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The econophysics in the Euromillions lottery

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Abstract
This paper provides a simple analysis of data available online concerning the Euromillions lottery. We show that the data support a model where only a fraction of the bettors convince other people to start playing. We can estimate how large this fraction is and what the number of new bettors brought into the game is. Also we found that bettors do not play randomly, and that consequently there is available information that any bettor can use in order to increase his expected returns.

1. Introduction

Physicists have long been interested in multidisciplinary fields. The current interest in economy and finance is not different. Physicists are particularly interested in unveiling the basic mechanisms ruling systems behaviour. This means that they are not simply concerned with applying mathematics and finding good fits for their experimental data, but also with understanding what may be at the source of their results. Physicists do not aim only to predict future outcomes, but also want to know when their predictions may fail and eventually new theories have to be developed. In this paper, we provide a simple analysis of data available online and show that the number of bettors on the Euromillions lottery displays a collective phenomenon. We show that after consecutive rollovers a fraction of players engage in persuasive behaviour that attracts more people into the game, leading to a kind of herding behaviour. Herding behaviour is a collective phenomenon which, in extreme cases, may have unpredictable consequences. In financial markets, it is associated with a well-known phenomenon: a crash. Crashes occur when all economical agents simultaneously lose their confidence in the market and decide to sell their assets at any price. This generalized loss of confidence may lead to irreversible economical changes with dramatic social consequences, such as a general loss of employment in the population. Hence, understanding how these collective modes emerge and how they can be controlled is a topic of major importance,

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to which physicists have recently been attracted. The reverse of a crash is a bubble, a euphoric period during which all agents are willing to buy assets at any price. Bubbles are desired, as they make many people rich, and hence it would be the dream of any speculator to know how to induce one. In many other areas of social sciences the same phenomenon reappears under different names: politicians dream of gaining instant credibility and popularity, brands strive to create fashion waves around their products, and collective transportation tries to avoid traffic jams.

Why do social systems display such well-defined patterns at certain times, and how can agents change suddenly and collectively their attitudes? Two conceptually different approaches are usually taken to approach these topical questions. Economists tend to look into the historical economical and social environment searching for crucial events that may have triggered collective changes [1]. This approach assumes that fluctuations in collective behaviour are triggered by events that are external to the system, such as an earthquake. This approach produces often hardly testable theories, as any moment in history is different from any other. In the context of the Euromillions lottery, one could argue, for instance, that during Christmas there are more people playing because they have more money. It may be possible that after many Christmas periods we could probably gather enough statistical data to support this view. However, even then, doubts would remain concerning those Christmas periods that did not fit this explanation. Other explanations would have then to be found to explain these odd Christmas events, which would again remain difficult to test. Within this framework quantitative predictions are difficult, if not impossible, to make.

Physicists tend to think differently about what causes market collective movements. Physicists think that collective phenomena emerge from the simple interaction rules that govern the dynamics of the individual agents [1, 2]. Physicists believe that it is the information produced within the game that causes the collective fluctuations. Hence, the purpose of the physicist is to identify the essential simple rules that have to be considered in order to model collective behaviour.

In principle, a good model should be able to fit any observed data, provided that the required conditions are met. With a good model it is possible to formalize a more intuitive picture to describe how the system behaves. If predictions fit well with the observed data and the model is sufficiently simple, then people tend to interpret reality in terms of the model's conceptions, forgetting that models are only physicists' descriptions of reality. Conceptual revolutions often arise when a new model produces predictions that fit better the observed data, contrary to expectations.

In this paper, we show that studying the available data on the Euromillions lottery can represent a toy educational example on how physicists like to approach social, economical and financial problems. By comparing alternative models, we are able to illustrate how theoretical modelling provides deeper insights into a system's behaviour. By confronting alternative models, we show how different conceptual pictures provide different descriptions of how a system behaves.

