A Note on Zeng's Technique for Color Reindexing of Palette-Based Images

Armando J. Pinho, *Member, IEEE,* and António J. R. Neves, *Student Member, IEEE*

***Abstract*—Palette reindexing is a well-known and very effective approach for improving the compression of color-indexed images. In this letter, we address the reindexing technique proposed by Zeng *et al.* and we show how its performance can be improved through a theoretically motivated choice of parameters. Experimental results show the practical appropriateness of the proposed modification.**

***Index Terms*—Color-indexed image compression, lossless image compression, palette reindexing.**

I. Introduction

COLOR-INDEXED images are represented by a matrix of indexes (the index image) and by a color-map or palette. The indexes in the matrix point to positions in the color-map and, therefore, establish the colors of the corresponding pixels. For a particular image, the mapping between index values and colors (typically, RGB triplets) is not unique—it can be arbitrarily permuted, as long as the corresponding index image is changed accordingly. However, for most continuous-tone image coding techniques, such as JPEG-LS [1], [2] or lossless JPEG 2000 [3], [4], these alternative representations are generally not equivalent, and sometimes have a dramatic impact on the compression performance.

With the aim of minimizing this drawback several techniques have been proposed. Basically, they rely on finding a suitable reordering of the color-map in such a way that the corresponding image of indexes becomes more amenable to compression. Regrettably, the problem of finding the optimal mapping seems to be computationally intractable [5], which motivated several suboptimal, lower complexity, proposals. Among those we find reindexing methods based on approximated solutions to the traveling salesman problem [6], [7], methods based on the maximization of the compression performance through a greedy index assignment [8], methods based on greedy pairwise merging heuristics [5], or methods as simple as those based on color reordering by luminance [9].

In this letter, we address Zeng's approach for color reindexing of palette-based images [8]. We provide a theoretical analysis of the technique, leading to a set of parameters that differs from the one originally suggested in [8]. Moreover, we provide experimental evidence of the practical appropriateness of this theoretically motivated modification.

Manuscript received March 21, 2003; revised May 6, 2003. This work was supported in part by the Fundação para a Ciência e a Tecnologia (FCT). The associate editor coordinating the review of this manuscript and approving it for publication was Prof. Amir Asif.

The authors are with the Departamento de Electrónica e Telecomunicações and also with the Instituto de Engenharia Electrónica e Telemática de Aveiro (IEETA), Universidade de Aveiro, 3810-193 Aveiro, Portugal (e-mail: ap@det.ua.pt/an@ieeta.pt).

Digital Object Identifier 10.1109/LSP.2003.821758

II. Zeng's Method for Palette Reindexing

The palette reindexing method proposed by Zeng *et al.* [8] is based on a fast and quite efficient one-step lookahead greedy approach, which aims at increasing the lossless compression efficiency of color-indexed images.

The algorithm starts by finding the symbol that is most frequently located adjacent to other (different) symbols, and the symbol that is most frequently found adjacent to it. This pair of symbols is the starting base of a symbol list P_n that will be constructed, one symbol at a time, during the operation of the reindexing algorithm. Let us denote by L_j the symbols already assigned to the symbol list and by S_i those still unassigned. Therefore, just before starting the iterations, $P_2 = (L_0, L_1)$, where L_0 and L_1 form the pair of symbols mentioned above.

New symbols can only be attached to the extremities of the list. Let us denote by S_L the symbol that satisfies

$$S_L = \arg \max_{S_i \notin P_N} D_L(S_i, N) \tag{1}$$

where

$$D_L(S_i, N) = \sum_{j=0}^{N-1} a_j C(S_i, L_j) \tag{2}$$

and by S_R the symbol satisfying

$$S_R = \arg \max_{S_i \notin P_N} D_R(S_i, N) \tag{3}$$

where

$$D_R(S_i, N) = \sum_{j=0}^{N-1} a_{N-j-1} C(S_i, L_j). \tag{4}$$

The function $C(S_i, S_j) = C(S_j, S_i)$ denotes the number of occurrences (measured on the initial index image) corresponding to pixels with symbol S_i that are spatially adjacent to pixels with symbol S_j. The weights a_j's control the impact of the $C(S_i, S_j)$ on $D_L(S_i, N)$ and $D_R(S_i, N)$, and the summations are performed over all the N symbols already located in the symbol list $P_N = (L_0, L_1, \ldots, L_{N-1})$.

