Hybrid models for hardware-in-the-loop simulation of hydraulic systems. Part 2: Experiments

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Abstract

The use of new control schemes for hydraulic systems has been the object of study during the last years. A simulated environment is the cheapest and fastest way of evaluating the relative merits of different control schemes for a given application. The real time simulation allows the parameterization and test of the performance of real controllers. This paper describes the setup of a real time simulation platform to perform hardware-in-the-loop simulation experiments with the hydraulic models proposed in a companion paper (Part 1). A set of parameterization techniques are proposed for the semi-empirical models of a valve controlled hydraulic cylinder. Manufacturers data sheets and/or experimental measurements were used to adjust the model parameters. Some of them were directly calculated and others were estimated through the use of optimization techniques. Closed loop control experiments were then performed on the real time simulation platform, and on the real system, in order to evaluate the real time performance of the developed models.

Keywords: fluid power, modeling, real time simulation, hardware-in-the-loop.

NOTATION

- \( A_1, A_2 \) cylinder chamber areas
- \( A_{s1}, A_{s2} \) pseudo sections
- \( F_{COn,p} \) Coulomb friction for negative/positive velocities
- \( F_f \) friction force
- \( F_L \) load applied force
- \( f_n, \omega_h \) natural (angular) frequency
- \( F_{SOn,p} \) Stribeck friction for negative/positive velocities
- \( g \) acceleration of gravity
- \( g_{lc} \) cylinder leakage conductance
- \( k_1, k_2, k_3, k_4, k_5 \) pseudo section parameters
- \( k_{s1}, k_{s2}, k_{s3}, k_{s4}, k_{s5} \) pseudo section parameters
- \( \overline{K}_{q0} \) flow gain \( \overline{x}_s = 0 \)
- \( \overline{K}_{p0} \) relative pressure gain at \( \overline{x}_s = 0 \)
- \( K_{vm,p} \) viscous friction for negative/positive velocities
- \( L \) cylinder maximum stroke
- \( L_h, L_a \) spool velocity and acceleration limits
- \( M \) connected mass (load, piston, rod)
- \( P_1, P_2 \) cylinder chamber relative pressures
- \( P_i \) relative pressure at valve port \( i \)
- \( P_L \) load pressure drop
- \( P_n \) nominal pressure drop
- \( Q_{i}, Q_j \) outlet ports volumetric flow rate
- \( q_{ij} \) volumetric flow rate from port \( i \) to port \( j \)
- \( Q_L \) load volumetric flow rate
- \( q_{lk} \) leakage volumetric flow rate
- \( q_{l0} \) leakage flow at \( x_s = 0 \)
- \( q_{lc} \) cylinder leakage volumetric flow rate
- \( Q_u \) nominal volumetric flow rate
- \( Q_e, Q_i \) tank and source volumetric flow rate
- \( \overline{u} \) normalized valve input, \( \overline{u} \in [-1, 1] \)
- \( V_{L1}, V_{L2} \) enclosed volumes at line 1 and 2
\( v_{Sn,p} \) Striebeck velocity for negative/positive velocities
\( v_p \) piston velocity
\( v_s \) Striebeck velocity
\( x_p \) piston position
\( \bar{x}_s \) normalized valve spool position, \( \bar{x}_s \in [-1,1] \)
\( z \) seal deformation (friction model)
\( \xi \) damping ratio
\( \beta \) oil bulk modulus
\( \beta_{e1} \) chamber 1 effective bulk modulus
\( \beta_{e2} \) chamber 2 effective bulk modulus
\( \Delta P_{ij} \) pressure drop between port \( i \) and port \( j \)
\( \Delta P_{m} \) pressure difference to middle point
\( \sigma_0 \) seal stiffness (friction model)
\( \sigma_1 \) seal damping coefficient (friction model)

1 INTRODUCTION

The use of new control schemes for hydraulic systems has been the object of study during the last years (1). It is commonly accepted that a simulated environment is the cheapest and fastest way for the evaluation of the relative merits of different control schemes for a given application. Modelling and real time simulation of complex systems still is, referring Burrows (2), an area to explore. In fact, and according to Lennevi et al (3), with the growing of computing power, more and more complex systems can be simulated in real time, with decreasing costs.

