

## Problem set 2

### Two Period Production Economy

#### REVIEW QUESTIONS

1. Explain what the Fisher separation theorem states. Do you expect it to hold exactly in the real world? Why?
2. Compare, with the help of a graph, the implications of a temporary output shock on consumption and investment, when: (a) the economy is open to capital flows; (b) the economy is closed to capital flows. Repeat the exercise considering a change in the productivity parameter  $A$  (in  $Q_2 = AK_2^{1/2}$ ).

#### EXERCISES

- 2.1. (Small open economy with production).** Consider an economy open to capital flows, facing a world interest rate equal to  $r^* = 0.25$ . In this economy, the production function is given by  $Q_t = 16K_t^{0.5}$ , the current capital stock is  $K_1 = 400$ , and the depreciation rate is equal to  $\delta = 0.15$ . Finally, assume that consumers live only two periods, and the preferences of the representative consumer are given by  $U = \ln C_1 + \frac{\ln C_2}{1+\rho}$ , with the rate of time preference equal to 0.25.
- a) Find out the optimal investment [60].
  - b) Given the optimal investment, find out the economy's life-time wealth [320].
  - c) Compute the optimal levels of current and future consumption [438].
  - d) Find out the values of: the trade balance in periods 1 and 2; production in periods 1 and 2; income in periods 1 and 2; current account in periods 1 and 2.
  - e) Finally, assume that this economy started out with no capital at all (that is,  $K_1 = Q_1 = 0$ ). Would the optimal investment change? What would be the new values of consumption today and in the future? [71]
- 2.2. (Transitory shock, open and closed).** Consider an economy open to capital flows, where current output is  $Q_1 = 300$  and future output depends on investment, according to the following production function:  $Q_2 = 22K_2^{0.5}$ . Capital depreciates fully each year. In this economy, the preferences of the representative consumer are given by  $U = \ln C_1 + \ln C_2$ . Further assume that agents in this economy are able to borrow and lend in the international markets at the interest rate 10%.
- a) Find out the optimal levels of consumption and of investment, as well as the implied trade balance. Describe the equilibrium in a graph.
  - b) Suppose current output was instead  $Q_1 = 200$ . What would happen to the current levels of investment, consumption, trade balance and current account in periods 1 and 2?

- c) If, simultaneously, the economy lost access to external finance, how different the impact of the output shock would be? Compare graphically with the case before and conclude.

**2.3. (Choque na produtividade, antecipado e não antecipado)** Considere uma economia onde a função de utilidade do consumidor típico é dada por:  $U = C_1 C_2$ . No período 1, a produção é igual a  $Q_1 = 100$ . Relativamente ao segundo período, a produção é uma função do stock de capital, de acordo com  $Q_2 = 20K^{0.5}$ , onde K se deprecia totalmente após um período. Esta economia está aberta ao exterior e a taxa de juro internacional é  $r=0\%$ .

- a) (Equilíbrio): (a1) Determine o nível óptimo de investimento e calcule o VAL respectivo; (a2) Descreva o impacto da decisão de investimento na restrição orçamental intertemporal da economia. (a3) Determine o perfil de consumo óptimo e represente graficamente; (a4) Neste equilíbrio, qual o valor da balança comercial no período 1 e o rendimento nacional no período 2?
- b) (Quebra de produtividade) Assuma agora que esta economia enfrentava um mau ano agrícola, de tal forma que a função de produção passava a ser  $Q_2 = 16K^{0.5}$ . Descreva o impacto desse choque no investimento, no perfil de consumo e nas contas externas: (e1) Admitindo que esse choque era conhecido antes de as decisões relativas ao período 1 terem sido tomadas; (e2) admitindo que os agentes apenas se apercebiam do choque após terem sido tomadas as decisões relativas ao período 1. (e3) Compare graficamente as duas situações.

**2.4. (Small economy with production: transitory shock).** Consider an economy where the preferences of the representative consumer are given by  $U = C_1 C_2$ , the production function is given by  $Q_2 = 10K_2^{1/2}$ , and capital depreciates fully each year. Further assume that there are no initial debts or liabilities, and that current output is  $Q_1 = 48$ .

