16^{th} ANNUAL WORKSHOP

on

APPLICATIONS AND GENERALIZATIONS OF COMPLEX ANALYSIS

Booklet of Abstracts

AVEIRO, March 21-22, 2014

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GENERAL INFORMATIONS

We welcome you all to the 16th Annual Workshop in Aveiro. The workshop will take place at room Sousa Pinto (second floor).

INTERNET ACCESS

Rooms are available for checking your e-mail trough a computer terminal

Login: EIC.2014@visit.uaveiro.eu
Password: aveiro2014

Important: do not change the password, as it is a multiple login.

Alternatively, if you possess a personal Laptop with WLAN you can use either the above login or your personal EDUROAM access at your home university.

COMPUTER TERMINALS

Computers terminal are available at the room 11.2.7, located in the 2nd floor. However, due to lecturing restrictions, please do confirm its availability before entering.

WHERE TO EAT?

Around the campus there exist several coffee bars where you can have also small meals such as sandwiches, snacks, fruits, ice creams, etc.

For lunch or dinner, there exist several possibilities:

- **Refeitório de Santiago**, monday to friday, lunch 12h to 14h30 / dinner 18h30 to 20h30, saturday and sunday, lunch 13h to 14h30 / dinner 19h to 20h30. Prices: from 2,50 to 5,00 euros
- **Edifício do Snack-bar and Self-Service (ground floor)**, monday to friday, lunch 12h to 14h30. Prices: from 2,50 to 5,00 euros
- **Restaurante Universitário (1st floor)**, monday to friday, lunch 12h to 14h30. Prices: buffet service 7,00 euros

In addition, several restaurants can be found all around the campus.

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SUPPORT

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We wish you all a happy stay and a good and fruitful workshop.

Aveiro, March 17, 2014

The Organizers

Uwe Kähler
Paula Cerejeiras

Departamento de Matemática
Universidade de Aveiro

ABSTRACTS - 21th March

10:00 *Opening Session*

10:15-11:00 Beyond the circular law: global asymptotics of Gabor-Toeplitz ensembles

Daniel Abreu
Acoustics Research Institute - Vienna, Austria

The transdisciplinar concept of "Universality" aims at understanding how the macroscopic laws of nature can be obtained from those ruling the microscopic world. Often, a system of interacting objects (for instance, electrons) can display an extreme complexity even at small dimensions. However, if one increases the dimension of the system further enough, the system organizes itself, leading to the laws of the macroscopic world (some intuition can be obtained from the central limit theorem in statistics).

We will explain basic ideas on how to random distribute points in a plane using determinantal point processes and illustrating with the model case of the Ginibre ensemble. In this case, the asymptotic distribution of the points approaches the uniform distribution of a disc (the famous "circle law"). We will present a new point process (Gabor-Toeplitz ensembles) based in time-frequency representations, which contains the Ginibre and other processes as special cases, including polyanalytic analogues. We have found a law of large numbers which go beyond the circle law: the limit distributions are uniform over domains wich can be quite irregular in shape and frontier regularity.

This is joint work with Karlheinz Gröchenig and José Luis Romero from the University of Vienna.

11:05-11:25 L-packets and depth for $SL_2(K)$ with K a local function field of characteristic 2

Sérgio Mendes
University Institute of Lisbon, Portugal

The use of unramied quasicharacters to create a complex structure is well established in number theory. Working with quasicharacters of the Weil group of a local field, Brodzki and Plymen

eqquiped the smooth dual of $GL(n)$ with the structure of complex algebraic variety which paved the way to a recent geometric conjecture on the reducibility of representations of p-adic groups.

Let G = $SL_2(K)$ with K a local function field of characteristic 2. Artin-Schreier theory for the field K leads to a parametrization of $L-$packets in the smooth dual of G. This is related to the geometric conjecture. The $L-$packets in the principal series are parametrized by quadratic extensions, and the supercuspidal $L-$packets by biquadratic extensions. Each supercuspidal packet is accompanied by a singleton packet for $SL_1(D)$. All the depths are computed.