Hardware-in-the-loop simulation (HILS) refers to a technology in which some of the components of a pure simulation are replaced with the respective hardware component. This type of procedure is useful, for example, to test a controller which, instead of being connected to the real equipment under control, is connected to a real time simulator. The controller must “think” that it is working with the real system and so the accuracy of the simulation and its electrical interfacing to the controller must be adequate. This technology provides a mean for testing control systems over the full range of operating conditions, including failure modes. Testing a control system prior to its use in a real plant can reduce the cost and the development cycle of the overall system. HILS have been used with success in the aerospace industry and is now emerging as a technique for testing electronic control units (4, 5). This procedure has been applied to solve some specific problems but is seldom used as a platform to test the real time behaviour of hardware components. The implementation of hardware-in-the-loop simulation is important for performance analysis of components or systems, and also for control algorithm validation. The real time code should be generated through model descriptions. This code can be executed afterwards in dedicated hardware, in order to guarantee enough performance for real time execution.

The following section presents the hardware setup for the valve and cylinder model parameterization and the real time simulation platform to perform hardware-in-the-loop simulation experiments.
2 HARDWARE SETUP AND HILS PLATFORM

A hydraulic apparatus that consists on a linear hydraulic actuator driven by a servo-solenoid valve, as shown in Fig. 1, was developed for the identification of model’s parameters and to perform hardware-in-the-loop simulation experiments. The system is equipped with a set of sensors to measure the system pressures and piston position. The valve ports and cylinder chambers pressures, \((P_1, P_2, P_s, P_t)\), are measured using four analogue pressure sensors. The cylinder rod position is acquired with a linear digital encoder with 1 \(\mu m\) of resolution. The velocity was obtained by differentiation of the position signal. All the sensors and the valve electrical input are connected to a low cost DSP based real time card (RTC) from dSPACE®(6), model DS1102, in such a way that real time control and data acquisition can be performed.

![Fig. 1 Hydraulic testbed (draft and real system).](image)

To perform hardware-in-the-loop simulation experiments (see section 4) two DS1102 boards, installed in two different personal computers, were used as shown in Fig. 2.

![Fig. 2 Hardware-in-the-loop simulation platform.](image)
The control algorithms run in one of the RTC, being the other responsible to run the real time simulation of the cylinder and valve models. The real system is then connected to the controller, through a double switch, to acquire the data used in the HILS performance evaluation.

3 PARAMETER IDENTIFICATION AND PARTIAL RESULTS

This section presents the strategies and experiments performed for the identification of the parameters of the hybrid models proposed in [part1] (7).

3.1 Valve model

3.1.1 Spool motion model parameters

The spool motion model reproduces the frequency response amplitudes with a second order model with acceleration and velocity saturation, with the phase lag adjusted with a delay.

![Fig. 3 Dynamic model for spool position with velocity and acceleration limits.](image)

The least squares method was used for the parameter estimation. The block diagram model, shown in Fig. 3, was simulated over a frequency range of 10 to 300 Hz in steps of 10 Hz. The parameters were adjusted using three Bode amplitude curves available in the manufacturer data sheet (5%, 25% and 50% of maximum amplitude). A variable frequency (and amplitude) sine wave (\( \Hat{u} \)) was applied to the input of the dynamic model. The output spool position (\( \Hat{x} \)) was then used to evaluate the output gain in dB (\( G_{sn} \)). This gain was then compared with the data sheet gain at the same frequency and amplitude (\( G_{r} \)). The cost function \( F \) is calculated for each set of model parameters (\( \omega_n, \xi, L_v, L_a \)). The model parameters were selected for the minimum value of the function \( F \). Because most of the valve action takes place near the middle position, a weighting factor of four was applied on the 5% quadratic error when calculating the cost function value.

\[
\Hat{u} = A_{in} \cdot \sin(2\pi \cdot f_n \cdot t) ; \quad A_{out} = \left| \Hat{r} \right|
\]

where \( n = \{1,2,...,29,30\} \), \( f_n = 10n \) and \( t \) is the variable time.

