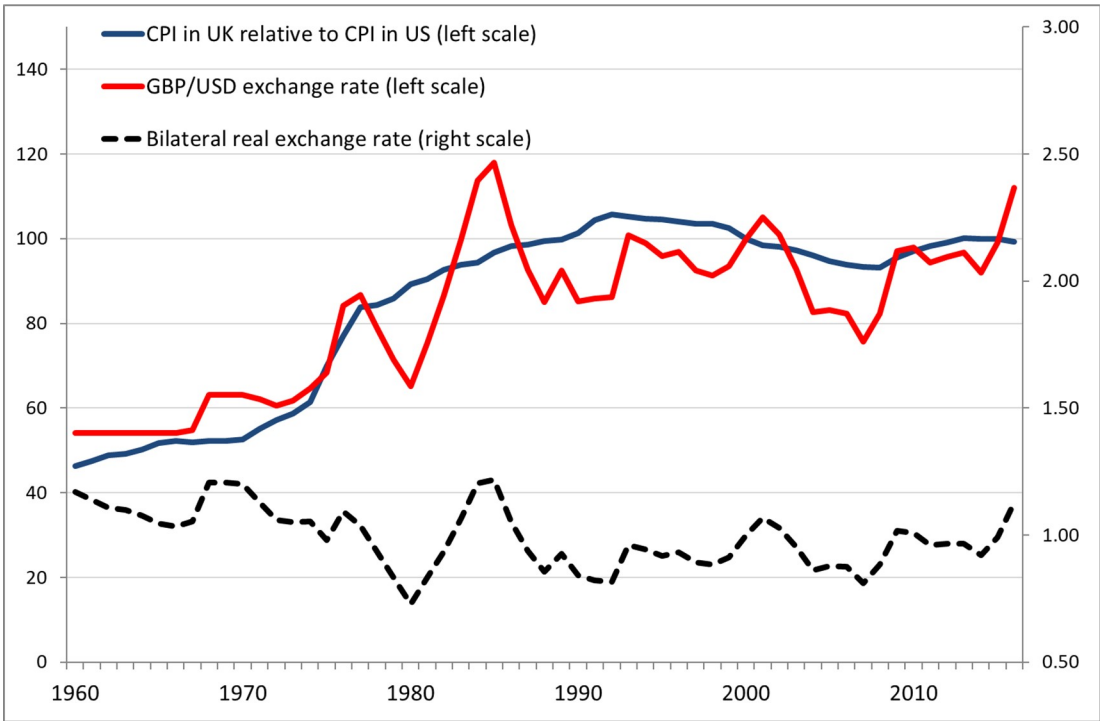
Box 2: Bilateral real exchange rates and the relative PPP hypothesis

To illustrate the PPP hypothesis with real world data, we refer to Figure 2. The figure displays the time change of three variables: the pound sterling-USD nominal exchange rate (e), the ratio of consumer price indexes in the UK and in the US (P/P^*), and the ratio between the two (the bilateral real exchange rate, $\theta = eP^*/P$).

The nominal exchange rate between the pound and the dollar (red line) was fixed along 1960-1971 (with two episodes of exchange rate realignment, in 1967 and 1968), After 1971, the sterling entered in a float and the exchange rate exhibits high volatility. The relative price level (blue line), in turn, evolved slowly over time, reflecting the short-run stickiness that characterizes prices in general. In the figure, we see that the two series (relative prices and nominal exchange rate) have evolved basically together in the long run, supporting the

relative PPP hypothesis. True, in the short run, price stickiness and other frictions prevented the two series from exactly matching each other each moment in time. Hence, in the short run, the real exchange rate has drifted up and down[4]. In the long run, however, the real exchange rate between the two countries has been more or less trendless, supporting the relative PPP hypothesis for these two countries, in this particular time spam.

Figure 2– Nominal exchange rate and relative CPI between UK and the United States, 1960-2016



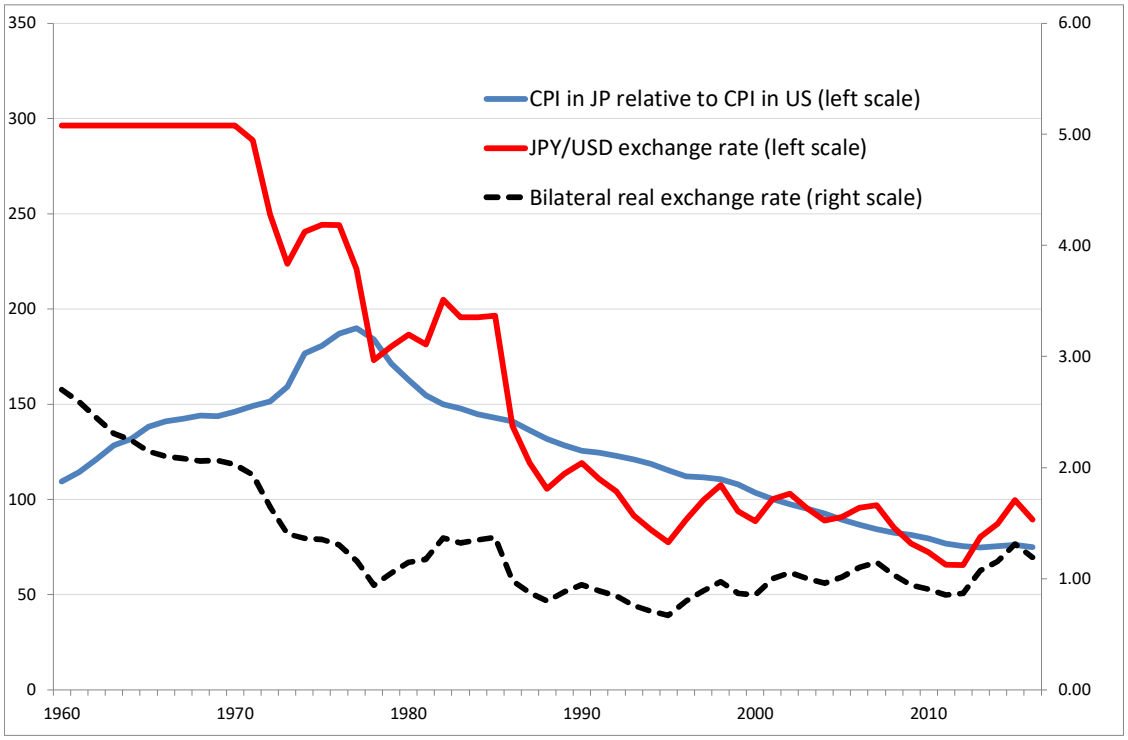
Source: AMECO.

[4] Frankel and Rose found that temporary deviations from PPP, such as those implied by volatile nominal exchange rates, tend to die away slowly over time, with half of the departure from PPP still remaining four years after the shock [Frankel, J., Rose, A., 1996. A panel project on Purchasing Power Parity: mean reversion within and between countries". Journal of International Economics 40, 209-224].

Figure 3 repeats the exercise, focusing on the bilateral real exchange rate between Japan and the US. In the figure, we see that during the period until 1971, when the bilateral nominal exchange rate remained fixed, consumer price inflation in Japan was much higher than that in the United States. Hence, before the collapse of the Bretton Woods system in 1971, the yean had already appreciated significantly in real terms, relative to the US dollar. When the bilateral exchange rate started floating, one could expect the yen to depreciate, to compensate for the accumulated inflation differential. However, this did not happen. Actually, the yen appreciated in nominal terms relative to the USD, at a time where inflation in Japan was still higher than that in the US. This means that the yen kept appreciating in real terms throughout the mid-1970s. After the mid-1970s, CPI inflation in Japan became lower than in the US, but this effect was offset by the nominal appreciation of the yen, implying that the bilateral real exchange rate remained roughly constant.