2. Rules of the game and available data

The Euromillions lottery is the biggest weekly lottery in Europe, assembling lotteries from eight countries. It started with only three countries, in February 2004, but 8 months later six more countries joined in. The rules of the Euromillions lottery are extremely simple. Each week, bettors place as many bets as they wish, paying a fixed amount of money per bet. A bet consists in guessing five numbers (from 1 to 50) and two stars (from 1 to 9). Every week a (supposedly) random key is drawn. The first prize (the jackpot) is shared by all the players

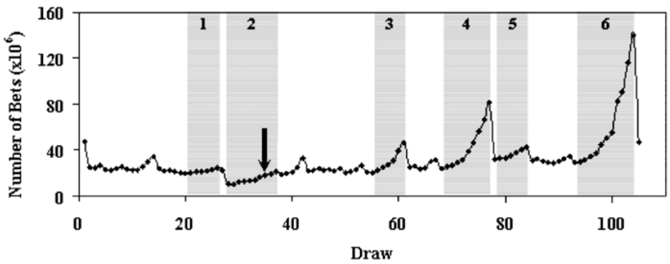

Figure 1. Amount of bets sold on each draw of the Euromillions lottery. The arrow indicates the moment when more countries joined the lottery. In grey we highlight the periods with the highest number of consecutive jackpots studied in this work. They are called rollovers 1, 2, . . . , 6.

who have guessed correctly all the numbers and all the stars. Lower prizes are shared by bettors who guessed a smaller number of numbers and stars. The prizes are calculated as a fraction of the total money gathered, and are shared equally by the number of people who guessed correctly the same number of numbers and stars. When there are no winners for a given prize, the prize rolls over to the next week. Typically this only happens with the first prize.

There are several websites where a considerable amount of information concerning the Euromillions lottery is available [3, 4]. Most of them provide historical records of the winning numbers, the number of people winning each prize and the prize values. In this paper, we only used data available online. There were two reasons for this. Firstly, from an educational perspective, it is certainly interesting to realize that much can be learnt by studying the information available online. Secondly, in any economical system, imperfect information is inevitably always present. Hence, not being able to gather all the possible information concerning the game should never be considered a handicap to studying the system. Actually, in most cases what is essential is to understand what the available information is and how the economic agents deal with it.

In figure 1, we show the number of bets sold for each draw of the Euromillions lottery since its beginnings in February 2004. The shaded regions highlight the six longest periods with consecutive rollovers, with at least six consecutive rollovers on each period. When consecutive rollovers take place, the first prize increases considerably and reaches sums comparable to the fortunes of the richest men in the world. Consequently, rollovers attract a considerable number of players. In these periods, a consistent growth in the number of bets is clearly visible. In this work, we will study how the number of bettors increases in these cases.

3. Modelling burst behaviour during consecutive rollovers

In order to model the empirical data, we consider how the number of bettors varies through consecutive rollovers. One could think that the media should play a crucial role. So, we start by questioning how the media influences people's decision to start playing. Consider a population with N individuals. From these there are N_{s} individuals susceptible to persuasive arguments presented by the media and N_{ns} are non-susceptible: $N = N_{\mathrm{s}} + N_{\mathrm{ns}}$. Susceptible individuals are those who have not played previously, but who may change their decision. If they decide to start playing then they become non-susceptible, and for the sake of simplicity we will assume that they will continue playing if a new rollover arises. Here we will assume that each individual buys one bet, so that it becomes equivalent to discuss the number of bettors or number of bets. This should not change qualitatively the results, provided bettors

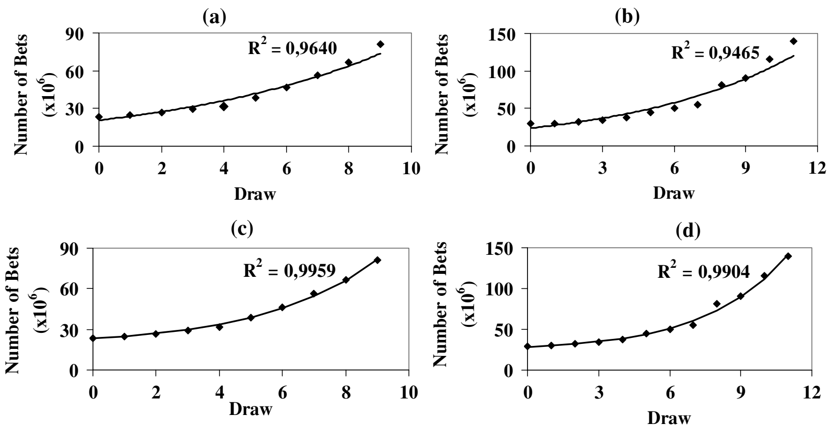

Figure 2. Analysis of consecutive rollovers 4 (left) and 6 (right). In these figures, $t = 0$ represents the first draw with no winners for the jackpot prize, in a series. In figures (a) and (b) we show how model 1 fits the empirical data. In figures (c) and (d), we show the fits obtained with model 2 for the same data.

buy a finite number of bets, which happens in practice. If a player buys more than one bet we can always think that he or she represents the role of several players in the model.