Joint work with Roger Plymen.

11:30-11:50 Paramonogenic Maps and Conjugate Harmonic Functions

Kerr Ballenger Craig Nolder
Florida State University, U.S.A.

Clifford valued functions decompose into even and odd parts, depending on the length of reduced Clifford basis elements. Using the Cauchy Riemann Dirac operator, the monogenic condition becomes a Cauchy Riemann system between the even and odd parts. These conjugate harmonic functions are unique only up to addition of a paramonogenic function (hyperholomorphic constant).

We show that, for a monogenic function in the upper half space $\mathbb{R}^{n+1}_+$, that the even and odd parts simultaneously have non-tangential boundary values in $\mathbb{R}^n$. This generalizes a result of E. Stein which occurs when the mapping is a paravector.

12:00-14:00 *Lunch-break*

14:30-15:15 Cauchy integral formulas for bicomplex pseudoanalytic functions and Schrödinger equations with complex coefficients

Vladislav V. Kravchenko
Cinvestav del IPN, Queretaro, Mexico

In the talk we show that the study of the Dirac system and second-order elliptic equations with complex-valued coefficients on the plane naturally leads to bicomplex Vekua-type equations. To the difference of complex pseudoanalytic (or generalized analytic) functions the theory of bicomplex pseudoanalytic functions has not been developed. Such basic facts as, e.g., the similarity principle or the Liouville theorem in general are no longer available due to the presence of zero divisors in the algebra of bicomplex numbers.

In the present work a theory of bicomplex pseudoanalytic formal powers including the Cauchy kernels and integral formulas is developed. Combining the approaches of L. Bers and I. N. Vekua with some additional ideas [1] we obtain the Cauchy integral formula in the bicomplex setting. In the classical complex situation this formula was obtained under the assumption that the involved Cauchy kernel is global, a very restrictive condition if one takes into account possible practical applications, especially when the equation itself is not defined on the whole plane. We show that the Cauchy integral formula remains valid with the Cauchy kernel from a wider class called here the reproducing Cauchy kernels. We give a complete characterization of this class. The results are new even for complex Vekua equations.

Useful connections between the Cauchy kernels and the fundamental solutions for the Schrödinger operators are established, allowing one to construct the Cauchy kernel from a fundamental solution and vice versa. Using these results we construct the fundamental solutions for the Darboux transformed Schrödinger operators.

Bicomplex pseudoanalytic functions of a hyperbolic variable are also considered. Their relation to transmutation operators from the theory of linear differential equations will be discussed [2].

[1] H. Campos, V. V. Kravchenko Fundamentals of bicomplex pseudoanalytic function theory: Cauchy integral formulas, negative formal powers and Schrödinger equations with complex coefficients. *Complex Analysis and Operator Theory*, 2013, Volume 7, Issue 2, pp 485-518.

[2] V. V. Kravchenko, S. M. Torba Analytic approximation of transmutation operators and applications to highly accurate solution of spectral problems. *Submitted*, available at arxiv.org.

15:20-15:40 The spectral theorem for unitary operators based on the S-spectrum

Fabrizio Colombo
Politecnico di Milano, Italy

In this talk we introduce the quaternionic Riesz projectors using the S-functional calculus. Then we show that the quaternionic spectral theorem for unitary operators is based on the S-spectrum. This result is proved using the Herglotz theorem in the quaternionic setting.

This is a joint work with D. Alpay, D. P. Kimsey, I. Sabadini.

