Deduction-Detachment Theorem in Hidden \( k \)-logics

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Abstract. Hidden \( k \)-logics, as a natural generalization of \( k \)-deductive systems, were introduced by Martins and Pigozzi in [4], where questions about specification and verification of software systems were investigated using tools from abstract algebraic logic (AAL). This generalization of the AAL theory has been successfully explored (cf. [3–5]). The theory of hidden \( k \)-logics provides a unified approach for several logical systems such as assertional logics, Boolean logics (1-dimensional multisorted logics with Boolean as the only visible sort, and with equality-test operations for some of the hidden sorts in place of equality predicates), equational and inequational logics, and the hidden versions of all of these.

In the onesorted case there is a precise meaning for a deductive system to possess the deduction detachment theorem (DDT) (cf. [1] and [2]). This has been intensively studied for several logical systems individually. In the abstract algebraic logic setting, the deduction theorem is examined through another perspective: sufficient and necessary conditions for DDT to hold are investigated for classes of deductive systems (cf. [1]). In this work, we propose and discuss a natural generalization of DDT for hidden \( k \)-logics and we develop an introductory theory.

We also consider an equivalence relation between hidden \( k \)-logics whose signatures may differ only on the set of visible sorts. Intuitively, two hidden \( k \)-logics are considered equivalent if there are two interpretations between them which are inverses of one another. Our work is inspired by the theory developed by Blok, Czelakowski and Pigozzi concerning equivalence between onesorted logics (cf. [1] and [2]). We show that DDT is preserved under our notion of equivalence of logics (i.e., if two sorted hidden \( k \)-logics are equivalent then one has DDT if and only the other has too).

Finally, we show how the underlying consequence relation of a hidden \( k \)-logic can be simplified in the presence of DDT.

Keywords: hidden \( k \)-logic, equivalence between logics, deduction-detachment theorem, Craig interpolation property.

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References