Contents

Preface xi
List of Contributors xiii

1 Introduction 1
JOHN J. BENEDETTO AND PAULO J. S. G. FERREIRA
1.1 The Classical Sampling Theorem .......................... 1
1.2 Non-Uniform Sampling and Frames ....................... 10
1.3 Outline of the Book ......................................... 21
   1.3.1 Sampling, Wavelets, and the Uncertainty Principle .... 22
   1.3.2 Sampling Topics from Mathematical Analysis ........ 23
   1.3.3 Sampling Tools and Applications .................... 25

2 On the Transmission Capacity of the “Ether” and Wire in Electrocommunications 29
V. A. KOTELENIKOV (TRANSLATED BY V. E. KATSNELSON)

I Sampling, Wavelets, and the Uncertainty Principle 49

3 Wavelets and Sampling 51
GILBERT G. WALTER
3.1 Introduction .................................................. 51
   3.1.1 Background ............................................. 51
   3.1.2 The RKHS Setting ..................................... 52
   3.1.3 The Wavelet Setting ................................... 53
3.2 Sampling in Other Spaces .................................. 54
   3.2.1 Impulse Train Convergence ............................. 55
   3.2.2 RKHS and Sampling .................................... 59
3.3 Sampling in Wavelet Subspaces ............................. 61
   3.3.1 Elements of Wavelet Theory ........................... 61
   3.3.2 Sampling Functions .................................... 62
   3.3.3 Sampled Values as Coefficients ..................... 65
3.4 Interpolating Multiscalelets ............................... 68
   3.4.1 Properties of Hermite Sampling Functions ........... 70
Contents

4 Embeddings and Uncertainty Principles for Generalized Modulation Spaces 75
    J. A. HOGAN AND J. D. LAKEY
    4.1 Introduction ................................................. 75
      4.1.1 Weighted Fourier Inequalities Imply Uncertainty Principle Inequalities ............ 77
      4.1.2 Sharp Constants and Endpoint Estimates ............ 78
      4.1.3 Embeddings for Modulation Spaces ............ 79
    4.2 The Short-Time Fourier Transform and Modulation Spaces 79
      4.2.1 Weighted Fourier Inequalities Imply Modulation Embeddings ...................... 82
      4.2.2 Representation Theory and the Link with Littlewood-Paley Theory ................. 83
      4.2.3 More Remarks on the Uncertainty Principle ............ 84
    4.3 Modulation Embeddings and Uncertainty Principles ............ 84
      4.3.1 Gröchenig’s Two-Sided Embedding Theorem ............ 84
      4.3.2 Lieb’s Sharp Single-Sided Embedding Theorem ............ 90
    4.4 Symmetric Localization ........................................ 91
    4.5 Generalized Modulation Spaces ................................ 95
      4.5.1 Generalized Square-Integrability, Frames, and the Metaplectic Group ............. 96
      4.5.2 The Metaplectic Group ...................... 97
      4.5.3 Generalized Square Integrability ............ 98
      4.5.4 Connection with Time-Frequency Localization ............ 99
      4.5.5 Metaplectic Frames ...................... 99
    4.6 Generalized Modulation Spaces and Embedding ............ 102

5 Sampling Theory for Certain Hilbert Spaces of Bandlimited Functions 109
    JEAN-PIERRE GABARDO
    5.1 Introduction ................................................. 109
    5.2 Notation ................................................. 110
    5.3 Positive-Definite Distributions on \((-\mathcal{R},\mathcal{R})\) .................... 111
    5.4 The Case Where \(\mathcal{M}_R(\mu)\) Has Finite Codimension in \(L^2_{\mu}\) ............ 118
    5.5 The General Case ............................................. 131
    5.6 Riesz Bases and Frames ........................................ 136