Thus, if one tested the relative PPP hypothesis between Japan in the United States using a series starting in the 1980s, eventually that assumption would not be rejected by the data. But if one started the sample period in 1960, rejection was certain: along the 1960's and 1970's, the yen appreciated significantly in real terms relative to the USD, meaning that Japan became more expensive relative to the US than it was before. Since this appreciation was not reverted in the decades that followed, we conjecture that it was a sustainable (equilibrium) phenomenon.

Figure 3 – Nominal exchange rate and relative CPI between Japan and the United States, 1960-2016



Source: AMECO.

6.3 Productivity and the real exchange rate

We argues that the absolute PPP hypothesis fails because price levels differ across countries and differences in price levels do not necessarily vanish over time. One reason why price levels do not converge in the long run was already advanced in the section above: barriers to trade such as tariffs and transportation costs prevent some goods to have the same price in different locations. The model that follows explores a limiting case along this avenue, by considering two types of goods: a tradable good, assumed perfectly mobile across countries and hence obeying to the law of one price, and a non-tradable good, with prohibitive transaction costs, and therefore not subject to international competition.

6.3.1 Traded and non-traded goods

The main reason why the relative PPP theory fails in the long run is the presence of non-traded goods. A non-traded good is one that can only be sold in the same economy where it is produced. Think, for example, in ordering a pizza: you could order a pizza from a foreign country, but probably it would be too expensive and most surely the pizza would be useless at the time of arrival. The same happens with many other goods, such as vehicle repairs, retail, personal services, residential housing, and entertainment. Either because of prohibitive transaction costs or because of physical impediments, these goods cannot in practice be imported from abroad or sold in foreign markets. Since producers of these goods are isolated from foreign competition, they are able to set the corresponding prices based on domestic considerations, only.

In contrast, a traded good is one that can be consumed in an economy other than where it was produced. Traded goods are goods that can be imported or exported. Examples of traded goods include agricultural commodities, fish, minerals, manufactures and some services, such as shipping. Because these goods are subject to international competition, their prices cannot deviate too much from the prices of similar goods abroad.

In practice, the distinction between a traded and non-traded good is not always obvious. Consider, for instance, the act of drinking a beer in a bar. A bottle of beer is clearly a good that can be traded internationally. Nevertheless, the action of drinking a beer in a bar, which comprises the whole atmosphere of the bar, is a non-traded good. Thus, the non-traded service provided by the bar includes a traded component, which is the beer itself.

Some goods will be traded or non-traded, depending on geographical, technological, cultural and political circumstances. For instance, goods with high transportation costs (including those with high weight as compared to value, such as cement, and those subject to fast deterioration, such as fresh vegetables) are more likely to be traded within a territory close to where they are produced. On the other hand, legal impediments to trade, such as tariffs, trade quotas and quality standards, can turn potential traded goods into de facto non-traded. One may say that the incidence of these barriers to trade in each particular product determines its degree of "tradability".

Technology also determines which goods can be traded internationally or not. In today's world, many services are moving from the non-traded category to traded, due to technological advances in telecommunications. An example is banking: today, you can

manage an account in a bank located in a foreign country as easily as you can manage an account held in a bank at home. The same applies to many other services, such as consultancy, training, and call centres. Technological progress is expanding the range of goods that are subject to international competition.

6.3.2 Non-traded goods and the real exchange rate

Consider an economy producing and consuming only two goods. A good that can be traded internationally (T) and a good that can only be consumed in the economy where it was produced (N). The Consumer Price Index is a weighted average of traded and non-traded good prices. Denoting by α the share of the traded good in domestic expenditure, the Consumer Price Index (P) becomes:

$$P = P_T^{\alpha} P_N^{1-\alpha} \qquad (10)$$

Postulating (for simplicity) that the same shares hold in the foreign economy, the foreign Consumer Price Index (P^*) will be:

$$P^* = (P_T^*)^{\alpha} (P_N^*)^{1-\alpha} \qquad (11)$$

Now, if we substitute these two expressions in (1), one obtains:

$$\theta = \frac{eP^*}{P} = \frac{e(P_T^*)^{\alpha}(P_N^*)^{1-\alpha}}{P_T^{\alpha} P_N^{1-\alpha}} = \left(\frac{eP_T^*}{P_T}\right)^{\alpha} \left(\frac{eP_N^*}{P_N}\right)^{1-\alpha} \qquad (12)$$

Since transaction costs are prohibitive for the non-traded good, the law of one price does not hold. As for the traded good, we rule out permanent deviations from the law of one price (assuming away transportation costs, tariffs, etc). We however allow for short term deviations from LOOP (resulting from price stickiness, information lags, and other delays in price adjustment). Thus:

$$P_T = eP_T^* (1+\chi) \qquad (3a)$$

In the long run, short term frictions vanish and $\chi = 0$.

Substituting (3a) in (12), one obtains:

$$\theta = (1+\chi)^{-\alpha} \left(\frac{eP_N^*}{P_N}\right)^{1-\alpha} \qquad (12a)$$

This expression states that the real exchange rate in each particular moment is determined by short-term deviations from the Law of One Price, and by the relative price of the non-traded good. The first component can be seen as an indicator of external *competitiveness*: whenever price stickiness prevents tradable-good prices at home to equal foreign prices, an arbitrage opportunity arises in the goods market, causing import or export booms. In light of (12a), the real exchange rate may appreciate for reasons other than price competitiveness: whenever the non-traded good price increases, this will come along with an exchange rate appreciation, without necessarily implying that the country is less competitive in terms of the goods that face external competition.

6.3.3 Wages, prices and productivity

Assume that both the tradable good and the non-tradable good are produced using labour only, under constant returns:

$$Q_T = zL_T \qquad (13)$$

$$Q_N = aL_N \qquad (14)$$

where z and *a* are productivity parameters, and subscripts refer to industries N and T[5].

Under perfect competition, firms maximize profits taking wages and prices as given. In the traded goods sector, the problem is:

[5] Although the model ignores the role of capital, in a broader interpretation you may think capital as being included in parameter z. Note that linearity between output and labour holds in a production function with constant returns to scale, provided that capital (the reproducible input) is set to expand proportionally to labour. In the cob-douglas case, assuming $Q_T = ZK_T^{\beta}L_T^{1-\beta}$, output can be rewritten as in (13), defining $z = Z\left(K_T/L_T\right)^{\beta} = Zk_T^{\beta}$. Assuming that capital is freely mobile internationally, the real interest rate will be exogenous and the capital-labour ratio k, will be constant over time, ensuring the linearity between production and labour. Along this modelling, see Obstfeld and Rogoff, 1996, chapter 8. [Obstfeld, M. Rogoff, K., 1996, Foundations of International Macroeconomics, MIT Press, Cambridge, MA].

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$$\max_{L_T} \Pi_T = P_T z L_T - W L_T$$

where Π_T stands for the firm' profits in the traded goods sector, and W is the wage rate, both in units of domestic currency.

The profit-maximizing problem in the non-traded goods sector is the following:

$$\max_{L_N} \Pi_N = P_N a L_N - W L_N$$

where Π_N stands for the firm' profits in the non-traded goods sector and w is the wage rate in units of domestic currency.