If we assume that a fraction α of the susceptible players starts playing after a rollover, and that on the first rollover in the series ($t = 0$) there were N_0 bettors, then after t consecutive rollovers the total number of bets $N_b(t)$ follows the equation

$$\begin{aligned} &N_b(t) = N_b(t-1) + \alpha[N - N_b(t-1)] = \alpha N + (1-\alpha)N_b(t-1) \\ &N_b(0) = N_0, \end{aligned}$$

that is,

$$N_b(t) = N - (N - N_0)(1-\alpha)^t \approx N - (N - N_0)\exp(-\alpha t). \tag{1}$$

It is clear that this model cannot explain the empirical data, because it leads to an exponential with a decreasing derivative. It is nevertheless an instructive example, because it shows that if the media had a direct impact on people's decisions then empirical data should behave differently. We could try to change this model slightly, to consider that the media changes the intensity of its performances along the time. However, this would only change α on each session, and would not change the general behaviour of the model.

A different model is then required. The empirical data in figure 2 suggest that an exponential growth takes place. Figures 2(a) and (b) show that in rollovers 4 and 6, an exponential function fits the data reasonably well. This is common to models that describe the propagation of a disease. So we consider that it is communication within the population that drives new people to start playing. Hence, we assume that on each session the number of new players is proportional to the number of people who played in the previous sessions:

$$\begin{aligned} &N_b(t) = N_b(t-1) + \alpha N_b(t-1) \\ &N_b(0) = N_0. \end{aligned}$$

Hence, we get

$$N_b(t) = N_0(1-\alpha)^t \approx N_0 \exp(\alpha t). \tag{2}$$

This model (model 1) explains the data considerably well because it agrees with the good fits obtained in figures 2(a) and (b). From a conceptual point of view this means that we

Table 1. Results from the fits with the two models for the six consecutive rollovers under study.

Rollovers	Number of draws	Model 1 $N_0(\times 10^6)$	Model 1 α	Model 1 R^2	Model 2 $N_i(\times 10^6)$	Model 2 $N_{p0}(\times 10^6)$	Model 2 α	Model 2 R^2	F	$F_{p=0.01}$
1	7	19.6	0.036	0.920	18.4	1.2	0.265	0.939	2.8	15.5
2	10	10.5	0.083	0.983	6.8	3.7	0.160	0.994	11.2	6.8
3	7	20.3	0.137	0.971	16.3	4.0	0.333	0.996	26.6	15.5
4	10	23.5	0.141	0.964	18.2	5.3	0.282	0.996	77.9	6.8
5	7	32.2	0.051	0.943	28.4	3.8	0.246	0.961	4.9	15.5
6	12	29.6	0.150	0.946	23.8	5.8	0.292	0.990	45.7	5.3

can describe the collective decision behaviour as a contagious propagation where every individual who played the game previously brings into the game a certain number of new players. The media may have an impact but only an indirect one. The media may work by stimulating communication among the members of the population and by making individuals more vulnerable. That is, it can change α, but not directly N_b.

If one looks more critically at figures 2(a) and (b), one may wonder if this is indeed the best model that describes the data. We note that the curvature of the fitted exponential does not seem to agree with the empirical points. In particular, the last points are not well fitted, especially in figure 2(b), where a larger number of points are available. We may thus question if a better model exists, that fits better both cases, such that the quality of the fits does not depend on the range of the fitted points.

A better model can indeed be found by assuming that the population has two components. There are a number of individuals who we call *indifferent*, N_i, who bet all the time and are indifferent relatively to all the other potential players (that is, they will not try to influence the other people's decisions). Hence, we assume that N_i stays approximately constant. The second component is made of bettors who try to influence other people's decisions, trying to bring them into the game. As in the previous model, once a player starts playing, he or she behaves in future sessions as a persuasive player and also continues playing. The number of persuasive players N_p is initially N_{p0}, and on average each persuasive player brings into the game α new players. It will be the population of persuasive players that grows exponentially in this model, i.e., $N_p(t+1) = N_p(t) + \alpha\, N_p(t)$. Hence, for the total number of bets $N_b(t) = N_i + N_p(t)$, we write

$$N_b(t+1) = N_b(t) + \alpha[N_b(t) - N_i]$$
$$N_b(0) = N_i + N_{p0}$$

which leads to the solution (model 2)

$$N_b(t) = N_i + N_{p0}(1+\alpha)^t \approx N_i + N_{p0}\exp(\alpha t). \quad (3)$$