6 Shannon-Type Wavelets and the Convergence of Their Associated Wavelet Series 139
    AHMED I. ZAYED
    6.1 Introduction ................................................. 139
    6.2 Preliminaries ................................................. 140
    6.3 Shannon’s Wavelet ............................................. 143
    6.4 Generalized Shannon Wavelets ................................ 144
    6.5 Pointwise Convergence of Shannon-type Wavelet Series ............ 151
II Sampling Topics from Mathematical Analysis 157

7 Non-Uniform Sampling in Higher Dimensions: From Trigonometric Polynomials to Bandlimited Functions 159
KARLHEINZ GRÖCHENIG
7.1 Introduction ............................................. 159
7.2 Non-Uniform Sampling With Trigonometric Polynomials .................................. 161
7.3 Toward Bandlimited Functions ...................................... 165
7.4 Proof of Theorem 7.1 ....................................... 171

8 The Analysis of Oscillatory Behavior in Signals Through Their Samples 177
RODOLFO H. TORRES
8.1 Introduction and Motivation ................................... 177
8.2 Besov Spaces of Functions .................................... 182
8.3 Sampling Theorem for Besov Spaces and Their Discrete Counterpart .................... 186
8.4 Other Characterizations of the Discrete Besov Spaces .................................. 189
8.5 Nonlinear Approximation of Band Limited Signals Using Samples .................... 192
8.6 Concluding Remarks .......................................... 196

9 Residue and Sampling Techniques in Deconvolution 197
STEPHEN CASEY AND DAVID WALNUT
9.1 Introduction .................................................. 197
9.1.1 Statement of the Problem .................................. 197
9.1.2 The Coprime Condition and Local Deconvolution .................................. 198
9.1.3 Other Types of Deconvolvers .................................. 199
9.1.4 Example: Cubes in $\mathbb{R}^d$ .................................. 200
9.1.5 Example: Balls in $\mathbb{R}^d$ .................................. 201
9.1.6 Practical Deconvolution ..................................... 202
9.2 Residue Methods for Deconvolution .................................. 203
9.3 Sampling Methods for Deconvolution .................................. 214
9.3.1 Deconvolution and Sets of Uniqueness in $C^{m-2}[-R, R]$ .......................... 214
9.3.2 Nonperiodic Frames and Bases for $L^2[-R, R]$ .................................. 219
9.3.3 Deconvolution and the Completeness of $\mathcal{F}$ .................................. 221

10 Sampling Theorems from the Iteration of Low Order Differential Operators 223
J. R. HIGGINS
10.1 Introduction .................................................. 223
10.2 Preliminary Facts and Methods .................................. 224
10.3 Examples of the Method ......................................... 227
10.4 Remarks and Open Problems ...................................... 231
Contents

11 Approximation of Continuous Functions by Rogosinski–Type Sampling Series 233
Andi Kivinukk
11.1 Notation and Introduction to the Sampling Series 233
11.2 Rogosinski-Type Sampling Series 236
11.3 Applications to Generalized Sampling Series 244
11.4 Conclusion 248

III Sampling Tools and Applications 249

12 Fast Fourier Transforms for Nonequispaced Data: A Tutorial 251
Daniel Potts, Gabriele Steidl, and Manfred Tasche
12.1 Introduction 251
12.2 NDFT for Nonequispaced Data Either in Time or Frequency Domain 254
12.3 NDFT for Nonequispaced Data in Time and Frequency Domain 262
12.4 Roundoff Errors 264
12.5 Fast Bessel Transform 269

13 Efficient Minimum Rate Sampling of Signals with Frequency Support over Non-Commensurable Sets 275
Cormac Herley and Ping Wah Wong
13.1 Introduction 275
13.2 Periodic Non-Uniform Sampling 277
13.3 Reconstruction of a Discrete-Time Signal from $N$ Samples Out of $M$ 279
13.4 Filter Design Using POCS 287
13.5 Minimum Rate Sampling of Multiband Signals 288
13.6 Slicing the Spectrum 292
13.6.1 Dividing the Bands into Slices 294
13.6.2 Freedom in Pairing 294
13.6.3 Pairing the Edges 295

