Processos e Concorrência 2015/16
Bloco de acetatos 6

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mCRL2: A toolset for process algebra

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- a generic **process algebra**, based on ACP (Bergstra & Klop, 82), in which other calculi can be **embedded**

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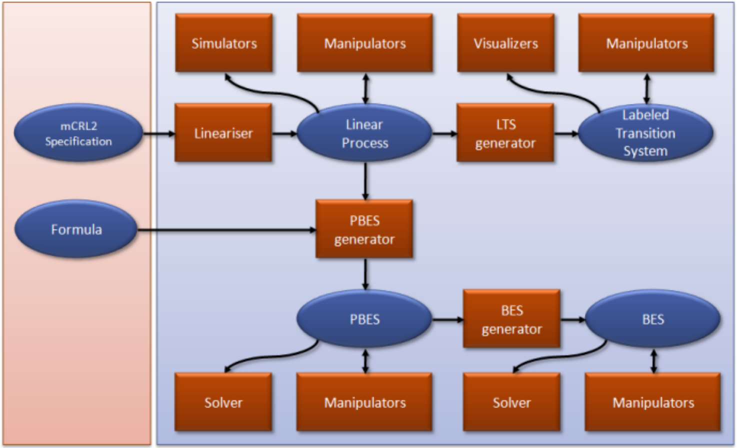
- a generic **process algebra**, based on ACP (Bergstra & Klop, 82), in which other calculi can be **embedded**
- extended with data and (real) time
- with an axiomatic semantics
- the full μ-**calculus** as a specification logic
- powerful toolset for **simulation** and **verification** of reactive systems

www.mcrl2.org

Toolset overview



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Interaction through multisets of actions

- A multiaction is an elementary unit of interaction that can **execute itself atomically in time** (no duration), after which it terminates successfully

$$\alpha \quad ::= \quad \tau \;\mid\; a \;\mid\; a(d) \;\mid\; \alpha \mid \alpha$$

- actions may be parametric on data
- the structure $\langle N, |, \tau \rangle$ forms an Abelian **monoid**

Sequential, non deterministic behaviour

The set $\mathbb{P}$ of processes is the set of all terms generated by the following BNF, for $a \in N$,

$$p \ ::= \ \alpha \ | \ \delta \ | \ p + p \ | \ p \cdot p \ | \ \mathsf{P}(d)$$

- choice: $+$
- sequential composition: $\cdot$
- inaction or deadlock: δ (it cannot even to terminate!)
- process references introduced through definitions of the form $\mathsf{P}(x : D) = p$, parametric on data

Axioms: : $+$, $\cdot$, δ

$A1$	$x + y$	$= y + x$
$A2$	$(x + y) + z$	$= x + (y + z)$
$A3$	$x + x$	$= x$
$A4$	$(x + y).z$	$= x.z + y.z$
$A5$	$(x.y).z$	$= x.(y.z)$
$A6$	$x + \delta$	$= x$
$A7$	$\delta \cdot x$	$= \delta$

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- the equality relation is sound: if $s = t$ holds for basic process terms, then $s \sim t$
- and complete: if $s \sim t$ holds for basic process terms, then $s = t$
- an axiomatic theory enables equational reasoning

Sequential Processes

Exercise

Describe the behaviour of

- $a.b.\delta.c + a$
- $(a + b).\delta.c$
- $(a + b).e + \delta.c$
- $a + (\delta + a)$
- $a.(b + c).d.(b + c)$

Axioms: : $+$, $\cdot$, δ

Exercise

- show that $\delta.(a+b) \; = \; \delta \cdot a + \delta \cdot b$
- show that $a + (\delta + a) \; = \; a$
- is it true that $a.(b+c) \; = \; a.b + a.c$?

Alternative composition

We have also this kind of processes:

$$c \to p \diamond q$$

where

- c is a boolean condition
- p and q are processes

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where

- c is a boolean condition
- p and q are processes

Axioms

Cond1	$true \to x \diamond y \; = \; x$
Cond2	$false \to x \diamond y \; = \; y$
Then	$c \to x \; = \; c \to x \diamond \delta$

mCRL2: A toolset for process algebra

Example

```
act    order, receive, keep, refund, return;

proc   Buy = order.OrderedItem

       OrderedItem = receive.ReceivedItem + refund.Buy;
       ReceivedItem = return.OrderedItem + keep;

init   Buy;
```

Example

Clock

```
act    set, alarm, reset;

proc   P = set.R
       R = reset.P + alarm.R

init   P
```

Example

A refined clock

```
act    set:N, alarm, reset, tick;

proc   P = (sum n:N . set(n).R(n)) + tick.P
       R(n:N) = reset.P + ((n == 0) -> alarm.R(0) <> tick.R(n-1))

init   P
```

Parallel composition

$\|$ = **interleaving** + **synchronization**

- modelling principle: **interaction** is the key element in software design
- modelling principle: (distributed, reactive) **architectures** are configurations of communicating black boxes
- mCRL2: supports flexible **synchronization** discipline ($\neq$ CCS)

$$p \ ::= \ \cdots \ | \ p \parallel p \ | \ p \mid p \ | \ p \mathbin{\lfloor\!\lfloor} p$$

Parallel composition

- parallel $p \parallel q$: interleaves and synchronises the actions of both processes.
- synchronisation $p \mid q$: synchronises the first actions of p and q and combines the remainder of p with q with $\parallel$, cf axiom:

$$(a.p) \mid (b.q) \sim (a \mid b) . (p \parallel q)$$

- left merge $p \mathbin{⌊\!⌊} q$: executes a first action of p and thereafter combines the remainder of p with q with $\parallel$.