In figures 2(c) and (d), we show two least-squares fits to this model for the longest consecutive rollovers (rollovers 4 and 6). We remark that these fits are considerably better than those obtained with the previous model. In table 1, we compare the correlation coefficients R^2 obtained with the two different models. R^2 was calculated using the standard formula $R^2 = SS_{reg}/SS_t$, where SS_{reg} is the regression sum of squares and SS_t is the total sum of squares [5] for the corresponding least-squares regression. R^2 is defined as the proportion of variance of the dependent variable that is explained by the fit. Except for rollovers 1 and 5, the R^2 values are all greater than 0.99, meaning that the fit explains over 99% of the total variation in the data about the average. This suggests that the new model fits the data very well (for a comparison with other typical experimental results, see for instance [6]). In any case, an improved fit was expected as the new model introduces an extra parameter. To test whether this

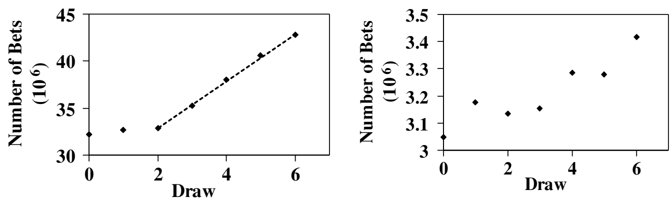

Figure 3. Consecutive rollover 3 for the total number of bets in Europe (left) and in Portugal only (right). The behaviour of the total number of bets does not suggest the existence of exponential bursts. The increase in the total number of bets in Europe displays a linear growth section.

extra parameter in the model produces significantly better fits we used a stepwise regression method, also known as an F-test. The F parameter was calculated according to the formula $F = (SSE1\text{-}SSE2) \times DF2/(SSE2 \times (DF2\text{-}DF1))$, where SSE_{i} is the sum of squared residuals for the model i, which has 2 ($i = 1$) or 3 ($i = 2$) parameters. DF_j are the degrees of freedom for model j, for each case, which is given by the number of data points minus the number of parameters. In the last column, we display the F threshold value for a confidence level of 99% ($p = 0.01$). We can conclude that the new model fits the data significantly better than the simple exponential model, except for rollovers 1 and 5.

We also display the N_0, N_i, $N_{\mathrm{p}0}$ parameters and the α values for the two cases. As should be expected, α values are larger in the second model. This happens because in the first model one assumes that all bettors can convince other people to start playing, while in the second model only a sub-population does it. Also we should remark that α values are not universal, but rather they depend on the rollover. However, they do not vary wildly, and in the last rollovers with good exponential fits, α seems to tend to a typical value around 0.3. More rollovers with exponential bursts would be required to clarify whether a typical value indeed exists.

Our analysis is based on a short range of data. To be able to clearly prove the existence of the proposed exponential behaviour, the number of bets should range at least over 2–3 orders of magnitude. This is impossible with the present data, and consequently we cannot eliminate other alternative models with the same number of parameters. As an example, rollovers 4 and 6 could also be well fitted with a second-order polynomial. In this case, however, no physical meaning could be associated with the polynomial coefficients. In particular, we verified that the linear coefficient of the fitted polynomial tends to be negative with the increasing number of consecutive rollovers. Mathematically this can be understood in the light of our proposed model: in order to fit the exponential behaviour with a second-order polynomial, the second-order coefficient has to increase if wider intervals are considered, and consequently the first-order coefficient must decrease in order to fit the initial points equally well. Physically, however, it seems unreasonable that for the longest consecutive rollovers, the number of bettors should decrease in the first rounds. Hence, our model seems a better candidate to describe the observed behaviour.

Rollovers 1 and 5 suggest that the set of exponential bursts is not guaranteed. The range of data does not allow definite conclusions, but fits with our model lead to poorer results and smaller α values. Hence, either no exponential bursts are taking place, or they are only slowly building up in such a way that they may be getting disguised by any temporal variation in the number of indifferent bettors (that we considered static). In figure 3, we show the evolution of the number of bets in the whole of Europe, and also in Portugal alone (a macroscopic sub-domain in the whole population). No exponential bursts seem present in either case. This did not happen for the other consecutive rollovers, where our model also produces good fits for the number of bets in Portugal.