14 Finite and Infinite-Dimensional Models for Oversampled Filter Banks 297
Thomas Strohmer
14.1 Introduction 297
14.1.1 Oversampled Filter Banks and Frames 299
14.2 Finite-Dimensional Models for Filter Banks 300
14.2.1 The Finite Section Method for Oversampled Filter Banks 301
14.2.2 Finite Sections, Laurent Operators, and Polyphase Matrices .......................... 302
14.2.3 Polyphase Matrices and “Perfect Symbol Calculus” in Finite Dimensions ................ 304
14.3 Convergence and Rates of Approximation for Finite Dimensional Approximations .................. 305
14.4 Rates of Approximation for Oversampled Filter Banks ........................................... 306
14.5 Convergence Using Polyphase Representation ....................................................... 309
14.5 Finite-Dimensional Approximation of Paraunitary Filter Banks Via $S^\perp$ ................. 313
14.6 Oversampled DFT Filter Banks — Beyond Polyphase Representation ........................ 315
14.6.1 Matrix Factorization of the Frame Operator ...................................................... 316

15 Statistical Aspects of Sampling for Noisy and Grouped Data 321
M. PAWLAK AND U. STADTMÜLLER
15.1 Introduction ................................................. 321
15.2 Sampling from Noisy Data and Signal Recovery ................................................. 323
15.3 Signal Recovery from Grouped Data ......................................................... 329
15.4 Statistical Accuracy ........................................... 331
15.5 Statistical Accuracy ........................................... 331
15.6 Statistical Accuracy ........................................... 338
15.6 Concluding Remarks ........................................... 338
15.6.1 Matrix Factorization of the Frame Operator ...................................................... 340

16 Reconstruction of MRI Images from Non-Uniform Sampling and Its Application to Intrascan Motion Correction in Functional MRI 349
MARC BOURgeois, FRANK T. A. W. WajER, DIRk VAN OrMondT, AND DANIELLE GRAVERon-DEmILLY
16.1 Introduction ................................................. 349
16.2 MRI Sampling Trajectories ................................................. 350
16.3 Motion Influence and Correction of Intrascan Motion Artifacts ...................................... 352
16.3.1 Physiological and Subject Motions ................................................. 352
16.3.2 Reduction of Motion Artifacts in Images ................................................. 353
16.3.3 Correction of Intrascan Motions in fMRI ................................................. 353
16.4 Image Reconstruction from Non-Uniform Sampling ................................................. 356
16.4.1 Gridding from a Non-Uniform Grid to a Cartesian Grid and Vice Versa ...................... 356
16.4.2 Density Correction and Voronoi Algorithm ................................................. 357
16.4.3 Limits of Gridding ................................................. 358
16.5 Bayesian Image Estimation ................................................. 359
16.5.1 Preamble ................................................. 359
16.5.2 The Most Probable Image ................................................. 360
Contents

16.5.3 The Likelihood .......................... 361
16.5.4 The Prior .............................. 361
16.5.5 Calculation of $T_1$ ...................... 364
16.6 Applications of Intrascan Motion Correction to fMRI ... 364
16.6.1 Shepp-Logan Simulation .................. 364
16.6.2 Simulated fMRI Experiment ............... 365
16.7 Conclusions .............................. 368

17 Efficient Sampling of the Rotation Invariant Radon Transform 371
LAURENT DESBAT AND CATHERINE MENNESSIER
17.1 Introduction ............................. 371
17.1.1 Principle .............................. 371
17.1.2 Inverse Problem Formalism ............... 372
17.2 Efficient Sampling in 2D Tomography ............ 375
17.2.1 Results in Standard Tomography .......... 375
17.2.2 Generalization to the Rotation Invariant
Radon Transform ............................. 377
17.3 Application to Doppler Imaging ............... 380
17.3.1 Null Inclination Case: $\alpha = 0$ ........ 380
17.3.2 Perpendicular Inclination Case: $\alpha = \pi/2$ 380
17.4 Discussion .............................. 382

References 385

Index 420