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Concerning the Nyquist Plots of Rational Functions of Nonzero Type

Paulo J. S. G. Ferreira

***Abstract*— The Nyquist plots of rational functions of type one or higher are often represented with branches that tend to infinity while approaching either the real or imaginary axis. It is shown that this fails to be true in general, a fact that appears to have little, if any, impact on the usual analysis. However, it has pedagogical interest since it explains the discrepancy between the shape of the Nyquist plots obtained analytically, or with the help of computer programs in the classroom, and the plots found in many standard textbooks. The discrepancy is most clear when the system type is at least two, in which case the branches may move infinitely further from *both* axes.**

***Index Terms*—Control engineering education, control systems, Nyquist plots.**

I. Introduction

The Nyquist plot of $H(s)$ is the locus, in the complex plane, of the points[1]

$$\{\mathrm{Re}H(j\omega),\, \mathrm{Im}H(j\omega)\} \tag{1}$$

as a function of ω. The traditional Nyquist plot of a rational function $H(s)$ of type one or higher often shows branches which tend to infinity while approaching either the real or imaginary axis. This is, in general, incorrect. The purpose of this note is to discuss and clarify this issue, since some standard textbooks on control theory do not address it.

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[1] The real and imaginary parts of a complex s are denoted by $\mathrm{Re}s$ and $\mathrm{Im}s$, respectively. The imaginary unit is denoted by j.

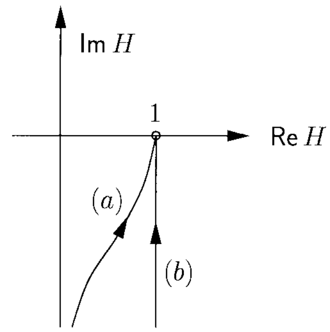

Fig. 1. Nyquist plot of $H(s) = (1+s)/s$: (a) the expected plot and (b) the correct plot.

A rational function of type $n \in \mathbb{N}$ can be written as

$$H(s) = \frac{P(s)}{s^n Q(s)} \tag{2}$$

where $P(s)$ and $Q(s)$ are polynomials. When the type is one or higher, $H(s)$ has poles at $s = 0$, and the Nyquist plot branches approach the point at infinity as $s = j\omega \to 0$. The phase angle subtended by a point on the branches then converges to a multiple of $\pi/2$. However, this does not mean that the branches approach one of the axes. As the following counterexamples show, for systems of type two the branches may even move further and further away from *both* axes as $\omega \to 0$.

II. Examples

Example 1: The Nyquist plot of

$$H(s) = \frac{s+1}{s}. \tag{3}$$

In this case

$$H(j\omega) = \frac{j\omega + 1}{j\omega} = 1 - j\,\frac{1}{\omega} \tag{4}$$

and consequently

$$\begin{cases} \mathrm{Re}H(j\omega) = 1, \\ \mathrm{Im}\,H(j\omega) = -\dfrac{1}{\omega}. \end{cases} \tag{5}$$

The Nyquist plot corresponding to $\omega \in \mathbb{R}^+$ is clearly a vertical half-line ending in (1, 0) [see Fig. 1(b)]. Usually, the student expects a plot similar to the one depicted in Fig. 1(a).

Example 2: The Nyquist plot of

$$H(s) = \frac{s+1}{s^2}. \tag{6}$$

In this case

$$H(j\omega) = \frac{j\omega + 1}{-\omega^2} \tag{7}$$

and thus

$$\begin{cases} \mathrm{Re}H(j\omega) = \dfrac{1}{\omega^2} \\ \mathrm{Im}H(j\omega) = -\dfrac{1}{\omega} \quad . \end{cases} \tag{8}$$

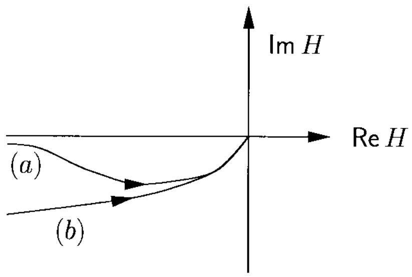

Fig. 2. Nyquist plot of $H(s) = (1+s)/s^2$: (a) the expected plot and (b) the correct plot, a parabola (there is no asymptote, the distance to both axes increases as the distance to the origin increases).