15:45-16:05 A geometric structure on the nullspace of higher spin Dirac operators of order three

Tim Raeymaekers David Eelbode
Ghent University, Belgium

The higher spin Dirac operators $\mathcal{Q}_\lambda$, acting on functions $f(x)$ on $\mathbb{R}^m$ taking values in arbitrary irreducible half-integer highest weight representations $\mathcal{S}_\lambda$ for Spin(m), should be seen as generalisations of the classical Dirac operator and the Rarita-Schwinger operator. In recent work, we developed an algorithm to decompose the kernel of these higher spin Dirac operators of general order into irreducible spin representations (i.e. the analogue of describing the space of monogenic polynomials for the Dirac operator). In doing so, we discovered an elegant hyperrectangular structure for this kernel. The problem at hand was thereby reduced to a combinatorical problem. In this talk, we will tackle the case of higher spin Dirac operators of order three: the geometrical structure of the kernel then reduces to a rectangular cuboid, which allows a nice visualisation. Nonetheless, the general algorithm can be well understood from this case. We also describe the related combinatorical problem.

16:10-16:40 *Coffee-break*

16:45-17:05 Self-mappings of the quaternionic unit ball and Nevanlinna-Pick interpolation problem

Irene Sabadini
Politecnico di Milano, Italy

I will present some results on slice regular functions mapping the quaternionic open unit ball $\mathbb{B}$ into itself. We characterize these functions in terms of their Taylor coefficients at the origin and identify them as contractive multipliers of the Hardy space $H^2(\mathbb{B})$. Then, we formulate and solve the Nevanlinna-Pick interpolation problem in the class of such functions presenting necessary and sufficient conditions for the existence and for the uniqueness of a solution.

This is a joint work with D. Alpay, V. Bolotnikov, F. Colombo

17:10-17:30 Set of operators with 0 in the closure of the numerical range

Cristina Diogo
University Institute of Lisbon, Portugal

Let $\mathscr{W}_{\{0\}}$ be the set of all bounded linear operators on a complex Hilbert space $\mathscr{H}$ which contain 0 in the closure of the numerical range, that is,

$$\mathscr{W}_{\{0\}} = \{A \in \mathcal{B}(\mathscr{H});\ 0 \in \overline{W(A)}\}.$$

Some properties of this set of operators will be presented. For instance, it will be shown that $\mathscr{W}_{\{0\}}$ is not closed under addition and multiplication. However, it has a less obvious algebraic structure, which can be described in the following way. Let $\mathcal{T} \subseteq \mathcal{B}(\mathscr{H})$ be a given set of operators. Then there exist the largest sets $\mathcal{Q}_{\mathcal{T}}, \mathcal{R}_{\mathcal{T}} \subseteq \mathcal{B}(\mathscr{H})$ such that

$$\mathcal{T}\mathcal{Q}_{\mathcal{T}} \subseteq \mathscr{W}_{\{0\}} \qquad \text{and} \qquad \mathcal{R}_{\mathcal{T}}\mathcal{T} \subseteq \mathscr{W}_{\{0\}},$$

where $\mathcal{T}\mathcal{Q}_{\mathcal{T}}$ is the set of all products TQ with $T \in \mathcal{T}$ and $Q \in \mathcal{Q}_{\mathcal{T}}$ and $\mathcal{R}_{\mathcal{T}}\mathcal{T}$ has a similar meaning. We characterize $\mathcal{Q}_{\mathcal{T}}$ for some particular sets $\mathcal{T} \subseteq \mathcal{B}(\mathscr{H})$.

17:35-17:55 On a singular integral operator with shift and conjugation

Rui Marreiros
University of Algarve, Portugal

On the Hilbert space $L_2(\mathbb{T})$ we consider the singular integral operator with non-Carleman shift and conjugation $K = P_+ + (aI + AC)P_-$, where $P_\pm$ are the Cauchy projectors, $A = \sum_{j=0}^{m} a_j U^j$, a, a_j, $j = \overline{1,m}$, are continuous functions on the unit circle $\mathbb{T}$, U is the shift operator and C is the operator of complex conjugation. Some estimates for the dimension of the kernel of the operator K are obtained.

This talk is based on a joint work with Ana Conceição.