The amplitude gain, \( G_S = 20 \cdot \log \left( \frac{A_{out}}{A_{in}} \right) \), is used to calculate the cost function:

\[
F(\omega_n, \xi, L_v, L_a) = 4 \sum_{n=1}^{30} \left( \frac{G_{sn} - G_{r}}{2} \right)^2 \left|_{\omega_n = 5\%} \right| + \sum_{n=1}^{30} \left( \frac{G_{sn} - G_{r}}{2} \right)^2 \left|_{\omega_n = 25\%} \right| + \sum_{n=1}^{30} \left( \frac{G_{sn} - G_{r}}{2} \right)^2 \left|_{\omega_n = 50\%} \right|
\]

where \( \omega_n \) is the frequency and \( G_{sn} \) is the calculated gain at each frequency.
The following parameter set minimizes the cost function for the selected model:

\[ \omega_c = 1007.01 \text{ rad/s}, \quad \xi = 0.48, \quad L_c = 125.56 \text{ s}^{-1}, \quad L_u = 81184.24 \text{ s}^{-2}. \]

The simulation results (dotted lines), presented in Fig. 4, show that the amplitude effects of non-modeled dynamic behaviour are more visible for frequencies higher than 200Hz.

Fig. 4  Datasheet and simulated (dotted lines) bode diagrams for amplitude and phase lag response (with courtesy of Eaton Corporation).

To adjust the phase curve for the different amplitudes a delay was used. The approach for the delay estimation was identical to the one used for the amplitude response parameters. Analyzing the results \(( \Delta t = 7.625 \times 10^{-7} \text{s} )\), presented in Fig. 4, it can be concluded that very good results are obtained for the 5% input variation (for the valve application frequency range).

### 3.1.2 Static parameters

The static model equations proposed in (7) for a symmetrical but unmatched valve uses four pseudo sections functions \( A_{s1}(\overline{x}_s), A_{s2}(\overline{x}_s), A_{t1}(\overline{x}_t) \) and \( A_{t2}(\overline{x}_t) \) as follows:

\[
\begin{align*}
A_{s1}(\overline{x}_s) &= k_{s1} \cdot \overline{x}_s + k_{s2} + \sqrt{k_{s3} \cdot \overline{x}_s^2 + k_{s4} \cdot \overline{x}_s^2 + k_{s5}} \\
A_{s2}(\overline{x}_s) &= -k_{s1} \cdot \overline{x}_s + k_{s2} + \sqrt{k_{s3} \cdot \overline{x}_s^2 - k_{s4} \cdot \overline{x}_s + k_{s5}} \\
A_{t1}(\overline{x}_t) &= k_{t1} \cdot \overline{x}_t + k_{t2} + \sqrt{k_{t3} \cdot \overline{x}_t^2 + k_{t4} \cdot \overline{x}_t^2 + k_{t5}} \\
A_{t2}(\overline{x}_t) &= -k_{t1} \cdot \overline{x}_t + k_{t2} + \sqrt{k_{t3} \cdot \overline{x}_t^2 - k_{t4} \cdot \overline{x}_t + k_{t5}}
\end{align*}
\]

(3)

The \( k_i \) parameters of \( A_{s1}(\overline{x}_s), A_{s2}(\overline{x}_s), A_{t1}(\overline{x}_t) \) and \( A_{t2}(\overline{x}_t) \) can be estimated in order to reproduce the valve pressure gain and the valve flow gain. The valve used has the following static measured characteristics: \( Q_n = 25.5 \text{ l/min}, \quad P_n = 35 \text{ bar}, \quad P_s = 70 \text{ bar}, \quad \overline{K}_{p0} = 28 \text{ l/min}, \quad \overline{K}_{rho} = 36.5, \quad q_{l0} = 1.36 \text{ l/min}. \)

where \( \overline{K}_{rho} \) is the relative pressure gain at \( \overline{x}_s = 0, \overline{K}_{p0} \) is the flow gain at \( \overline{x}_s = 0, \overline{P}_n \) is the nominal pressure drop, \( q_{l0} \) is the leakage flow at \( \overline{x}_s = 0, Q_n \) is the nominal volumetric flow rate and \( P_n \) is the source pressure.
A characteristic of this type of valve is that the chamber pressures may not intercept at $P_s/2$, as can be seen in Fig. 5a. The actual valve has the interception point at 43 bar for $P_s = 70$ bar, thus having a difference of $\Delta P_m = 8$ bar relative to $P_s/2$. Using equations (2) and (4) (presented in Part 1) and considering that ports 1 and 2 are closed (pressure gain measurement), that is $Q_1 = Q_2 = 0$, the following relation can be set for the pressure difference at middle point:

$$\Delta P_m = \frac{P_s A_{22} (x)^2 - A_{21} (x)^2}{2 A_{22} (x)^2 + A_{21} (x)^2}$$  \hspace{1cm} (4)