The corresponding first-order-conditions are:

$$\frac{d\Pi_T}{dL_T} = 0 \Leftrightarrow \frac{W}{P_T} = z \qquad (15)$$

$$\frac{d\Pi_N}{dL_N} = 0 \Leftrightarrow \frac{W}{P_N} = a \qquad (16)$$

Equations (15) and (16) state that the marginal product of labour in each industry shall be equal to the corresponding real wage rate.

6.3.4 Arbitrage in the labour market

To find out the equilibrium in a competitive economy, we consider a frictionless labour market, where labour is homogeneous, and workers can move at no cost from one industry to the other (remember, we are focusing on the long run).

Absence of arbitrage opportunities in the labour market implies that the nominal wage rate must be equal in both sectors:

$$W = P_N a = P_T z \qquad (17)$$

This condition is very important, because it determines the relative price of tradable and non-tradable goods:

$$\frac{P_T}{P_N} = \frac{a}{z} \qquad (18)$$

The relative price P_T/P_N tells how many units of the non-traded (home) good one must abdicate in exchange for one unit of the traded (international) good and is often referred to as the *internal* real exchange rate. When the internal real exchange rate decreases, this means that the traded good will cost less units of the non-traded good, so the home country is experiencing a real exchange rate appreciation.

The arbitrage condition (17) establishes a channel through which changes in the productivity in the tradable good sector impact on the price of non-tradable good. To see how this effect materializes, remember that the price of the traded good is determined in the international economy (equation 3a) while the price of the non-traded good is determined domestically. Using (3a), one may rewrite equation (17) as follows:

$$\frac{W}{e(1+\chi)} = P_T^* z \qquad (17a)$$

This equation reveals the long-run constraint underlying the choice of wages and of the nominal exchange rate: in the long run $\chi = 0$. Hence, given the parameters in the right-hand side, which are exogenous, the wage rate in units of foreign currency (W/e) is uniquely defined. For instance, when the productivity in the traded good sector, z, increases, the wage rate in units of foreign currency must increase.

6.3.5 The Balassa-Samuelson proposition

When one compares the cost of living in different locations, we observe a tendency for goods and services to be more expensive in rich countries than in poor countries (figure 1). One explanation for this was formulated by Bela Balassa and Paul Samuelson[6]. In short, the authors contended that the cost of living tends to be higher in rich countries than in poor

[6] Balassa, B. (1964), "The Purchasing Power Parity Doctrine: A Reappraisal", Journal of Political Economy 72 (6): 584–596. Samuelson, P. A. (1964), "Theoretical Notes on Trade Problems", Review of Economics and Statistics 46 (2): 145–154.

countries because productivity in traded goods is higher in rich countries than in poor countries.

To see this in terms of our model, let's solve (18) for P_N , obtaining $P_N = zP_T/a$. Then, assume that a similar condition holds for the foreign economy, $P_N^* = z^* P_T^*/a^*$, and substitute both in (12). This gives:

$$\theta = \frac{eP^*}{P} = \left(\frac{eP_T^*}{P_T}\right)^{\alpha}\left(\frac{eP_N^*}{P_N}\right)^{1-\alpha} = \left(\frac{eP_T^*}{P_T}\right)\left(\frac{z}{z^*}\frac{a^*}{a}\right)^{1-\alpha} \qquad (19)$$

Using the LOOP to eliminate the price of the tradable good, the *equilibrium real* exchange rate becomes:

$$\tilde{\theta} = \frac{eP^*}{P} = \left(\frac{z^*}{z}\frac{a}{a^*}\right)^{1-\alpha} \qquad (20)$$

Equation (19) states that the equilibrium real exchange rate shall reflect cross-country differences in the marginal products of labour in the tradable good and in the non-tradable good sectors. For instance, if the home country has a lower cost of living than the foreign country, this can be accounted for by a *lower* marginal product of labour in the traded goods sector or by a *higher* marginal product of labour in the non-traded goods sector.

The question that immediately arises is how productivity differences look like in the real world. Balassa and Samuelson contended that rich countries have better technologies than poor countries, but that productivity differences are higher in traded goods than in non-traded goods. Thus, rich countries should observe higher living costs than poor countries.

Note however, that equation (20) applies in the long run only, after all prices have adjusted to their equilibrium levels. In the short run, nominal frictions prevent the law of one price from holding exactly. Using (3a) in (19), the *actual* real exchange rate each moment in time will be:

$$\theta = \frac{eP^*}{P} = (1+\chi)^{-1}\left(\frac{z^*}{z}\frac{a}{a^*}\right)^{1-\alpha} \qquad (20a)$$

The parameter χ measures the real exchange rate gap, ie, the extent to which the actual real exchange rate departs form the fundamental one, as implied by productivity differences.

6.3.6 Implications for Purchasing Power Parity

The theory of purchasing power parity states, in its relative formulation, that nominal exchange rates and price levels should move together, in a way that the real exchange rate remains constant over time. As we see in equation (20), the equilibrium real exchange rate will remain constant over time only in case home productivities and foreign productivities evolve in parallel in the domestic economy and abroad.

If instead a country is catching up in terms of productivity in the tradable good sector, then the country is expected to experiment a real exchange rate appreciation. There will be a departure from relative PPP and this has nothing to do with price misalignments: simply, wages and the purchasing power of home workers must increase because workers are becoming more productive.

6.3.7 Nominal avenues

Fast technological progress in traded good sectors is a characteristic of "catch up" economies. These are economies that start out poor and that, following some major change in the economic, social or political spheres, engage in a process of technological convergence towards rich countries. Example of catch up economies include the Southeast Asian miracles, the Eastern European countries after the fall of the Berlin wall, India and China.

As we already saw, the real exchange rate appreciation that comes along with productivity growth (z) is not a matter of concern for competitiveness: it is because labour becomes more productive that the purchasing power of workers increases. This move is an equilibrium phenomenon and there is nothing wrong about it.

Still, central banks may be concerned with the inflationary impact of a sustained increase in wages and in the prices of non-traded goods. If a central bank fears that its commitment with low inflation is at stake, it may prefer to achieve the unavoidable real exchange rate appreciation by letting the nominal exchange rate appreciate (remember, from

17a, that the adjustment can either occur through an increase in wages or through a decrease in the nominal exchange rate). In any case, what a central bank cannot do is to commit at the same time with price stability and nominal exchange rate stability: if the country is catching up in terms of productivity, the adjustment in the real exchange rate must take place in one manner or the other[7].

A more difficult situation occurs when the country is diverging, with technological change being slower than abroad. In that case, the real exchange rate must depreciate, which requires a decline in nominal wages or, in alternative, a nominal exchange rate depreciation. The difficulty in adjusting downwards nominal wages is one of the reasons why many fixed exchange rate regimes collapse.

6.4 Demand side effects

In this section, we enrich the model with the demand side. We account for the government sector, and also for the possibility of aggregate demand exceeding the value of domestic production.

6.4.1 The demand side

We assume that households enjoy consuming tradable and the non-tradable good. The utility function of the representative household is:

$$U(C_T, C_N) = \alpha \ln C_T + (1-\alpha) \ln C_N \qquad (21)$$

[7] An interesting example of a tension between nominal avenues occurred in some Eastern European countries in the 1990s, during the run up to the EMU: these countries were experimenting fast technological progress, but at the same time they were committed with nominal exchange rate stability and with low inflation, to be entitled with EMU membership. Of course, it would be impossible to meet these two criteria at the same time (Szarpáry, G. Transition Countries' Choice of Exchange Rate Regime in the Run-Up to EMU Membership, Finance and Development, June 2001).

where C_T and C_N refer to the consumption of T and N, respectively. The slopes of the indifference curves are obtained by totally differentiating (21) at an unchanged utility level, that is:

$$MRS_{T,N} = -\left.\frac{dC_N}{dC_T}\right|_{dU=0} = \left(\frac{\alpha}{1-\alpha}\right)\frac{C_N}{C_T} \qquad (22)$$

The marginal rate of substitution (MRS) measures the opportunity cost of the tradable good in units of foregone non-tradable good, given the household' subjective preferences.