The new symbol list will be given by $(S_L, L_0, L_1, \ldots, L_{N-1})$ if $D_L(S_L, N) > D_R(S_R, N)$ or by $(L_0, L_1, \ldots, L_{N-1}, S_R)$ otherwise. Finally, the symbols in the symbol list are relabeled, creating $P_{N+1} = (L_0, L_1, \ldots, L_N)$. This iterative process continues until all symbols are assigned to the symbol list. Then, the reindexed image is constructed by applying the mapping

$L_j \mapsto j$ to all image pixels, and changing the color-map accordingly.

In [8], Zeng *et al.* suggest that a reasonable choice for the weights a_j's is given by

$$a_j = \log_2\left(1 + \frac{1}{j+1}\right)$$

where $(j+1)$ corresponds to the physical distance between the current left end position of the symbol list and the position of symbol L_j.

In Section III, we prove that the optimal weights used in the process of choosing the next symbol are given by $a_j = 1$ if an exponential distribution is assumed for the differences between the neighboring pixels (this is a widely accepted model for the prediction residuals of continuous-tone images [2]). We also show that the process for determining the correct side of the list for attaching new symbols requires different weights that decrease linearly with the value of j.

III. Theoretical Analysis

In the analysis that follows, we consider the entropy of the absolute differences between the neighboring pixels as an indicator of the degree of compressibility of an image. This seems to be a reasonable assumption, specially if prediction-based compression methods are intended to be used after the reindexing.

Let us assume that N symbols have already been moved to the symbol list, i.e., $P_N = (L_0, L_1, \ldots, L_{N-1})$. According to the greedy strategy of Zeng's algorithm, the next symbol S_i that should integrate the list is the one that implies the largest increase in code length if its choice is postponed to the next iteration. For a memoryless source, the number of bits required to represent the occurrence of a given symbol S is given by $-\log_2 P(S)$, where $P(S)$ denotes the probability of occurrence of S. Therefore, S_i should maximize

$$\begin{aligned} D_L(S_i, N) &= \sum_{j=0}^{N-1} \log_2 P(j+1) C(S_i, L_j) \\ &\quad - \sum_{j=0}^{N-1} \log_2 P(j+2) C(S_i, L_j) \\ &= \sum_{j=0}^{N-1} \log_2 \frac{P(j+1)}{P(j+2)} C(S_i, L_j) \end{aligned} \quad (5)$$

or

$$\begin{aligned} D_R(S_i, N) &= \sum_{j=0}^{N-1} \log_2 P(N-j) C(S_i, L_j) \\ &\quad - \sum_{j=0}^{N-1} \log_2 P(N-j+1) C(S_i, L_j) \\ &= \sum_{j=0}^{N-1} \log_2 \frac{P(N-j)}{P(N-j+1)} C(S_i, L_j) \end{aligned} \quad (6)$$

where $P(k)$ denotes the probability of occurrence of a difference of k units between two neighboring pixels. Notice that we can identify in these relations the weights mentioned in (2) and (4) as

$$a_j = \log_2 \frac{P(j+1)}{P(j+2)} \quad a_{N-j-1} = \log_2 \frac{P(N-j)}{P(N-j+1)}. \quad (7)$$

For exponentially distributed residuals, i.e., considering

$$P(k) = A\theta^k, \qquad 0 < \theta < 1,\ 0 \le k < M \quad (8)$$

(7) reduces to

$$a_j = a_{N-j-1} = \log_2 \frac{A\theta^{j+1}}{A\theta^{j+2}} = -\log_2 \theta. \quad (9)$$