Using again (2), (4) (of Part 1) and $Q_1 = Q_2 = 0$, the relative load pressure is given by:

$$\bar{P}_l (x) = \frac{A_{21} (x)^2}{A_{21} (x)^2 + A_{21} (x)^2} = \frac{A_{22} (x)^2}{A_{22} (x)^2 + A_{22} (x)^2}$$  \hspace{1cm} (5)

where $P_l = P_s - P_t$ and $\bar{P}_l = \frac{P_t}{P_s}$.

The pressure gain is then defined as:

$$K_{pl} = \frac{\partial P_l (x)}{\partial x_s} \bigg|_{x_s=0}$$  \hspace{1cm} (6)

When measuring the flow gain, that is connecting port 1 to port 2 (see Fig. 3 of Part 1) with a null resistance, the load flow rate, $Q_L$, can be expressed by $Q_1$ or by $Q_2$. Then using (2), (4) (of Part 1) and the chamber pressure difference to middle point, $\Delta P_m$, the load flow is given by the following equation when $Q_L = Q_1$:

$$Q_L (x) = A_{21} (x) \sqrt{\frac{P_s}{2} - \Delta P_m} - A_{22} (x) \sqrt{\frac{P_s}{2} + \Delta P_m}$$  \hspace{1cm} (7)

and the flow gain can be written as:

$$\bar{K}_{qL} = \frac{\partial Q_L (x)}{\partial x_s} \bigg|_{x_s=0} = \frac{\partial A_{21} (x)}{\partial x_s} \bigg|_{x_s=0} \sqrt{\frac{P_s}{2} - \Delta P_m} - \frac{\partial A_{22} (x)}{\partial x_s} \bigg|_{x_s=0} \sqrt{\frac{P_s}{2} + \Delta P_m}$$  \hspace{1cm} (8)

Using $Q_L = Q_2$, the flow gain can also be expressed by:

$$\bar{K}_{qL} = \frac{\partial A_{22} (x)}{\partial x_s} \bigg|_{x_s=0} \sqrt{\frac{P_s}{2} + \Delta P_m} - \frac{\partial A_{21} (x)}{\partial x_s} \bigg|_{x_s=0} \sqrt{\frac{P_s}{2} - \Delta P_m}$$  \hspace{1cm} (9)

The flow outside the origin area can be adjusted with the nominal flow ($Q_n$) and nominal pressure ($P_n$), that can be measured for a specific valve or are available in the manufacturer’s data sheet, as $A_{11} (x) = A_{21} (x) \approx 0$ for $x = 1$.  


The leakage flow can be expressed as a function of the relative valve chamber pressures, \( P_1 = P_1 / P_s \) and \( P_2 = P_2 / P_s \) using (2) and (3) (of part 1) when \( Q_1 = Q_2 = 0 \):

\[
\frac{Q}{\sqrt{P_s}} = A_{i1}(\bar{x}) \bigg|_{\bar{x}=1} 
\]

\[
\frac{Q}{\sqrt{P_s}} = A_{i2}(\bar{x}) \bigg|_{\bar{x}=1} 
\]

Assuming the conditions of \( P_1 \approx 1 \) and \( P_2 \approx 0 \), and using (12) and (13), new relations can be stated for the leakage flow and leakage flow derivative at a certain spool position. If the leakage flow curve is available, a measurement at a certain position \( \bar{x} > 0 \) can be used, otherwise the leakage at \( \bar{x} = 1 \) can be set to a very small value or even zero.