Government expenditures in traded and non-traded good (G_T and G_N) are assumed exogenous. The traded good can be imported or exported giving rise to deficits or surpluses in the trade balance (TB):

$$Q_N = C_N + G_N \qquad (23)$$

$$Q_T = C_T + G_T + TB \qquad (24)$$

6.4.2 External balance

Since our model is static, it cannot account for foreign borrowing or lending. Thus, we impose the current account to be equal to zero. We label this situation as of external balance:

$$CA = TB + X = 0 \qquad (25)$$

The term X in our model accounts for the possibility of the economy running a deficit in the trade balance financed with income generated abroad. X can stand for the balance of primary income (NFIA) or for the balance of secondary income (NUT) of the balance of payments. Adding X to domestic production, we obtain the national disposable income of domestic residents. The national disposable income corresponds to the amount an economy can spend without incurring in foreign borrowing or lending.

Consider, for instance, the case of unilateral transfer (say, emigrant' remittances or external aid). With positive unilateral transfers, the country can run a deficit in the trade balance without accumulating liabilities against non-residents. A similar reasoning holds for primary income received from abroad, say, because of temporary work undertaken by

residents in the foreign economy. For the purposes of writing the model, it is indifferent the exact nature of this extra income. The only point is that any imbalance in the TB must be financed within the context of the CA, so that the country does not need to borrow or lend. Since this extra source of income, X, is exogenous, the TB is also exogenous and equal to -X.

6.4.3 The Production Possibility Frontier (PPF)

The supply side of this economy is described by equations (13), (14), plus a restriction stating that the total labour used in the two sectors is equal to total labour in the economy:

$$L_T + L_N = \bar{L} \qquad (26)$$

Equation (26) could be presented as an inequality (less or equal), allowing for unemployment. Since however we are restricting attention to the long run equilibrium where prices and wages are, by definition, flexible, the economy will always operate at full employment (or *internal balance*).

Using (13), (14) and (26), the production possibilities frontier (PPF) of this economy becomes:

$$\bar{L} = \frac{Q_N}{a} + \frac{Q_T}{z} \quad . \qquad (27)$$

The slope of the PPF is the Marginal Rate of Transformation (MRT):

$$MRT_{T,N} = -\frac{dQ_N}{dQ_T}\Big|_{d\bar{L}=0} = \frac{a}{z} \qquad (28)$$

The MRT measures the opportunity cost of producing one extra unit of the traded good in terms of the non-traded good. More precisely, it gives the number of units of non-traded good that must be sacrificed in order to expand the production of the traded good by one unit, given the technology.

Substituting (23), (24), and (25) in the PPF (27), we obtain the Consumption Possibility Frontier (CPF):

$$C_T + \frac{C_N}{a} = z\left[\bar{L} - \frac{G_N}{a}\right] - G_T + X \qquad (29)$$

The right-hand side of (29) corresponds to the income available for the household to spend, after the "confiscation" corresponding to government consumption and adding the income received from abroad. The consumption possibilities frontier gives the maximum possible C_T for each level of C_N, given the exogenous variables of the model.

6.4.4 Equilibrium

Since our model contains no market failures, we know that the competitive equilibrium and the central planner solution are the same. This can be obtained choosing C_T and C_N to maximize (23) subject to (29). The first order conditions of this problem deliver the budget constraint, (29), plus the well-known equality between the MRS and the MRT:

$$\left(\frac{\alpha}{1-\alpha}\right)\frac{C_N}{C_T}=\frac{a}{z} \qquad (30)$$

Solving together (29) and (30), one obtains the optimal consumption of tradable and non-tradable goods:

$$C_N=(1-\alpha)a\left[\left(\bar{L}-\frac{G_N}{a}\right)+\frac{X-G_T}{z}\right] \qquad (31)$$

$$C_T=\alpha\left[z\left(\bar{L}-\frac{G_N}{a}\right)-G_T+X\right] \qquad (32)$$

Replacing (30) and (31) in (23a) and (24a), and using (13) and (14), we obtain the corresponding allocation of employment:

$$L_N=(1-\alpha)\left[\bar{L}+\frac{X-G_T}{z}\right]+\frac{\alpha}{a}G_N \qquad (33)$$

$$L_T=\alpha\left(\bar{L}-\frac{G_N}{a}\right)+(1-\alpha)\left(\frac{X-G_T}{z}\right) \qquad (34)$$

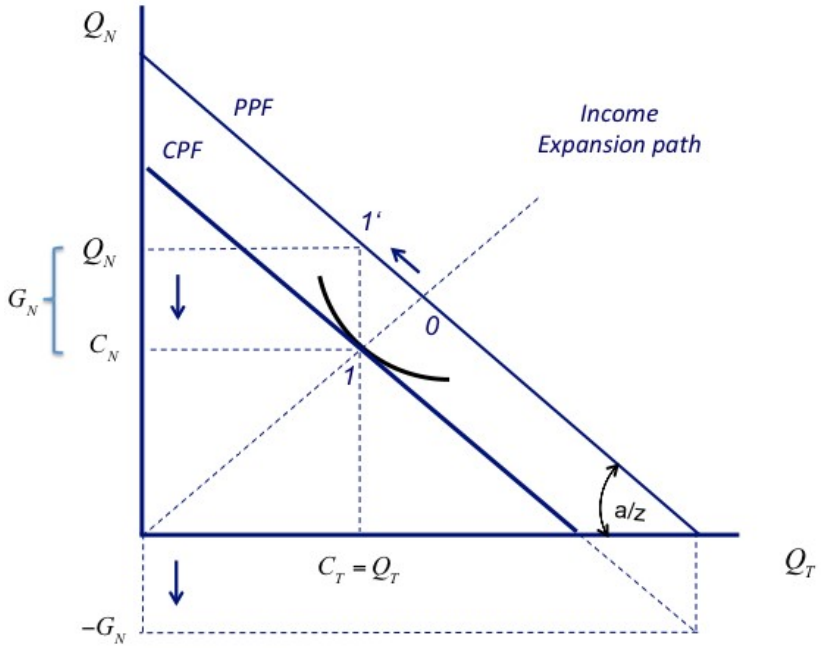
6.4.5 What happen when government spending increases?

When government expenditures increase, less units of output are available for consumers. Hence, the consumption possibilities frontier (CPF) shifts downwards. Since the

tradable and the non-tradable good are both normal, the consumer responds to the increase in government expenditures reducing the consumption of both goods.

Figure 4 illustrates the case in which the govern buys the non-tradable good (for simplicity, in the figure we assume $G_T = X = 0$). The equilibrium with $G_N = 0$ is described by point 0. In that case, the PPF is equal to the CPF and production is equal to private consumption in both sectors. When G_N increases and turns positive, the CPF shifts *vertically* downwards by the amount of government purchases. This causes the consumption point to move from 0 to point 1, along the consumer income expansion path. In the new equilibrium, production of both goods must equal total demand, private and public. Because total demand for the tradable good decreased and the total demand for the non-tradable goof increased, production moved away from point 0 to point 1' along the PPF. Consequently, employment shifts towards the non-tradable good sector.

Figure 4 – Impact of government spending in non-tradable goods (linear case)



Conversely, when the government directs its purchases towards the tradable good, the CPF will shift *horizontally* to the left, causing a fall in the private consumption of both goods. Total demand will be increasing for tradable goods and decreasing for no-tradable goods. In response, Thus, employment will move away from the non-tradable good sector towards the tradable good sector.