19:30 *Conference dinner* - Restaurante *O Mercantel*

ABSTRACTS - 22th March

10:15 - 11:00 Harmonic analysis on the Möbius gyrogroup

Milton Ferreira
CIDMA, Polytechnic Institute of Leiria, Portugal

In this talk we will intend to give a new vision on the harmonic analysis of symmetric spaces through the study of the harmonic analysis on the Möbius gyrogroup of the Poincaré ball (with arbitrary radius t) of the n-dimensional real hyperbolic space. The following aspects will be considered: the σ_t-translation, the σ_t-convolution, the eigenfunctions of the σ_t-Laplace-Beltrami operator, the σ_t-Helgason Fourier transform, its inverse transform and Parseval and Plancherel's Theorems. In the Euclidean limit $t \to +\infty$ to the space of zero curvature we show that we obtain the standard theory of harmonic analysis on the Euclidean space $\mathbb{R}^n$, establishing the link between hyperbolic and Euclidean harmonic analysis. Finally, considering the 1-dimensional case, we will construct the hyperbolic Gabor transform and the hyperbolic wavelet transform associated to representation theory of the hyperbolic Heisenberg group and the hyperbolic affine group, respectively. The extension of these hyperbolic transforms to higher dimensions seems to be not possible due to the lack of an appropriate algebraic structure.

11:05-11:25 Reproducing Kernels in Hermitian Clifford Analysis

Michael Wutzig
Ghent University, Belgium

In this talk we look at some models of higher dimensional Klein bottles in $\mathbb{R}^n$ that are endowed with pin structures. We develop explicit formulas for the fundamental solution to the time independent Klein-Gordon operator in terms of pseudo multiperiodic series. Finally we present some integral formulas to address inhomogeneous BVP for this operator on this class of non-orientable manifolds

11:30-11:50 Symbolic computation on the kernel of some classes of singular integral operators

Ana Conceição
University of Algarve, Portugal

The main goal of this talk is to show how symbolic computation can be used to compute the kernel of some classes of singular integral operators. In addition, some interesting estimates for the dimension of the kernel of singular integral operators with non-Carleman shift and conjugation are obtained. The methods developed rely on innovative techniques of Operator Theory and have a great potential of extension to more complex and general problems. Some nontrivial examples computed with the computer algebra system *Mathematica* are presented.

This talk is based on a joint work with Rui C. Marreiros.

[1] Ana C. Conceição, José C. Pereira: An Overview of Symbolic Computation on Operator Theory, *Proceedings of the 1st International Conference on Algebraic and Symbolic Computation - SYMCOMP 2013 (ECCOMAS Thematic Conference) (pen drive), Instituto Superior Tcnico, Lisbon, Portugal*, (2013), IP02, 39–71.

[2] Ana C. Conceição, Viktor G. Kravchenko, José C. Pereira: Computing some classes of Cauchy type singular integrals with *Mathematica* software, *Advances in Computational Mathematics*, 39 (2) (2013), 273–288.

[3] Viktor G. Kravchenko, Rui C. Marreiros, Juan S. Rodriguez: An estimate for the number of solutions of an homogeneous generalized Riemann boundary value problem with shift, *Operator Theory: Advances and Applications*, 220 (2012), 163–177.

[4] Ana C. Conceição, Viktor G. Kravchenko, José C. Pereira: Rational Functions Factorization Algorithm: a symbolic computation for the scalar and matrix cases, *Proceedings of the 1st National Conference on Symbolic Computation in Education and Research - CSEI 2012 (CD-ROM), Instituto Superior Tcnico, Lisbon, Portugal*, (2012), P02, 1–13.

[5] Ana C. Conceição, Viktor G. Kravchenko: About explicit factorization of some classes of non-rational matrix functions, *Mathematische Nachrichten*, 280 (9-10) (2007), 1022–1034.