\[
q_{\alpha}(\bar{x}) = q_{\alpha1} + q_{\alpha2} = A_{i1}(\bar{x})\sqrt{P_s(1 - \bar{P}_1)} + A_{i2}(\bar{x})\sqrt{P_s(1 - \bar{P}_2)} \quad (12) 
\]

\[
q_{\alpha}(\bar{x}) = q_{\alpha0} = A_{i1}(\bar{x})\sqrt{P_s\bar{P}_1} + A_{i2}(\bar{x})\sqrt{P_s\bar{P}_2} \quad (13) 
\]

Using (3) for the pseudo section functions, ten equations can be stated to solve for the \( k_i \) and \( k_{it} \) pseudo section parameters model. Thus, using (4), (5), (8), (9), (10), (11), (14), (15), (16) and (17), and considering that the leakage flows and their derivatives are zero for \( \bar{x} = 1 \) (where \( \bar{P}_1 = 1 \) and \( \bar{P}_2 = 0 \)), the pseudo-section equation parameters (see equation (3)), \( k_i \), are the following:

\[
k_{i1} = -2.136, \quad k_{i2} = 1.602 \times 10^{-2}, \quad k_{i3} = 4.561, \quad k_{i4} = -8.084 \times 10^{-2}, \quad k_{i5} = 1.563 \times 10^{-2}, 
\]

\[
k_{it1} = -2.145, \quad k_{it2} = 7.276 \times 10^{-3}, \quad k_{it3} = 4.602, \quad k_{it4} = -4.208 \times 10^{-2}, \quad k_{it5} = 1.092 \times 10^{-2}. 
\]

The results for the relative pressures and load flow rates obtained from the simulation of the static valve model are presented in Fig. 5.
3.2 Cylinder model

3.2.1 The effective bulk modulus

The effective bulk modulus, \( \beta_e \), was estimated through the comparison of the maximum cylinder piston acceleration and its occurring frequency (natural frequency of the system) with the results of the simulation of a linear version of the whole system. The experimental block diagram used to measure the natural frequency is shown in Fig. 6. The system has to run in closed loop around \( X_{po} \) because of the different cylinder areas and because of the difficulties to set the valve middle position. The system is linearized around \( [P_{10}, P_{20}, X_{po}, V_{po}, X_{o0}]^T \). These values are obtained from the steady state conditions of velocity and acceleration equal to zero, which occur for \( X_{o0} = 0.009 \) where \( P_1 = P_{10} \) and \( P_2 = P_{20} \). In this situation the resulting force is zero, that is, \( Ap_{10} + M \cdot g - A_2p_{20} = 0 \) and, with \( P_e = P_{10} + P_{20} \) (9), the equilibrium pressures are given by:

\[
P_{10} = \frac{A_2p_e - M \cdot g}{A_1 + A_2} \quad \text{and} \quad P_{20} = \frac{A_1p_e + M \cdot g}{A_1 + A_2}
\]

where \( g \) is the acceleration of gravity.
At the position $X_{p0} = 82\text{mm}$ the chamber volumes are almost the same. The cylinder areas are $A_1 = 1.2566 \cdot 10^{-3} \text{m}^2$ and $A_2 = 8.7650 \cdot 10^{-4} \text{m}^2$.

The valve static characteristics at the linearized points have the following measured values:

$k_{q1} = \frac{\partial Q_1}{\partial X_{p1}} \bigg|_{X_{p1}} = 4.76 \cdot 10^4 \frac{P}{V} \text{m}^3 \text{s}^{-1}$; $k_{q2} = \frac{\partial Q_2}{\partial X_{p2}} \bigg|_{X_{p2}} = 4.76 \cdot 10^4 \frac{P}{V} \text{m}^3 \text{s}^{-1}$

$k_{p1} = \frac{\partial P_1}{\partial X_{p1}} \bigg|_{X_{p1}} = 19 \cdot P \text{ Pa}$; $k_{p2} = \frac{\partial P_2}{\partial X_{p2}} \bigg|_{X_{p2}} = -15.6 \cdot P \text{ Pa}$

where $k_{q1}$ is the flow gain at $X_{p1}$, $k_{q2}$ is the flow gain at $X_{p2}$, $k_{p1}$ and $k_{p2}$ are the pressure gains (at in chamber 1 and at $X_{p2}$, and $k_{p2}$ is the pressure gain in chamber 2 at $X_{p2}$). The flow-pressure coefficients are then defined as:

$$k_{q2} = -\frac{k_{q2}}{k_{p1}}; \quad k_{q1} = \frac{k_{q1}}{k_{p1}} \tag{19}$$