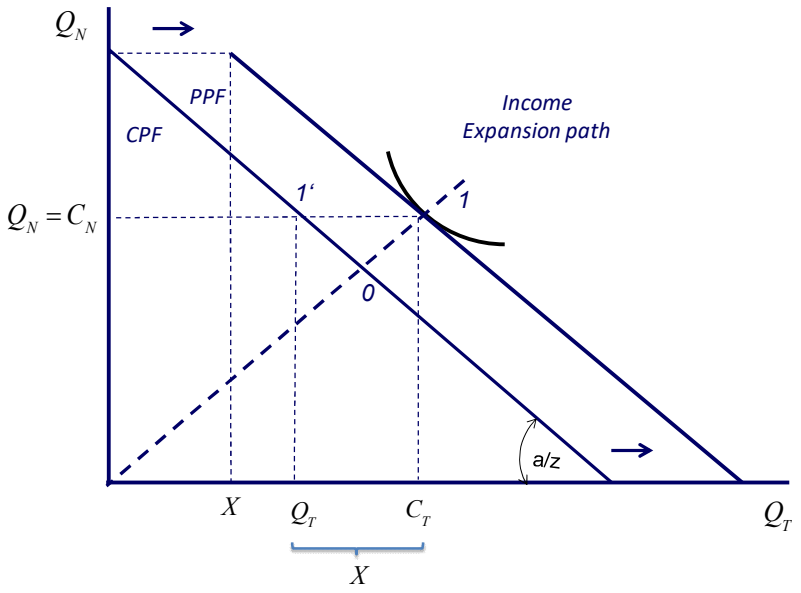
6.4.6 The Transfer Problem

The term X in our model accounts for the possibility of the economy running a deficit in the trade balance financed with income from abroad. Consider, for instance, that our economy is recipient of emigrant remittances. In Figure 5 we illustrate this case (for simplicity, it is assumed that $G_T = G_N = 0$).

In the figure, the equilibrium with $X = 0$ is described by point 0. In that case, the PPF is equal to the CPF and production is equal to private consumption in both sectors. When X>0, more units of the tradable good is available for consumers. This means that the consumption possibilities frontier (CPF) shifts *horizontally* to the right, ahead of the PPF, by the amount of the transferred income. Because both goods are normal, the household expands the consumption of both goods. The consumption point moves from point 0 to point 1, along the income expansion path.

In the new equilibrium, the production of tradable good is equal to consumption minus imports, that are financed by the international transfer. This allows labour to be released from the tradable good sector to produce more of non-tradable goods, which demand has expanded. Thus, production moves away from point 0 to point 1' along the PPF. In the end, the higher demand for non-traded goods caused a reallocation of resources away from tradable goods production towards non-tradable goods.

Figure 5 – Impact of unilateral transfer (linear case)



6.4.7 Immiserizing growth

We saw that when the economy is recipient of income generated abroad (say, unilateral transfers) employment moves away from the tradable good sector towards the non-tradable good sector. In the absence of market failures, there should be no problem at all. After all, the reallocation of resources is just following the consumer' preferences...

Unfortunately, in the real world, market failures may turn the competitive equilibrium socially undesirable. In the case of countries that are recipient of large transfers from abroad, a matter of concern for policymakers is a "dynamic externality" called "learning by doing" [8]. This externality arises when productivity growth in a given industry is a positive function of the economy' cumulative experience in that industry. If, as it is likely to be the case, learning by doing is more important in tradable goods industries than in no-tradable goods, a specialization in non-tradable goods may lead to lower growth.

In the framework above, a learning-by-doing externality could be modelled postulating future (period 2) productivity in tradables to depend on employment today (period 1). That is:

$$z_2 = z_2\left(L_{1T}\right), \text{ with } z' > 0$$

This equation states that the more a country gets specialized in traded goods today, the more the production possibility frontier will shift upwards in the second period (biased towards the traded good sector), allowing the country to enjoy a Balassa-Samuelsson effect, and thereby higher real wages and utility.

Thus, a country that starts out recipient of significant transfers from abroad because of the bias in production towards non-traded goods, is more likely to miss benefits of experience, engaging in a kind of immiserizing growth. Similar concerns arise to countries

[8] A recent discussion in Ostry, J., Ghosh, A., Korinek, A., 2012. Multilateral aspects of managing the capital account, IMF Staff Discussion note, September 7.

which exports are concentrated in a given commodity, which production does not involve significative learning effects (see box 3, on "Dutch disease").

Box 3. Dutch disease

A reallocation of employment towards the non-traded good sector such as the one described in figure 5 is a common phenomenon in commodity exporting countries, following a terms of trade improvement.

To see this, let's modify the model above to account for a "third sector", say oil, distinct from what we have labelled as tradable (manufactures, T) and non-tradable (N). For simplicity, we assume that this oil sector employs no workers, so it does not impact on the production possibilities frontier between N and T. We also assume that households preferences apply to T and N, only, not to oil, that is entirely exported. Under these simplifying assumptions, the only role of the natural resource sector is to deliver some extra income, X, that can increase or decrease according to changes in terms of trade.

Thus, when terms of trade improve, the aggregate demand expands, causing the economy to move to point 1 and 1' in Figure 5. The only difference in respect to the transfer case is that excess demand for non-oil traded goods (T), is now matched by oil exports, so the trade balance (oil inclusive) is zero[9].

The deindustrialization that often comes along with a natural resource boom was coined as "Dutch Disease" by the Economist magazine, in 1977. The phenomenon was subsequently modelled in the works of Corden and Neary[10]. The label "Dutch Disease" was inspired in the case of Netherlands in the 1960s. At that time, Netherlands discovered natural

[9] In the real world, the expansion in aggregate demand often comes ahead of oil production, because there is a phase of investment financed with a current account deficit. To explore such case one would need however an inter-temporal model, which is beyond the scope of this note.

[10] Corden WM (1981). "The exchange rate, monetary policy and North Sea oil. Oxford Economic papers 23-46. Corden WM (1984). "Boom Sector and Dutch Disease Economics: Survey and Consolidation". Oxford Economic Papers 36: 362. Corden WM, Neary JP (1982). "Booming Sector and De-industrialisation in a Small Open Economy". The Economic Journal 92 (December): 825–848.

gas in the North Sea. Such a discovery gave rise to sizeable export revenues, meaning that the country external budget constraint was substantially relaxed. The phenomenon was labelled "disease" because the resource discovery impacted negatively on traditional manufactures, leading to de-industrialization. Episodes of "Dutch Disease" have been identified in many commodity exporters, especially oil exporting countries.

The Dutch disease is seen as a problem because it leads to the contraction of the manufactures sector, and consequently to the loss of important skills, and learning by doing. On the other hand, large swings in the oil prices give rise to phases of expansion and contraction of the non-tradable good sector, with impact on unemployment. Because of the harmful effects of the Dutch Disease, many governments in commodity exporting countries have created wealth funds, in order to save abroad in good times and to spend the accumulated savings when the terms of trade deteriorate. These funds allow governments to smooth their spending, mitigating the impact of dramatic changes of the terms of trade on a country' pattern of production.

6.5 Aggregate demand and the real exchange rate

The model above assumes that the marginal product of labour is constant. That assumption may be reasonable in the very long run, when labour and capital move together ensuring the verification of constant returns to scale. In a shorter term horizon, however, the capital stock may not be easy to change. In that case, the production functions will exhibit diminishing returns, causing the PPF to be concave. In that case, the relative price of non-tradable goods (the internal real exchange rate) will be influenced both by supply factors and demand factors.

