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Asymptotic Behavior of the Minima to a Class of Optimization Problems for Differential Inclusions[1]

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Abstract. Denoting by $S_k(\xi_k)$ the set of solutions of the Cauchy problem $\dot{x} \in F_k(t, x)$, $x(0) = \xi_k$, for $k \in \mathbb{N} \cup \{\infty\}$, we prove that, under appropriate assumptions, the sequence $\{S_k(\xi_k)\}_{k\in\mathbb{N}}$ converges to $S_\infty(\xi_\infty)$ in the Kuratowski sense as well as in the Mosco sense. This result together with some facts from Γ-convergence theory are used to prove a result concerning the existence and the asymptotic behavior of the minima to the optimization problem

$$\min \int_0^T [g_k(t, x(t)) + h_k(t, \dot{x}(t))]\, dt + \psi_k(\xi), \qquad x \in S_k(\xi),\ \xi \in \mathcal{K},$$

with $\mathcal{K}$ a compact subset of R^n.

Key Words. Optimization problems, functionals of Bolza type, differential inclusions, Γ-convergence, Kuratowski convergence, Mosco convergence.

1. Introduction

One of the important problems in optimization theory is that of stability of optimal solutions under the perturbations of the dynamics of the system, as well as of the cost functional. In this paper, we consider a sequence of problems indexed by $k \in \bar{\mathbb{N}} = \mathbb{N} \cup \{\infty\}$ (the limit problem, $k = +\infty$, being the unperturbed problem),

$$\min \int_0^T [g_k(t, x(t)) + h_k(t, \dot{x}(t))]\, dt + \psi_k(\xi), \qquad x \in S_k(\xi),\ \xi \in \mathcal{K}, \tag{1}$$

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