12:00-14:00 *Lunch-break*

14:30-15:15 Decomposing polynomials in a matrix variable

`David Eelbode`
`Antwerp University, Belgium`

When considering polynomials in one vector variable in $\mathbb{R}^m$, it is well-known how to decompose them into harmonic polynomials (i.e. irreducible pieces for the action of the rotation group). This is basically an algebraic problem, solved in terms of the generators for the simple Lie algebra $\mathfrak{sl}(2)$. In this talk we will consider a similar problem for polynomials in several variables. Despite the

fact that the abstract representation theory behind this question is fairly easy (Howe duality in k variables is classical), it turns out that the projection problem is not that easy. We will explain how to solve this, and give a few applications.

15:20-15:40 Fredholmness and index of simplest singular integral operators with two slowly oscillating shifts

Alexei Karlovich
Universidade Nova de Lisboa, Portugal

Let α and β be orientation-preserving diffeomorphisms (shifts) of $\mathbb{R}_+ = (0, \infty)$ onto itself with the only fixed points 0 and ∞, where the derivatives α' and β' may have discontinuities of slowly oscillating type at 0 and ∞. For $p \in (1, \infty)$, we consider the weighted shift operators U_α and U_β given on the Lebesgue space $L^p(\mathbb{R}_+)$ by $U_\alpha f = (\alpha')^{1/p}(f \circ \alpha)$ and $U_\beta f = (\beta')^{1/p}(f \circ \beta)$. We apply the theory of Mellin pseudodifferential operators with symbols of limited smoothness to study the simplest singular integral operators with two shifts $A_{ij} = U_\alpha^i P_+ + U_\beta^j P_-$ on the space $L^p(\mathbb{R}_+)$, where $P_\pm = (I \pm S)/2$ are operators associated to the Cauchy singular integral operator S, and $i, j \in \mathbb{Z}$. We prove that all A_{ij} are Fredholm operators on $L^p(\mathbb{R}_+)$ and have zero indices.

This is a joint work with Yuri Karlovich and Amarino Lebre.

15:45-16:05 An algebraic method to study the kernel of Toeplitz operators

Maria Teresa Malheiro
University of Minho, Portugal

We show that the kernel of a Toeplitz operator T_G can be characterized, and conditions for its invertibility obtained, if its symbol G belongs to a $\mathfrak{Q}$-class C_{Q_1,Q_2}. This means that G satisfies a condition of the form $G^T Q_1 G = \det G.Q_2$ where Q_1 and Q_2 are invertible, symmetric 2×2 matrix functions. We show moreover that by using different $\mathfrak{Q}$-classes, this may be achieved by simple algebraic methods.

This is based on a joint work with Cristina Câmara.

16:10-16:40 *Coffee-break*

16:45-17:05 The Klein Gordon equation on Klein bottles with pin structures

Sören Kraußhar
Universität Erfurt, Germany

In this talk we look at some models of higher dimensional Klein bottles in $\mathbb{R}^n$ that are endowed with pin structures. We develop explicit formulas for the fundamental solution to the time independent Klein-Gordon operator in terms of pseudo multiperiodic series. Finally we present some integral formulas to address inhomogeneous BVP for this operator on this class of non-orientable manifolds

17:10-17:30 On quaternionic analysis and the scattering problem for the 2D Helmholtz equation

Baruch Schneider
Izmir University of Economics, Turkey

The theory of function with quaternionic values and two real variables has relationships with other fields of mathematics, in particular the scattering theory. We study scattering problem in the quaternionic setting using the theory of Helmholtz equation. The aim to talk is to explain some questions in classical scattering theory.

This is joint work with Ricardo Abreu Blaya, Juan Bory Reyes and Ramon Rodriguez Dagnino.

17:35-17:55 Ideals of operators and spectral properties on compact Lie groups

Julio Delgado
Imperial College London, U.K.

Given a compact Lie group G, in this talk we first present a characterisation of Schatten ideals of operators on $L^2(G)$ and some applications to the analysis of differential operators. In the second part we consider nuclear(Grothendieck) operators on $L^p(G)$ spaces and establish sufficient conditions for the membership to those ideals. We also relate our results with the Grothendieck-Lidskii trace formula.

This is a joint work with Michael Ruzhansky.

18:00 *Closing Session*