The linearized cylinder and valve equations are expressed in state space format and were simulated in the Simulink® (10) environment.

$$\begin{bmatrix} \delta p_1 \\ \delta p_2 \\ \delta v_p \\ \delta x_p \end{bmatrix} = \begin{bmatrix} -\frac{\beta_p A_1}{V_1} & 0 & -\frac{\beta_p A_2}{V_2} & 0 \\ 0 & -\frac{\beta_p v_p}{V_2} & \frac{\beta_p A_2}{V_2} & 0 \\ \frac{A_1}{M} & \frac{A_2}{M} & -\frac{f}{M} & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} \delta p_1 \\ \delta p_2 \\ \delta v_p \\ \delta x_p \end{bmatrix} + \begin{bmatrix} \frac{\beta_p A_1}{V_1} & 0 \\ \frac{\beta_p v_p}{V_2} & \frac{\beta_p A_2}{V_2} \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \delta x_{p1} \\ \delta x_{p2} \end{bmatrix} \tag{20}$$

where $V_1 = V_{01} + A_1 \cdot X_{p0}$, $V_2 = V_{02} + A_2 \cdot (L - X_{p0})$ are the equilibrium volumes and $V_{01} = 3 \cdot 10^{-5} \text{m}^3$ and $V_{02} = 5 \cdot 10^{-5} \text{m}^3$ are used for the lines, dead volumes and valve chambers volumes. $M$ represents all the mass in motion and is $M = 80 \text{Kg}$. The linearized version of the friction model (Part 1) (7) is only valid for small displacements ($< 15 \text{\mu m}$) where the seal deformation (variable $z$) is equal to the piston.
displacement \((x_p)\), that is, it is assumed that the piston is in the stiction state. The friction factor \(f\) is then intended to model all the friction effects (valve and cylinder) that occur when the spool velocity sign have fast changes.

The effective bulk modulus, \(\beta_e\), and friction factor, \(f\), were estimated by an optimization process that minimizes the distance (in the acceleration/frequency plane) between the real and simulated piston maximum acceleration. The amplitudes of the acceleration signals measured with \(\beta_m = 7.7 \times 10^4 Pa\), \(\beta_{rz} = 9.6 \times 10^4 Pa\) and \(f = 8100 Nsm^{-1}\) are presented in Fig. 7. The piston acceleration was measured with a high bandwidth accelerometer from 1 to 130Hz. The experience shown in Fig. 6 was repeated for several source pressures in order to evaluate the effective bulk modulus as a function of the pressure. Figure 8 shows the evolution of the real \(\beta_e\) with the chamber pressure and \(\beta_e\) calculated with equation (21). The parameters for the equation (21) were obtained by optimization and have the values: \(B = 9.71 \times 10^{-10}\) and \(C = 1.15 \times 10^{-3}\).

\[
\beta_e = \frac{10^5 + P}{B \cdot P + C}
\]  

(21)

where \(B \left[ Pa^{-1}\right]\) and \(C\) are constants related to the oil characteristics of the model proposed by (10).

3.2.2 The cylinder leakage conductance

The internal cylinder leakage is assumed laminar and is represented by a conductance defined as:

\[
g_{le} = \frac{q_{le}}{(P_i - P_2)}. \]

(22)

The leakage flow rate is measured indirectly in the following way. In the initial piston position, \(x_p = 0\), the port 2 of the valve was trapped, the port 1 is open to atmosphere and the cylinder was allowed to run in a free way with a heavy load. The piston position and chamber pressure were measured over a long period \((\Delta t)\) of time in order to obtain a constant velocity in steady state, \(v_{ps}\). The leakage conductance can then be calculated by:

\[
g_{le} = \frac{v_{ps} \cdot A_2}{P_2}
\]

(23)
The piston position and the pressure in chamber 2 were measured for a period of time \( \Delta t = 200 \text{s} \). The total displacement, in this period, was \( 1504 \mu \text{m} \), and the pressure mean value was \( P_2 = 5.28 \text{bar} \).

The value obtained for the internal leakage conductance was: \( g_{ac} = 1.248 \cdot 10^{-14} \text{m}^3\text{s}^{-1}\text{Pa}^{-1} \).