The model can be however be easily adapted to describe the case with diminishing returns to labour. The production functions become:

$$Q_T = zF\left(L_T\right) \qquad \text{with} \qquad F_L > 0\,,\ F_{LL} < 0 \qquad (13a)$$

$$Q_N = aG\left(L_N\right) \qquad \text{with} \qquad G_L > 0\,,\ G_{LL} < 0 \qquad (14a)$$

Under diminishing returns, a reallocation of labour from one sector to the other implies ever increasing opportunity costs in terms of the foregone output.

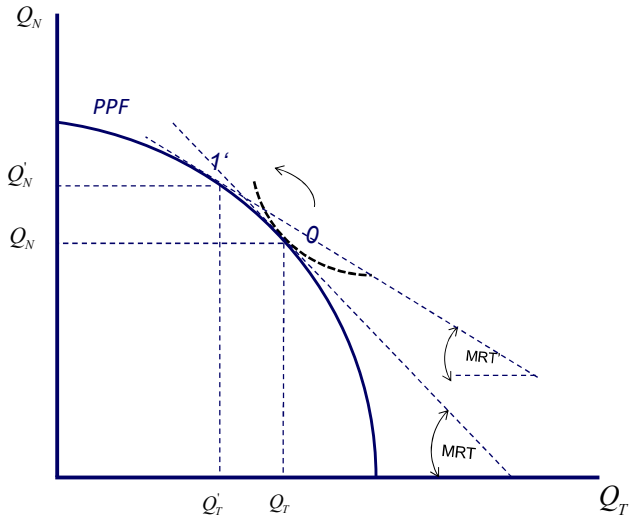
Totally differentiating (13a), (14a), we get $dQ_T = zF_L dL_T$ and $dQ_N = zG_L dL_N$. Since the labour force is exogenous, the constraint (26) implies: $dL_T + dL_N = 0$. Solving together, we get

$$MRT_{T,N} = -\frac{dQ_N}{dQ_T}\Big|_{d\bar{L}=0} = \frac{aG_L}{zF_L} \qquad (28a)$$

The difference relative to (28) is that the MRT is no longer constant. As we move along the PPF from right to left, employment expands in the non-tradable sector, causing the marginal product of labour in non-tradables G_L to decline and the marginal product of labour in the tradable sector F_L to increase. Thus, the marginal rate of transformation decreases as we move from right to left along the PPF.

This is illustrated with the concave PPF in Figure 6. For instance, when one moves from point 0 to point 1' in Figure 5, the MRT decreases, meaning that more units of the tradable good will be foregone in order to produce one more unit of the traded good. This is a direct consequence of assuming diminishing returns.

Figure 6 – The PPF under diminishing returns

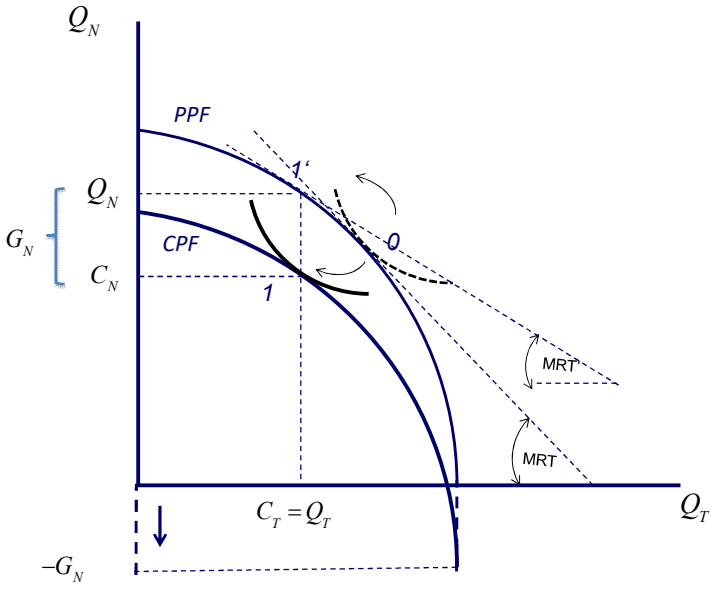


Under the assumption of diminishing returns, one can briefly re-examine the implications of changes in key parameters.

In figure 7 we analyse the case of an increase in government spending directed towards the non-tradable good. The figure basically mimics figure 4, with the exception that the PPF is concave. As before the case without government is described by point 0, with consumption equal to production. Government spending in the non-tradable good shifts the consumption possibilities frontier downwards, vertically.

In the new equilibrium, production is in point 1', and private consumption is in point 1. The difference is captured by the government consumption. In the new equilibrium, the relative price of the non-tradable good is higher than before, reflecting the fact that moving resources away from tradables to non-tradables implied increasing opportunity costs. Also note that the slope of the indifference curve in point 1 is not the same as in point 0 (point 0 and 1 are not along the same income expansion path). The fact that the relative price of non-tradable good increased caused a substitution effect in consumption, away from non-tradales towards the tradable good.

Figure 7 – Government expenditures and terms of trade

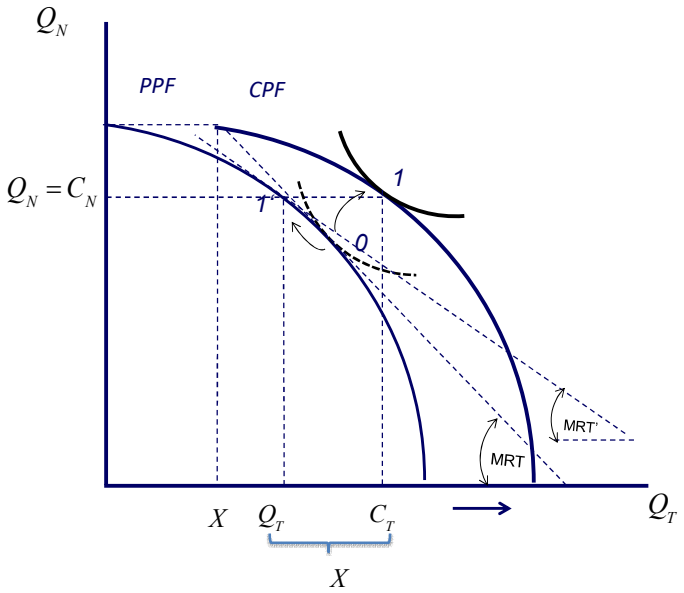


In sum, all the conclusion of figure 4 are maintained, except one: the reallocation of production from the tradable good sector towards the non-tradable good sector came along with an increase in the relative price of the non-tradable good (internal exchange rate appreciation) which in turn, as we know, implies an appreciation of the (external) real exchange rate.

Figure 8 re-examines the case of an international transfer (or of a dutch disease), assuming that the PPF is concave. As before, the international transfer causes the CPF to shift rightwards horizontally by the amount X. Being richer, consumers respond shifting the consumption point from 0 to 1. In point 1, the production of non-tradable good must equal the consumption of non-tradable good, and the difference between consumption and production of the tradable good is equal to the deficit in the trade balance. The novelty relative to figure 5 is that the reallocation of production away from tradable goods to non-tradable goods came along with an increase in the relative price of the non-tradable good or, in other words, a real exchange rate appreciation[11].

[11] Evidence on the relationship between unilateral transfers and real exchange rate appreciation and de-industrialization include: Rajan, R, Subramanian, A., 2009. Aid, Dutch disease and manufacturing growth, Journal of Development Economics 94(1), 106-118. Lartey, E., Mandelam, F., Acosta, P., 2008. Remittances, Exchange rate regimes and the Dutch Disease: a panel data analysis. Federal Reserve Bank of Atlanta, Working Paper Series 12-2008.