Since the a_j are independent of j, then they can be all set equal to one. However, $a_j = 1$ leads to $D_L(S_i, N) = D_R(S_i, N)$, $\forall S_i$, which prevents the determination of the correct side of the symbol list to which the new symbol should be attached. This drawback can be overcome by noting that the difference in code length due to choosing the left end-side instead of the right end-side is given by

$$\begin{aligned} \Delta &= \sum_{j=0}^{N-1} \log_2 P(j+1) C(S, L_j) \\ &\quad - \sum_{j=0}^{N-1} \log_2 P(N-j) C(S, L_j) \\ &= \sum_{j=0}^{N-1} b_j C(S, L_j) \end{aligned} \quad (10)$$

with

$$b_j = \log_2 \frac{P(j+1)}{P(N-j)}. \quad (11)$$

Therefore, we should choose the left end-side if $\Delta > 0$, otherwise we choose the right end-side.

Assuming an exponential distribution for the residuals, (11) reduces to

$$b_j = \log_2 \frac{A\theta^{j+1}}{A\theta^{N-j}} = (2j - N + 1)\log_2 \theta \quad (12)$$

i.e., the parameter b_j decreases linearly with j (notice that $\log_2 \theta < 0$).

IV. Experimental Results and Conclusion

To show the practical appropriateness of the theoretical analysis presented in the previous Section, we collected a number of color-indexed images of various sizes and number of colors, both from synthetic and natural origins. These images have been reindexed using Zeng's original method [8] and using the modification proposed in this letter, being afterward compressed using a JPEG-LS codec. Table I presents the compression results that have been obtained (both in terms of number of bytes and bits per pixel), which include the size of the color-maps. For reference, GIF file sizes are also included.

Analyzing the results presented in Table I, we conclude that the modification proposed in this letter is indeed effective. In fact, for all but two of the test images ("pc" and "party8"), compression improvements have occurred (seven of them

TABLE I
COMPRESSION RESULTS, USING A JPEG-LS CODEC, OF A NUMBER OF SYNTHETIC AND NATURAL COLOR-INDEXED IMAGES. THE "ORIGINAL" VALUES REFER TO THE TECHNIQUE PROPOSED IN [8], WHEREAS "PROPOSED" REFERS TO THE MODIFICATION PROPOSED IN THIS LETTER. "GAIN" INDICATES THE PERCENTAGE OF COMPRESSION OF THE "PROPOSED" IN RELATION TO THE "ORIGINAL" APPROACHES. FOR REFERENCE, GIF FILE SIZES ARE ALSO INCLUDED. ALL COMPRESSION VALUES INCLUDE THE SIZE OF THE COLOR-MAP. BEST RESULTS ARE DISPLAYED IN BOLDFACE

Image	Colors	GIF		Original		Proposed		Gain (%)
		Size	bpp	Size	bpp	Size	bpp	
pc	6	363,538	0.845	319,837	**0.743**	320,479	0.745	-0.2
books	7	11,246	1.580	10,458	**1.469**	10,458	**1.469**	0.0
music	8	1,955	1.269	1,634	1.060	1,620	**1.051**	0.9
winaw	10	18,647	0.506	16,569	**0.450**	16,569	**0.450**	0.0
party8	12	8,070	0.429	5,987	**0.318**	5,993	0.318	-0.1
netscape	32	16,232	2.121	13,707	1.791	13,405	**1.752**	2.2
sea_dusk	46	6,362	0.323	3,732	**0.189**	3,732	**0.189**	0.0
benjerry	48	4,386	1.254	4,036	1.154	3,977	**1.137**	1.5
gate	84	22,521	2.957	19,705	2.587	19,543	**2.566**	0.8
descent	122	23,618	2.952	23,547	2.943	22,834	**2.854**	3.0
sunset	204	100,180	2.608	98,705	2.570	88,610	**2.307**	10.2
yahoo	229	6,967	2.053	6,100	1.798	6,072	**1.789**	0.5
airplane	256	185,382	5.657	165,679	5.056	145,657	**4.445**	12.1
anemone	256	268,002	6.304	246,842	5.806	211,103	**4.966**	14.5
arial	256	309,125	6.824	311,241	6.871	280,074	**6.183**	10.0
baboon	256	256,509	7.828	232,568	7.097	212,881	**6.496**	8.5
bike3	256	431,564	4.810	441,040	4.915	372,720	**4.154**	15.5
boat	256	231,240	7.056	198,192	6.048	190,834	**5.823**	3.7
clegg	256	510,319	5.699	524,993	5.863	488,553	**5.456**	6.9
cwheel	256	166,154	**2.769**	183,521	3.058	172,718	2.878	5.9
fractal	256	336,804	6.951	300,091	6.193	282,417	**5.828**	5.9
frymire	256	413,965	**2.680**	558,994	3.619	521,446	3.376	6.7
ghouse	256	299,965	4.999	290,505	4.841	272,465	**4.541**	6.2
girl	256	214,938	6.559	187,681	5.727	172,202	**5.255**	8.2
house	256	47,028	5.740	42,441	5.180	39,767	**4.854**	6.3
lena	256	214,171	6.535	187,133	5.710	165,457	**5.049**	11.6
monarch	256	243,211	4.948	212,621	4.325	192,548	**3.917**	9.4
peppers	256	195,224	5.957	181,691	5.544	164,481	**5.019**	9.5
serrano	256	180,861	**2.897**	211,844	3.393	204,369	3.273	3.5
tulips	256	270,129	5.495	232,211	4.724	198,226	**4.032**	14.6