### 3.2.3 Friction model

The static parameters \( (F_{CO}, F_S, \dot{v}_S, k_s) \) and the dynamic parameters \( (\sigma_0, \sigma_1) \) estimation, for the LuGre friction model (see section 3 in part 1, (7)), are presented below.

#### 3.2.3.1 Static parameter identification

The static parameters are estimated by the velocity/friction force curve measured with constant velocities. The sample time was \( 5 \mu\text{s} \), with the friction forces and velocities being calculated with 20 samples in order to minimize the noise effects. The experiments at constant velocities were performed with a closed loop velocity control.

At constant velocity (a steady state situation, \( a_p = 0 \)), with the platform in the horizontal position, the friction force can be measured through the chamber pressures (see (5) in Part 1):

\[
F_f = P_1A_1 - P_2A_2
\]  

(24)

The friction force was measured for constant velocities between \( -0.2 \text{ms}^{-1} \) and \( 0.2 \text{ms}^{-1} \).

At constant velocity the state variable \( z \) of the friction model, equations (10), (11) and (12) of Part 1, is constant, \( dz/dt = 0 \) and the friction force can be estimated by:

\[
\hat{F}_f = \left( \hat{F}_{CO} + (\hat{F}_S - \hat{F}_{CO})e^{(\dot{v}_S/\hat{v}_S)^2} \right) \text{sign}(\dot{v}_p) + \hat{K}_s \dot{v}_p
\]  

(25)

where \( \hat{F}_{CO}, \hat{F}_S, \dot{v}_S, \hat{K}_s \) are the estimated static parameters and \( \hat{F}_f \) is the estimated friction force.

For the parameter estimation the least squares method was used for the cost function \( cf \):

\[
(cf = \sum_{i=1}^{n}(F_f(\dot{v}_i) - F_f(\dot{v}_i))^2
\]  

(26)

where \( \dot{v}_i \) are the measured velocities.

The cost function was calculated for each parameter set \( (\hat{F}_{CO}, \hat{F}_S, \dot{v}_S, \hat{K}_s) \) given by the Simplex algorithm used in the \texttt{fminsearch} function of the Matlab\textsuperscript{®} (12) optimization toolbox (13). Initial values for the parameters were obtained from the measured velocity versus friction force curve. The static parameters to be used are those that minimize the cost function.

For a symmetrical friction model, that is, having the same model parameters for positive and negative velocities the estimated parameters are:

\[
\hat{F}_{CO} = 101.8 \text{N}; \ (\hat{F}_S - \hat{F}_{CO}) = 153.0 \text{N}; \ \dot{v}_s = 0.019 \text{ms}^{-1}; \ \hat{K}_s = 1090 \text{Nms}^{-1}
\]
As the measured friction forces denote different parameters for different sign of velocity, the model can be enhanced with different parameters for negative and positive velocities.

The new estimated parameters are then:

\[ \hat{F}_{c0n} = 89.26N; \ (\hat{F}_{s} - \hat{F}_{c0n}) = 160.3N; \ \hat{\nu}_{m} = 0.0251ms^{-1}; \ \hat{K}_{m} = 1387Nsm^{-1} \]

\[ \hat{F}_{c0p} = 110.2N; \ (\hat{F}_{s} - \hat{F}_{c0p}) = 150.8N; \ \hat{\nu}_{p} = 0.0152ms^{-1}; \ \hat{K}_{p} = 818.4Nsm^{-1} \]

where the indices \( n \) and \( p \) are used for negative and positive velocities.

The comparison of the measured static friction forces and the ones obtained from the symmetric and non-symmetric static friction model is presented in Fig. 9.

![Friction force versus velocity steady state curves](image)

**Fig. 9** Friction force versus velocity steady state curves

### 3.2.3.2 The dynamic parameters identification

The strategy for the identification of the dynamic parameters \( \sigma_{e} \) and \( \sigma_{l} \) consists on matching the real hydraulic force with the equivalent hydraulic force, obtained at the same conditions, from the simulation of the non-linear valve plus cylinder model. The non-symmetric static friction parameters were used. The identification use open loop experiments, with the valve and cylinder models, enhancing the visibility of the dynamic parameters. The system, working in the horizontal direction, was excited with a sinusoidal signal with sufficient amplitude to lead the system in and out of the stiction state, that is the resulting force should be, during simulation, bigger and lower than the break away force.