Parallel composition

A semantic parenthesis

Lemma: There is no sound and complete finite axiomatisation for this process algebra with $\|$ modulo bisimilarity [F. Moller, 1990].

Solution: combine two auxiliar operators:

- left merge: $\mathbin{\lfloor\!\lfloor}$
- synchronous product: $|$

such that

$$p \parallel t \;\sim\; (p \mathbin{\lfloor\!\lfloor} t + t \mathbin{\lfloor\!\lfloor} p) + p \mid t$$

Parallel composition

An example

$$\boxed{a \cdot b \parallel c \cdot d}$$

Parallel composition

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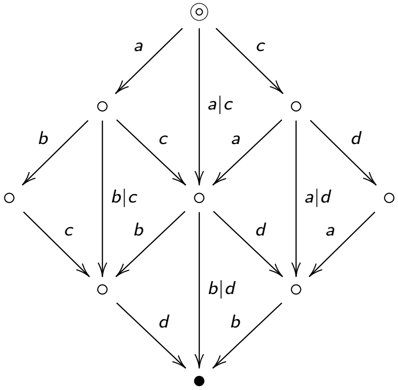

Parallel composition

An example

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Parallel composition

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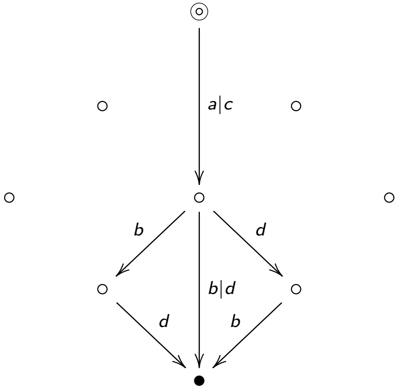


Interaction

Communication $\Gamma_C(p)$ (com)

- applies a **communication function** C forcing action synchronization and renaming to a new action:

$$a_1 \mid \cdots \mid a_n \;\rightarrow\; c$$

- data parameters are retained in action c, e.g.

$$\Gamma_{\{a|b\rightarrow c\}}(a(8) \mid b(8)) \;=\; c(8)$$
$$\Gamma_{\{a|b\rightarrow c\}}(a(12) \mid b(8)) \;=\; a(12) \mid b(8)$$
$$\Gamma_{\{a|b\rightarrow c\}}(a(8) \mid a(12) \mid b(8)) \;=\; a(12) \mid c(8)$$

- left hand-sides in C must be disjoint: e.g., $\{a \mid b \rightarrow c, a \mid d \rightarrow j\}$ is not allowed

Interface control

Restriction: $\nabla_B(p)$ (`allow`)

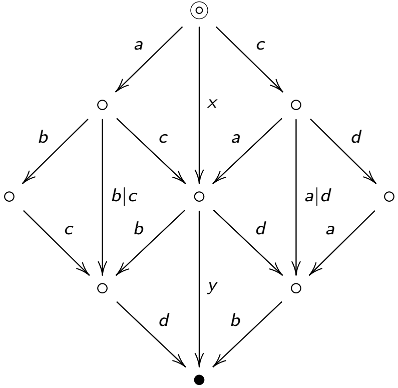
- specifies which actions are allowed to occur
- disregards the data parameters of actions

$$\nabla_{\{d,b|c\}}(d(12) + a(8) + (b(false, 4) \mid c)) \;=\; d(12) + (b(false, 4) \mid c)$$

- τ is always allowed to occur

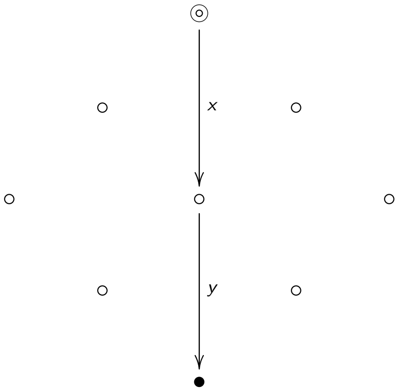
Discuss: $\nabla_{\{x,y\}}(\Gamma_{\{a|c->x,b|d->y\}}(a.b \parallel c.d))$

An example



$$\Gamma_{\{a|c->x,b|d->y\}}(a.b \parallel c.d)$$

An example



$$\nabla_{\{x,y\}}(\Gamma_{\{a|c->x,b|d->y\}}(a.b \parallel c.d))$$

Interface control

Block: $\partial_B(p)$ (`block`)

- specifies which actions are not allowed to occur
- disregards the data parameters of actions

$$\partial_{\{b\}}(d(12) + a(8) + (b(\mathit{false}, 4) \mid c)) \;=\; d(12) + a(8)$$