Figure 8 – The transfer problem and terms of trade



This model reveals an *exception to the rule that the cost of living tends to be higher in rich countries than in poor countries*. Poor oil exporting countries such as Angola and Nigeria face a very high cost of living, especially in the main cities, not because productive in manufactures is high, but instead because they are recipient of large oil export revenues, which gives rise to an high demand for non-tradable goods, causing the respective prices to increase

6.6 Summary

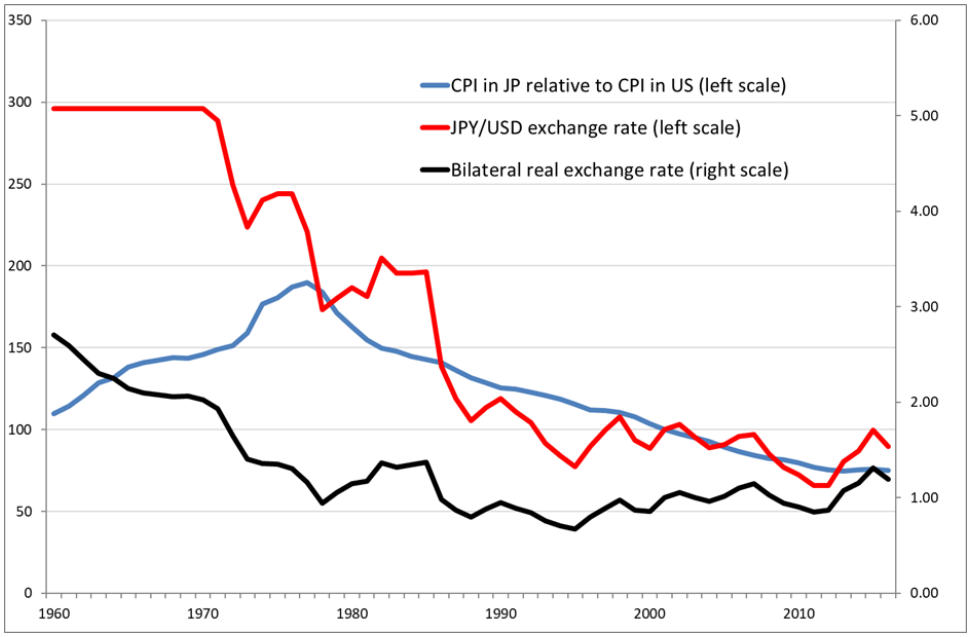
- The observation that costs of living differ significantly across countries contradicts the absolute PPP theory.
- The relative PPP theory assumes that cross-country differences in costs of living are constant over time. This theory provides a reasonable benchmark to forecasting the real exchange rate in the absence of real shocks. Yet when shocks affecting the economy are real - such as tariffs, productivity or in preferences - the long run real exchange rate is expected to change. In that case, the PPP theory fails, even in its relative formulation.

- Since tradable goods are subject to international competition, large and sustained differences in price levels across countries can only be accounted for by non-traded goods.
- In a small open economy, the wage rate in units of foreign currency is determined by the productivity in the tradable goods sector. Hence, when productivity in tradables increase, either the nominal wage must increase, or the nominal exchange rate must appreciate. In any case, there will be a real exchange rate appreciation. When, in alternative, productivity in the non-tradable goods sector increases, the price of non-tradable good decreases, delivering a real exchange rate depreciation.
- The level of aggregate demand influences the pattern of production: since the non-tradable good cannot be imported, a higher aggregate demand – say financed with a unilateral transfers from abroad – must involve the reallocation of labour away from the tradable good sector towards the non-tradable good sector. This phenomenon is lReview Questions and Exercises

Review questions

1. Suppose the assumptions needed for LOOP hold. As an example, assume that there are only two goods, say bread and milk, and that their prices are USD 1.0 and USD 2.0 and €1.00 and €2.00, respectively in the US and in the Eurozone, with the euro-dollar exchange rate equal to 1. Is this possible in this case for absolute PPP not to hold?

2. The following graph describes the evolution of the nominal exchange rate, the relative CPI and the bilateral real exchange rate between the Unites Stares and Japan. Are these figures in accordance to the PPP hypothesis? Why?



3. Comment: "The Purchasing Power Parity is a reasonable predictor of nominal exchange rates in a context where nominal shocks dominate, but not in the presence of large real shocks".

4. The Republic of Korbut is a small open economy with free capital movements that has been subject to a rise in the productivity of traded goods. Explain the options and the trade-offs involved concerning the choice of the exchange rate policy. Can real appreciation be avoided? If the primary objective of the monetary authorities was to control inflation which policy should be followed?

5. A usual practice in high inflation countries is to set-up labour contracts indexed to exchange rate, that is w=w(e). Referring to the TNT model, explain the extent to which this practice may undermine the macroeconomic adjustment.

6. Along the last decades, a number of countries (notably China) have pursued a policy of undervalued real exchange rates, keeping the domestic demand repressed and accumulating current account surpluses. Can you provide a rational for this policy?

7. Consider a small open economy under a fixed exchange rate regime. This economy has initially two sectors: one of traded goods and other of non-traded goods. Starting from a situation of internal and external balance, describe the adjustment process of that economy following the discovery of an important mineral resource (third sector). Which policies shall the authorities adopt to minimize the impact of that discovery?

Exercises

8. [**LOOP**] Consider a world with a single homogeneous good, which can either be produced domestically or abroad under conditions of perfect competition. Initially, the world price of this good is 100 USD and the price of the USD in terms of domestic currency (pesos) is 2.

   a) Suppose the price of the good in the domestic economy was initially 190 pesos. In the absence of trade costs, what do you think it would happen?

   b) In the real life, do you believe the adjustment described in a) would be instantaneous? Why?

   c) Suppose now that transport and other trade costs amounted to 20% of the price of the good. In this case, how would the non-arbitrage condition hold for exports and for imports? Find out the implied band for the real exchange rate.

9. (**Absolute PPP**) Consider a world with two economies, Pesoland and Poundland, where the respective currencies are Peso and Pound. In Poundland, there are N=1000 workers and the production level is Y=1000 units. In Pesoland, there are N=100 workers and Y=50 units. The Central Bank of Poundoland issues M=1000. To simplify, assume along the whole exercise that PPP holds and that Money velocity is unitary.

   a) How much should one unit of output cost in Poundoland (in Pounds)?
   b) How much should be the average wage rate of one worker in Poundland (in pounds)?
   c) If in Pesoland M was equal to 50000 pesos, how much would be prices and wages (in pesos)?
   d) Assuming that both countries produce exactly the same good and that this good can be both exported and imported, how much should be the price of a peso in terms of pounds?
   e) Keeping the previous assumptions, where is purchasing power higher? Why?
   f) Now imagine that the central bank of Pesoland decided to duplicate the quantity of money in its economy. What would happen to prices, wages and the exchange rate?
   g) Explain what would happen to prices, nominal wages and the exchange rate if labour productivity in Pesoland declined by one half. If the objective of the central bank was to keep inflation at zero, what should it do?

10. (**Balassa Samuelson effect and Exchange Rate regimes**). Consider a small open economy producing a tradable good (T) and a non-tradable (N) good. The corresponding production functions are $Y_T = aL_T$ and $Y_N = bL_N$. Assume that the foreign prices of these goods are $P_T^* = P_N^* = 1$ and that the nominal exchange rate is $e = 1$. Finally, assume the weight of each good in the consumer price index is 50%. Define w as the nominal wage rate, P_T as the price of T, P_N as the price of N and θ as the real exchange rate.