The possibility of the existence of non-exponential increments is interesting, because recent studies in the physics of networks have suggested that for scale-free networks no threshold is required for propagation of an epidemic [7]. Given that acquaintance networks typically display scale-free properties [8], it would seem reasonable to think that exponential bursts could arise all the time. As this is not the case, this can mean that, in these cases, the media did not choose their targets critically enough. Looking at table 1, we can also realize that the initial number of bettors or the initial number of persuasive bettors (or their ratio) do not seem to be enough to decide the type of outbreak. Hence, a variation in the topology of the social network may have been crucial. This possibility is actually quite reasonable if we note that both rollovers took place during summer months (July, for rollover 1, and August and September for rollover 5). Hence, it is likely that marketing campaigns were unable to efficiently select the most suitable targets due to changes in the social network. Another possible explanation for the behaviour of rollover 5 has been called the ‘prize fatigue effect’ [9]: as rollover 5 is very close to rollover 4, one could hypothesize that a refractory period took place as bettors lost motivation after the big breakdown in the jackpot prize that happened when rollover 4 terminated. After the discussion in section 1, it is clear that in order to test this type of hypothesis, a more complete data set and development of other models (for instance to describe the evolution of the social network) would be required. Nevertheless, a prize fatigue effect would not explain the behaviour in rollover 1.

In some areas, triggering exponential bursts can be the main goal. For instance, being able to communicate efficiently with a whole population is certainly socially advantageous. In marketing, theory recommends word-of-mouth techniques to implement the exponentially quick spread of information. Word-of-mouth is a highly valued communication method because it uses personal communication which is perceived as more credible. With the advent of the internet, word-of-mouth techniques gained new forms and a new name highlights its speed: viral marketing. Viral marketing has been pointed as the main source of success of Yahoo and Hotmail free e-mail services [10] and it was certainly extremely successful in the early days of the internet. Today, viral marketing is also a cause of concern. Due to its easy execution and low cost, viral marketing leads to massive e-mail spamming which threatens efficient e-mail communication. For these reasons understanding how decision bursts can be triggered and avoided goes beyond academic curiosity. Recently, some studies have addressed how burst behaviour can be altered due to network topology that shapes how agents communicate, or by the individual decision dynamics [11–13].

Finally, we should point out that our simple model is considerably different from typical quantitative economical models for this problem. Here, we considered that the most important ingredient was the propagation of persuasive behaviour. In typical quantitative economical models each agent decides whether to play or not depending on how he values the price of the bet relative to his chances of winning [9, 14]. Hence, economical models emphasize the role of players as individual agents. In contrast, our results suggest that bettors’ decisions in the Euromillions lottery may not crucially depend on the price of the bets. Rather, they may be an emergent phenomenon arising from agents’ word-of-mouth type of interactions.

4. Information production and rational behaviour

One of the key features of game theory is that bettors’ strategies can produce information that other players can use in their favour in future trials of the game. It then becomes relevant to know whether players of the Euromillions lottery use biased strategies that may

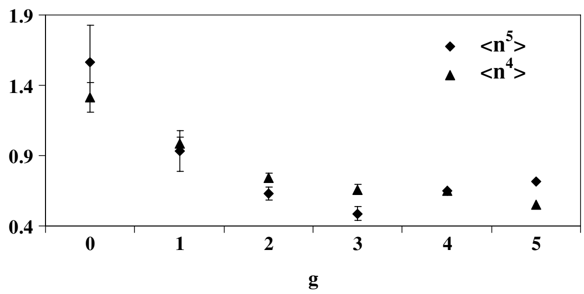

Figure 4. Average normalized frequencies as a function of the number g, of numbers in the key that are greater than $D = 31$. $\langle n^5 \rangle$ (or $\langle n^4 \rangle$) is a measure of the number of individuals who guessed correctly 5 (or 4) numbers in the key. $\langle n^5 \rangle$ and $\langle n^4 \rangle$ should be 1 if players used random guesses. Here, we see that if the key has 0 numbers greater than 31 then the number of players who guessed 5 or 4 numbers correctly is considerably larger than in the random case. Error bars were calculated with the standard errors, with a significance level of 68%. Points with $g = 4$ or $g = 5$ have no error bars because no statistically significant number of samples existed in these cases.

become detectable by any other bettor, and hence whether anti-correlated strategies (that is, strategies that counter the former ones) can be defined. Here, we will show that players of the Euromillions lottery seem to choose some numbers more often than others.