Letting $x = \mathrm{Re}H(j\omega)$ and $y = ImH(j\omega)$ we see, upon eliminating ω, that

$$x = -y^2. \tag{9}$$

This parabola is sketched in Fig. 2(b), whereas Fig. 2(a) depicts the expected plot.

These discrepancies can be explained easily. If $H(s)$ is of nonzero type

$$H(s) = \frac{P(s)}{s^n Q(s)} \tag{10}$$

with $n \in \mathbb{N}$. Consequently,

$$\lim_{\omega \to 0} \frac{P(j\omega)}{(j\omega)^n Q(j\omega)} = \lim_{\omega \to 0} M(\omega)\, e^{j\phi(\omega)} \tag{11}$$

where

$$M(\omega) = \left| \frac{P(j\omega)}{(j\omega)^n Q(j\omega)} \right| \tag{12}$$

and $\phi(\omega)$ is the phase angle of

$$\frac{P(j\omega)}{(j\omega)^n Q(j\omega)}. \tag{13}$$

If $P(s)$ and $Q(s)$ have real coefficients we may assume without loss of generality that $P(0)$ and $Q(0)$ are nonzero reals, meaning that

$$\lim_{\omega \to 0} \phi(\omega) = \text{integer} \times \frac{\pi}{2}. \tag{14}$$

The function $M(\omega)$ clearly increases without bound as $\omega \to 0$. However, $M(\omega) \to \infty$ and (14) together do not necessarily mean that the Nyquist plot of $H(s)$ approaches the real or imaginary axis as $\omega \to 0$. The phase angle of a complex point s which approaches the point at infinity may converge to a multiple of $\pi/2$ without its real or imaginary parts converging to zero.

The converse proposition is obviously true: if the limit point is a point belonging to one of the axes, then the phase angle must converge to a multiple of $\pi/2$. Thus, when the limit point is not the point at infinity, the branches will indeed end on one of the axes.

III. Remarks

Nyquist plots found in textbooks depict in a clear way the global behavior of the function $H(s)$ infinitely far from the origin, in terms of its magnitude and phase.

For systems of nonzero type, the branches of the plot do not necessarily have to approach an axis, and for systems of type two and higher the branches may even deviate infinitely from both of them. This behavior is not readily apparent analytically to the student, but becomes so with the help of computer software.

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Using Contests to Teach Design to EE Juniors

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***Abstract*—Most electrical engineering programs have a capstone design course, but lack a suitable design experience in the junior year. This makes the capstone course very difficult for students and compromises its pedagogical aims. A good design experience offers opportunities for learning to identify key operational concepts, to identify and remedy procedural and factual knowledge deficits, and to exercise judgment. Design problem should be open-ended, moderately difficult, and common to all groups. We use a design contest as a vehicle for teaching design in the junior-year analog electronics course, in lieu of conventional laboratories. Students design and build analog circuitry to autonomously control a small robotic vehicle. The contest culminates in a competitive tournament. Students' questionnaire responses indicate that the contest is a useful learning tool, increasing interest in electrical engineering and well worth the time spent. They indicate that contests are preferable to conventional labs for learning and understanding course material, for motivating them, and for providing an engineering experience.**

***Index Terms*— Capstone course, design contest, design course, integrated design, juniors.**

I. Introduction

To GAIN mastery of the discipline, an electrical engineer (indeed any engineer) requires:

1) factual knowledge;
2) knowledge of engineering procedures;
3) the ability to identify key concepts;
4) the ability to acquire new knowledge;
5) judgment to use incomplete/contradictory information.

The normal engineering curriculum addresses items 1) and 2) through didactic learning. Items 3)–5) are developed largely through design experience gained on-the-job during co-op and internship placements or after graduation, not in the classroom. This is in part because it is difficult to teach concept identification, knowledge acquisition, and judgment other than through practice. It is also very difficult to assess students' performance in these areas principally because

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