    1. Assume first that $a = b = 1$

a) Find out the labor demand equations in the two sectors.

b) Compute the equilibrium for the wage rate, the price level and the real exchange rate. [w=1, P=1, θ=1]

2. Consider an increase in the productivity of the tradable good from $a=1$ to $a=4$.

c) Describe the implications of such a shift on P_T, P_N, w and the equilibrium real exchange rate, assuming that the nominal exchange rate was fixed. [w=4, P=2, θ=0.5]

d) If instead the central bank' goal was to keep the inflation rate equal to zero, what should happen to prices and to the nominal exchange rate? [e=0.5]

e) What should happen to the real exchange rate if the productivity shock was instead on parameter b?

11. (**Balassa Samuelson effect and Exchange Rate regimes**): Consider a small open economy producing a tradable good (*T*) and a non-tradable (*N*) good. The corresponding production functions are $Y_T = aL_T$ and $Y_N = L_N$. Assume that the foreign prices of these goods are $P_T^* = P_N^* = 1$ and that the nominal exchange rate is $e = 100$. Finally, assume the weight of each good in the consumer price index is 50%. Define w as the nominal wage rate, P_T as the price of T, P_N as the price of N and θ as the real exchange rate.

1. Assume first that $a = 4$.

a) Find out the labor demand equations in the two sectors.

b) Compute the equilibrium wage rate, the corresponding price level and the real exchange rate. [w=400, P=200, θ=0.5]

c) Now suppose that the nominal exchange rate depreciated to $e = 400$. What would happen to the price level and to the real exchange rate? Was PPP a good theory in that case? In absolute terms or in relative terms? [w=1600, P=800, θ=0.5]

2. Departing again from $e = 100$, examine the impact of a fall in the productivity of the tradable good from $a = 4$ to $a = 1$.

d) Describe the implications of such a shift on P_T, P_N, and the equilibrium real exchange rate, assuming that the nominal exchange rate was fixed. [w=100, P=100, θ=1]

e) If non-tradable good prices were sticky, how could the central bank ease the adjustment process setting the nominal exchange rate? [e=400]

12. (**Balassa-Samuelson and the PPP exchange rate**) Consider a small open economy producing a tradable good (*T*) and a non-tradable (*N*) good. The corresponding production functions are $Y_T = aL_T$ and $Y_N = L_N$, with $a = 1$. Define w as the nominal wage rate, P_T as the price of *T*, P_N as the price of *N* and θ as the real exchange rate. The weight of

each good in the consumer price index is 50%. The foreign price of the tradable good is $P_T^* = 1$ and the nominal exchange rate in this economy is $e = 100$.

a) Assuming that firms maximize profits under perfect competition, find out the equilibrium wage rate in this economy in units of domestic currency, as well as the prices of the two goods and the consumer price index. [w=100, P=100]

b) Now assume that the foreign economy is similar to the home economy, except in that $a^* = 4$. What would be the wage rate there, in units of foreign currency? [w*=4, P*=2]

c) Find out the equilibrium real exchange rate between the two economies? Would absolute PPP hold in this case? Why? [θ=2]

d) On the basis of your findings, how much would be the purchasing power of workers at home relative to that of workers abroad? Explain the exchange rate measure used in this international comparison. [1/2]

13. (**Dutch disease, government expenditures**) Consider a small open economy producing a tradable good (T) and a non-tradable (N) good. The corresponding production functions are $Y_T = zL_T$ and $Y_N = L_N$, , $\overline{L} = 400$ and . Define w as the nominal wage rate, P_T as the price of T, P_N as the price of N and θ as the real exchange rate. The households utility is given by $U(C_T, C_N) = C_T C_N$. The foreign price level is $P^* = 1$ and the nominal exchange rate in this economy is $e = 10$. In this economy, the government only consumes non-tradable goods, G_N, and consumption there is a unilateral transfer from abroad amounting to X.

a) Find out the expressions of the PPF and CPF and plot them in a graph, assuming that (a1) $z = 1$, $X = 0$ and $G_N = 0$; (a2) $z = 16$, $G_N = 0$ and $X = 0$; (a3) $z = 1$, $G_N = 100$ and $X = 0$; (a4) $z = 1$, $G_N = 0$ and $X = 200$.

b) Find out the impact on the price level and on the internal and external real exchange rates of a change in: b1) productivity from $z = 1$ to $z = 16$; (b2) the nominal exchange rate from $e = 10$ to $e = 20$.

c) Describe the optimal consumption and employment level in each sector when: (c1) $z = 1$, $X = 0$ and $G_N = 0$; (c2) $z = 16$, $G_N = 0$ and $X = 0$; (c3) $z = 1$, $G_N = 100$ and $X = 0$; (c4) $z = 1$, $G_N = 0$ and $X = 200$.

14. [**Monetary avenues**] Consider a small open economy producing a tradable good (T) and a non-tradable (N) good. The corresponding production functions are $Y_T = aL_T$ and $Y_N = bL_N$, where $L_T = 60$ and $L_N = 60$ are immobile across sectors. In this economy, the demand functions are $C_T = M/2P_T$, $C_N = M/2P_N$, where M=120 denotes for nominal money balances, and the weight of each good in the consumer price index is 50%. Finally, assume that the TB is always zero, and foreign prices are $P_T^* = P_N^* = 1$.

a) Assume first that $a = b = 1$. Find out the nominal exchange rate, the domestic price level, and the real exchange rate.

b) (Money-target): Now consider the case of a productivity increase in the tradable good sector, from $a=1$ to $a=2$. Assuming that the money supply was kept constant, what would be the equilibrium levels of: (a1) the nominal exchange rate; (a2) the price level; (a3) the real exchange rate. [0.5; ¾; 2/3]

c) (inflation-target) Considering the same productivity shock, analyse what should happen if the central bank wanted to keep the inflation rate at zero. In particular, compute the implied levels of: (b1) the exchange rate; (b2) money supply; (b3) real exchange rate [2/3; 160; 2/3].

d) (exchange rate target) Finally, consider the case in which the central bank wanted the exchange rate to remain fixed after the productivity shock. In particular, compute the implied levels of: (c1) the exchange rate; (c2) money supply; (c3) real exchange rate [1; 240; 2/3].

e) Based on this exercise, explain why the EMU entry criteria, of a stable exchange rate with the euro and low inflation could not suit the enlargement countries in the East.

15. **(RER and aggregate demand)** Consider a small open economy producing a tradable-good (T) and a non-tradable (N) good. The corresponding production functions are $Y_T = aL_T^{1/2}$ and $Y_N = L_N$. In this economy, there are 100 workers, and prices are flexible, so full employment is always met. Assume that the foreign prices of these goods are $P_T^* = P_N^* = 1$ and that the nominal exchange rate is $e=1$. Finally, assume the weight of each good in the consumer price index is 50%. Define w as the nominal wage rate, P_T as the price of T, P_N as the price of N and θ as the real exchange rate.

    a) Find out the labour demand equations in the two sectors.

    b) Find out the expressions for the wage rate, the price level and the real exchange rate, as a function of the unknown parameters a and L_T.

    c) Assume that the steady state in this economy is characterized by $a=1$ and $L_T = 64$. Find out what the long run equilibrium real exchange rate will be.

    d) Now, consider the effects of a permanent external transfer amounting to X=6. What would happen to the real exchange rate in this case? Distinguish the phenomenon described in this exercise from the Balassa-Samuelson effect.