As humans like to attach a 'rational' meaning to all actions in their lives, we decided to analyse whether the numbers they chose correlate with dates. In order to do this we considered, for draw i, how many numbers within the 5 numbers in the correct key were greater than D. We called this number g_i. Given the number of players N_i^5 (or N_i^4) who guessed correctly 5 (or 4) numbers on draw i, we calculated for each session the fraction of players who guessed correctly 5 or 4 numbers: $f_i^5 = N_i^5 / N_i \left(f_i^4 = N_i^4 / N_i\right)$. Here, N_i is the number of bets on session i. Finally, we divided these numbers by the theoretical probabilities of guessing all 5 or 4 numbers correctly by random guessing obtaining normalized frequencies n_i^5(or n_i^5). The normalized frequencies should be close to 1. The bigger they are the more likely it is that more players make the same guess and hence will have to share prizes.

In figure 4, we show how the average normalized frequencies, $\langle n^5 \rangle$ and $\langle n^4 \rangle$ varied with g_i, with $D = 31$. The reason we also calculated $\langle n^4 \rangle$ is because the smaller number of players guessing correctly the five numbers makes the variance in n_i^5 increase considerably. Hence, $\langle n^4 \rangle$ has a smaller associated standard error. Nevertheless, a similar trend is visible on both in $\langle n^5 \rangle$ and $\langle n^4 \rangle$. Figure 4 agrees with the idea that players prefer to play numbers related to dates. A player using bets with two numbers greater than 31 has, on average to share prizes with less than a half of people than a player who uses in his or her guesses all numbers smaller than 31. In other words, his expected returns are at least twice as large. These results show that, although all keys are in principle equiprobable, some have larger expected returns than others, because players do not play randomly. Of course, if players start using this information, then interesting dynamics may result in future issues of the game, as described in minority games recently studied by physicists [3]. The game could even become efficient, which, according to the economical nomenclature means that no information is available to any player from the outcomes of the game.

In order to test whether this kind of rational behaviour took place, we divided the series chronologically in two halves, and performed the same analysis as before. Figure 5 displays $\langle n^4 \rangle(g)$ for the two sub-series. The two series follow a similar trend indicating that biases still persist on the second sub-series. Apart from the first point ($g = 0$), for which bigger

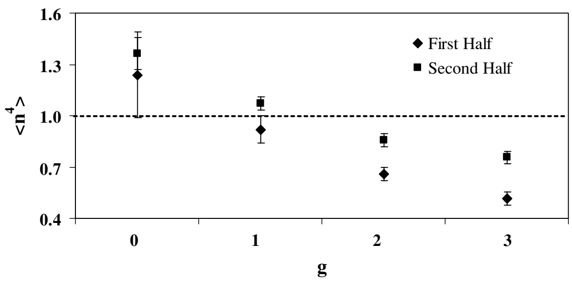

Figure 5. The same analysis as in figure 4, but now for the two sub-series obtained by dividing the initial series chronologically into two halves.

errors exist, the second sub-series has the values of $\langle n^4 \rangle(g)$ closer to 1. This could indicate that players have been adapting, playing rationally and using the available information in their favour. There is however a second more likely explanation for this reduced trend in the observed biases, namely the possibility of buying a randomly generated bet from the seller. This commercial possibility was only recently introduced which makes it difficult to draw conclusions with the available data, at present. In any case, the previous analysis clearly showed that there is information produced by the players in the game, which can be used by any player who is aware of it.

5. Conclusions

In this work we showed how it is possible, by constructing models with simple dynamical equations, to build an intuitive picture of the behaviour of a real economical system. We also showed how different types of parameters can be defined to gain insight on how players make their decisions.

More complete sets of data could provide deeper insights into how bursts in collective social movements such as in the Euromillions lottery arise, how they propagate and what is the role of the topology of social interactions. In this sense we hope this paper may serve as a stepping stone that may stimulate many students and teachers of physics to discuss multidisciplinary topics and to develop new approaches and research projects. Interestingly, we believe that the Euromillions lottery could provide a nice 'laboratory' to study the emergence of collective social behaviours.

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