An optimization method, as the one used for the static parameters estimation, was used to identify the dynamic parameters. The cost function is:

\[
 cf\left(F_{e}, F_{lm}, \hat{\sigma}\right) = \sum_{k=1}^{n} \left(F_{e}(k) - F_{lm}(k, \hat{\sigma})\right)^{2}
\]  

(27)
where \( F_h(k) \) is the \( k \) sample of the real hydraulic force (sample time equal to 10ms) and \( F_{\text{sim}}(k, \hat{\sigma}) \) is the hydraulic force that results from the model simulation with the same initial conditions, for the same time instant.

The utilization of the hydraulic force as the comparison force results from the following simplification. The net acceleration force if given by:

\[
M \frac{dv_p}{dt} = F_h - F_f
\]  \quad (28)

With the mass used at the tests (piston plus rod), as the maximum values of \( M \frac{dv_p}{dt} \) are less then 0.05N, this force is negligible when compared with the hydraulic and friction forces.

The comparison of the hydraulic forces, the cylinder chamber pressures, the spool position and the piston velocity, when simulating the model with the non-symmetrical friction static parameters, is presented in Fig. 10. The estimated dynamic parameters are \( \sigma_0 = 2.114 \cdot 10^7 \text{Nm}^{-1} \) and \( \sigma_1 = 2.914 \cdot 10^3 \text{Nsm}^{-1} \).

![Fig. 10](image)

a) System and model chamber pressures.  
b) Piston position and valve spool position.  
c) Hydraulic force and piston velocity.
4 HARDWARE-IN-THE-LOOP SIMULATION EXPERIMENTS

A Simulink® block implementing the cylinder hybrid statechart (7) and the valve model was used. The model was simulated in real time with a third order explicit solver and a fixed step size of 0.5 ms.

Closed loop position control experiments, with point-to-point position trajectory as the input reference signal, were performed. Fig. 11 shows the comparisons between the two experiments when controlled by a proportional control, with the proportional constant equal to 50, and a moved mass of 80Kg.

The experiment, presented in the Fig. 12, intend to evaluate the performance of the real and simulated system when the desired input trajectories are steps and ramps. In this experiment a pressure source with $P_s = 120\text{bar}$ was used with a proportional gain of $K_p = 100$. 

**Fig. 11** HILS experimental and simulated results.
a) Piston position (reference, real and simulated).

b) Piston velocity (real and simulated).

c) Valve input signal (real and simulated).

Fig. 12 Experimental and simulation results for input trajectories with high frequency contents.

From the results of the above experiments it can be said that the system model presents a satisfactory performance at small velocities and at trajectories with high frequency content.

A similar set of simulations was produced using a commercial library of hydraulic components (14). The cylinder end stops are modelled with the usual spring and damper components \( \text{spring stiffness equal to } 10^{10} \text{ N/m} \) and the damper coefficient equal to \( 10^{10} \text{ N/m s}^{-1} \) and the seal friction model only considers the viscous friction component. The overall system needs a third order fixed step solver with a \( 2 \mu \text{s} \) step size in order to run properly, negating real-time operation is not possible with low cost hardware.

5 CONCLUSIONS

A model of a hydraulic system, composed by a high performance proportional valve and a hydraulic cylinder presented in Part 1 (7), was fully parameterized. Most of the used models are semi-empirical with their parameters being calculated with simple methods or by optimization. The static valve parameters are calculated by solving a non-linear equation system. This equation system is specified in
order to reproduce the relevant static characteristics available from the manufacturer data or from experimental measurements. The parameters of the dynamic part of the valve and of the friction model are determined with optimization techniques.

The main goal of this work was to obtain not too complex models allowing their use in hardware-in-the-loop experiments. The developed models are reasonably accurate and the whole system can be simulated in real time with a third order explicit solver with a fixed step size of 0.5ms. Closed loop position control experiments were performed with the overall model running in a low cost real time card from dSPACE®. The results were compared with the behaviour of the real system, with the comparison being very satisfactory.

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REFERENCES


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