- a) Assume first that the economy is closed to capital flows. Find out the optimal consumption and investment patterns, as well as the implied marginal product of capital. Represent in a graph.
- a) Now assume that the economy opened to capital flows and that the international interest rate was  $r^* = 0$ . Describe the impact of trade openness on current consumption, investment and trade balance.
- b) Explain why financial openness improves efficiency in the world allocation of capital.
- c) Temporary shock: With all remaining parameters equal, analyse graphically and numerically the impact of a temporary output shock that drives up current GDP to  $Q_1 = 75$ , when: (c1) The economy is closed to capital flows; (c2) the economy is open.

**2.5. (Productivity shock, open and closed economies).** Consider a one-good economy with no government, where NIIP is initially zero. The representative consumer lives for two periods and has a lifetime utility function given by:  $U = C_1 C_2$ . In period 1, there is a pre-determined amount of output, equal to

$Q_1 = 192$ . As for the second period, there is no exogenous output, but there are investment opportunities, as described by the following production function,  $Q_2 = 16K^{0.5}$ , where K depreciates fully after one period. Further assume that this economy is able to borrow and lend in the international markets at the interest rate  $r=0\%$ .

- a) Compute the optimal investment plan, as well as the corresponding Net Present Value.
- b) Represent graphically the impact of the investment opportunity on the economy inter-temporal budget constraint.
- c) Find out the optimal consumption path, as well as the implied trade balance.
- d) Now assume that the economy was blessed with a productivity surge, so that the production function shifted to  $Q_2 = 20K^{0.5}$ . Describe the impact of this shock on: (d1) optimal investment; (d2) NPV; (d3) consumption in period 1 and 2; (d4) trade balance in period 1 and 2. (d5) Current account balance in period 1 and 2. (d6) Use a graph to compare with the previous equilibrium.
- e) Finally, consider the case where the economy was closed to financial trade. If the productivity term increased from  $A=16$  to  $A=20$  as described above, what would be the implications? Find out the optimal values of consumption and of investment after the productivity shock, and compare to d). Explain with the help of a graph.
- f) Which of the two cases, (d) or (e), is more in accordance to the findings of Feldstein and Horioka?

**3. (Fiscal policy in a production economy):** Consider an economy where the preferences of the representative consumer are given by  $U = C_1C_2$  and the production function is given by  $Q_2 = 20K_2^{1/2}$ . Further assume that there are no initial debts or liabilities, and that current output is  $Q_1 = 300$ .

- a) Assume first that this economy was closed to capital flows. Referring to period 1, what would be the optimal consumption, investment and savings in this case? And the interest rate? [A: 200, 100, 100, 0%].
- b) If the economy opened to capital flows and the interest rate abroad was  $r^* = 0$ , would the equilibrium change? Why?
- c) Now suppose that the government decided to launch  $G_1 = T_1 = 108$  holding  $G_2 = T_2 = 0$ .
  - c1) Assuming full access to foreign credit, describe the impact of the policy shift on: investment, consumption, private savings, government savings, trade balance. In this case, which agent would hold foreign liabilities in period 1? [A: 100, 146, 46, -54].
  - c2) Assuming that no agent in this economy had access to credit abroad, what would be the impact of the policy shift on: investment, consumption, private savings, government savings, interest rate [A: 64, 128, 64, 0, 25%].
- d) Departing from c) suppose that the government decided to size down the current taxes to zero, in exchange for an increase in taxation in period 2 the enough to clear all government debt.

d1) Assuming full access to foreign credit, describe the impact of the policy shift on: investment, consumption, private savings, government savings, trade balance. In this case, which agent would hold foreign liabilities in period 1? [A: 100, 146, 154, -108; -54].

d2) Assuming that no agent in this economy had access to credit abroad, what would be the impact of the policy shift on: investment, consumption, private savings, government savings, interest rate [A: 64, 128, 172, -108, 25%].

e) Based on your results, distinguish the impact of public spending and of tax policy when the economy has access to foreign finance and when it has not.

**2.6. [The Jorgensen formula]** Consider a two-period economy open to capital flows, facing a world *nominal* interest rate equal to  $i^*$ . In this economy, future output depends on the stock of capital stock, according to  $Q_2 = zF(K_2)$ . The current capital stock is pre-determined, and capital depreciates at the rate  $\delta$ . Finally, assume that output prices and the price of capital evolve according to  $P_2 = P_1(1 + \pi)$  and  $P_2^K = P_1^K(1 + \pi_K)$ . Prove that the first order condition of NPV maximization can be written as  $zF_K = R/P_1$ , where  $R = P_1^K(r + \delta - \pi_K)$  is the user cost of capital, and the real interest rate is such that  $1 + i = (1 + r)(P_2/P_1)$ .