with a gain of 10% or more). Moreover, the reduction in compression rate verified in the "pc" and "party8" images is negligible (-0.2% and -0.1%, respectively). We note that, although appropriate for modeling the prediction residuals of continuous-tone images, the exponential distribution might not be appropriate for some images. In fact, the differences in the compression gains among the images and, particularly, the reduction in compression rate that was verified in images "pc" and "party8," might be directly related to the level of matching attained between the model and the particular image. Further studies need to be conducted in order to assess this issue.

The experimental results presented also suggest that, in general, the larger the number of colors in the image, the larger seems to be the compression improvement. This behavior may indicate that the reindexing technique is less affected by suboptimal parameters when the number of colors to reindex is small. However, this still remains to be verified.

ACKNOWLEDGMENT

The authors would like to thank W. Zeng for providing the source code of the reindexing technique.

REFERENCES

[1] ISO, "Information technology—Lossless and near-lossless compression of continuous-tone still images,", ISO/IEC 14495-1 and ITU Rec. T.87, 1999.

[2] M. J. Weinberger, G. Seroussi, and G. Sapiro, "The LOCO-I lossless image compression algorithm: Principles and standardization into JPEG-LS," *IEEE Trans. Image Processing*, vol. 9, pp. 1309–1324, Aug. 2000.

[3] ISO, "Information tchnology—JPEG 2000 image coding system,", ISO/IEC International Standard 15444-1, ITU-T Rec. T.800, 2000.

[4] A. Skodras, C. Christopoulos, and T. Ebrahimi, "The JPEG 2000 still image compression standard," *IEEE Signal Processing Mag.*, vol. 18, no. 5, pp. 36–58, Sept. 2001.

[5] N. D. Memon and A. Venkateswaran, "On ordering color maps for lossless predictive coding," *IEEE Trans. Image Processing*, vol. 5, pp. 1522–1527, Nov. 1996.

[6] S. Battiato, G. Gallo, G. Impoco, and F. Stanco, "A color reindexing algorithm for lossless compression of digital images," in *Proc. IEEE Spring Conf. Computer Graphics*, Budmerice, Slovakia, Apr. 2001, pp. 104–108.

[7] A. Spira and D. Malah, "Improved lossless compression of color-mapped images by an approximate solution of the traveling salesman problem," in *Proc. IEEE Int. Conf. Acoustics, Speech, and Signal Processing*, Salt Lake City, UT, May 2001, pp. 1797–1800.

[8] W. Zeng, J. Li, and S. Lei, "An efficient color re-indexing scheme for palette-based compression," in *Proc. 7th IEEE Int. Conf. Image Processing*, Vancouver, Canada, Sept. 2000, pp. 476–479.

[9] A. Zaccarin and B. Liu, "A novel approach for coding color quantized images," *IEEE Trans. Image Processing*, vol. 2, pp. 442–453, Oct. 1993.