- the effect is that of renaming to δ
- τ cannot be blocked

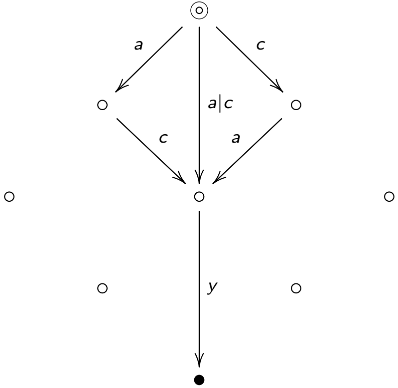
An example

$$\boxed{\partial_{\{b,d\}}(\Gamma_{\{b|d->y\}}(a.b \parallel c.d))}$$

Interface control

An example

$$\boxed{\partial_{\{b,d\}}(\Gamma_{\{b|d->y\}}(a.b \parallel c.d))}$$



Interface control

Enforce communication

- $\nabla_{\{c\}}(\Gamma_{\{a|b \to c\}}(p))$

Interface control

Enforce communication

- $\nabla_{\{c\}}(\Gamma_{\{a|b \to c\}}(p))$
- $\partial_{\{a,b\}}(\Gamma_{\{a|b \to c\}}(p))$

Renaming $\rho_M(p)$ (rename)

- renames actions in p according to a mapping M
- also disregards the data parameters, but when a renaming is applied the values of data parameters are retained:

$$\begin{aligned}\rho_{\{d \to h\}}&(d(12) + s(8) \mid d(\mathit{false}) + d.a.d(7)) \\ &= h(12) + s(8) \mid h(\mathit{false}) + h.a.h(7)\end{aligned}$$

- τ cannot be renamed

Interface control

Hiding $\tau_H(p)$ (hide)

- hides (or renames to τ) all actions in H in all multiactions of p.
- disregards the data parameters

$$\begin{aligned} &\tau_{\{d\}}(d(12) + s(8) \mid d(\mathit{false}) + h.a.d(7)) \\ &\quad = \tau + s(8) \mid \tau + h.a.\tau \;=\; \tau + s(8) + h.a.\tau \end{aligned}$$

- τ and δ cannot be renamed

An example

$$\boxed{\tau_{\{a\}}(\Gamma_{\{b|d->y\}}(a.b \parallel c.d))}$$

An example

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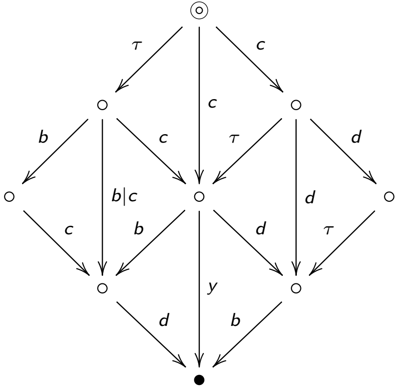

Example

New buffers from old

```
act    inn,outt,ia,ib,oa,ob,c :  Bool;

proc   BufferS = sum n:  Bool.inn(n).outt(n).BufferS;

       BufferA = rename({inn -> ia, outt -> oa}, BufferS);
       BufferB = rename({inn -> ib, outt -> ob}, BufferS);

       S = allow({ia,ob,c}, comm({oa|ib -> c}, BufferA || BufferB));

init   hide({c}, S);
```

Data types

- Equalities: equality, inequality, conditional (`if(-,-,-)`)
- Basic types: booleans, naturals, reals, integers, ... with the usual operators
- Sets, multisets, sequences ... with the usual operators
- Function definition, including the λ-notation
- Inductive types: as in

```
sort   BTree = struct leaf(Pos) | node(BTree, BTree)
```

Signatures and definitions

Sorts, functions, constants, variables ...

```
sort   S, A;

cons   s,t:S, b:set(A);

map    f:  S x S -> A;
       c:  A;

var    x:S;

eqn    f(x,s) = s;
```

Signatures and definitions

A full functional language ...

```
sort   BTree = struct leaf(Pos) | node(BTree, BTree);

map    flatten:  BTree -> List(Pos);

var    n:Pos, t,r:BTree;

eqn    flatten(leaf(n)) = [n];
       flatten(node(t,r)) = flatten(t) ++ flatten(r);
```

Processes with data

Why?

- Precise modeling of real-life systems
- Data allows for finite specifications of infinite systems

How?

- data and processes parametrized
- summation over data types: $\sum_{n:N} s(n)$
- processes conditional on data: $b \to p \diamond q$

A counter

```
act    up, down;
       setcounter:Pos;

proc   Ctr(x:Pos) = up.Ctr(x+1)
             + (x>0) -> down.Ctr(x-1)
             + sum m:Pos.(setcounter(m).Ctr(m))

init   Ctr(345);
```

Examples

A dynamic binary tree

```
act    left,right;

map    N:Pos;

eqn    N = 512;

proc   X(n:Pos)=(n<=N)->(left.X(2*n)+right.X(2*n+1))<>delta;

init   X(1);
```