Waves and Total Distributivity

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Abstract

For *total* $\mathscr{K}$ with defining adjunction $\bigvee \dashv Y: \mathscr{K} \to \widehat{\mathscr{K}} = \mathbf{set}^{\mathscr{K}^{\mathrm{op}}}$ the *wave functor* $W: \mathscr{K} \to \widehat{\mathscr{K}}$ is well-defined by $W(A)(K) = \widetilde{\mathscr{K}}(K, A) = \mathbf{set}^{\widehat{\mathscr{K}}}(\mathscr{K}(A, \bigvee -), [K, -])$ where $[K, -]$ is evaluation at K. Also there are natural transformations $\beta: W \bigvee \to 1_{\widehat{\mathscr{K}}}$ and $\gamma: \bigvee W \to 1_{\mathscr{K}}$ satisfying $\bigvee \beta = \gamma \bigvee$ and $\beta W = W\gamma$. A total $\mathscr{K}$ is *totally distributive* if $\bigvee$ has a left adjoint. $\mathscr{K}$ is totally distributive iff γ is invertible iff $W \dashv \bigvee$.

For total $\mathscr{K}$ there is an associative composition of waves. Composition becomes an arrow $\bullet: \widetilde{\mathscr{K}} \circ_{\mathscr{K}} \widetilde{\mathscr{K}} \to \widetilde{\mathscr{K}}$. Also there is an augmentation $(\bar{-}): \widetilde{\mathscr{K}}(-, -) \to \mathscr{K}(-, -)$ corresponding to a natural $\delta: W \to Y$ constructed via β. We will show that if $\mathscr{K}$ is totally distributive then $\bullet: \widetilde{\mathscr{K}} \circ_{\mathscr{K}} \widetilde{\mathscr{K}} \to \widetilde{\mathscr{K}}$ is invertible and then $\widetilde{\mathscr{K}}$ supports an idempotent comonad structure. In fact, $\widetilde{\mathscr{K}} \circ_{\mathscr{K}} \widetilde{\mathscr{K}} = \widetilde{\mathscr{K}} \circ_{\widetilde{\mathscr{K}}} \widetilde{\mathscr{K}}$ so that $\bullet$ is the coequalizer of $\bullet\mathscr{K}$ and $\mathscr{K}\bullet$ making $\widetilde{\mathscr{K}}$ a *taxon* in the sense of Koslowski [KOS]. For a small taxon $\mathscr{T}$, the category of interpolative modules $\mathbf{iMod}(\mathbf{1}, \mathscr{T})$ is totally distributive [MRW]. Here we show, for totally distributive $\mathscr{K}$, there is an equivalence $\mathscr{K} \to \mathbf{iMod}(\mathbf{1}, \widetilde{\mathscr{K}})$.

References

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Total categories

Locally small $\mathscr{K}$ is *total* [S&W] if $Y\colon \mathscr{K} \to \widehat{\mathscr{K}} = \mathbf{set}^{\mathscr{K}^{\mathrm{op}}}$ has a left adjoint

- Write $\langle \eta, \epsilon \colon \bigvee \dashv Y\colon \mathscr{K} \to \widehat{\mathscr{K}} \rangle$ for such an adjunction
- For $P \in \widehat{\mathscr{K}}$, $\bigvee P = \mathrm{colim}(\mathrm{el}P \to \mathscr{K}) = \int^K [K, P] \cdot K$
- $([K, P] = P(K))$ For $K \overset{x}{\dashrightarrow} P$, write $K \xrightarrow{i_x} \bigvee P$
- For $K \overset{x}{\dashrightarrow} P \xrightarrow{t} Q$,

$$\begin{tikzcd} K \arrow[r, "i_x"] \arrow[rr, bend right, "i_{tx}"'] & \bigvee P \arrow[r, "\bigvee t"] & \bigvee Q \end{tikzcd}$$

- For $K \overset{f}{\dashrightarrow} A_* = \mathscr{K}(-, A)$, that is $K \xrightarrow{f} A$ in $\mathscr{K}$

$$\begin{tikzcd} K \arrow[r, "i_f"] \arrow[rr, bend right, "f"'] & \bigvee A_* \arrow[r, "\epsilon A"] & A \end{tikzcd}$$

Total $\mathscr{K}$ is *totally distributive* [R&W] if $\bigvee$ has a left adjoint

Waves

For total $\mathscr{K}$, there is a $W : \mathscr{K} \to \widehat{\mathscr{K}}$ given by

$W(A)(K) = \mathbf{set}^{\widehat{\mathscr{K}}}(\mathscr{K}(A, \bigvee -), [K, -]) =: \widetilde{\mathscr{K}}(K, A)$ the waves (wavy arrows[J&J]) from K to A

$$\begin{tikzcd}
K \arrow[r, squiggly, "\omega"] \arrow[d, dotted, "\omega P(f) = \omega(f)"'] & A \arrow[d, "f"] \\
P & \bigvee P
\end{tikzcd}$$

$$\begin{tikzcd}
K \arrow[r, squiggly, "\omega"] \arrow[rr, squiggly, bend left, "a\omega"] \arrow[d, dotted, "a\omega(g) = \omega(ga)"'] & A \arrow[r, "a"] \arrow[d, "ga"] & B \arrow[d, "g"] \\
P & \bigvee P & \bigvee P
\end{tikzcd}$$

$$
\begin{tikzcd}
L \arrow[r, "k"] \arrow[d, dotted, "\omega k(f)=\omega(f)k"'] \arrow[rr, squiggly, bend left, "\omega k"] & K \arrow[r, squiggly, "\omega"] \arrow[d, dotted, "\omega(f)"'] & A \arrow[d, "f"] \\
P & P & \bigvee P
\end{tikzcd}
$$

$$
\begin{tikzcd}
K \arrow[r, squiggly, "\omega"] \arrow[d, dotted, "\omega(f)"'] \arrow[dd, dotted, bend right, "\omega(\bigvee(t)f)"'] & A \arrow[d, "f"] \\
P \arrow[d, "t"] & \bigvee P \arrow[d, "\bigvee t"] \\
Q & \bigvee Q
\end{tikzcd}
$$

$$
\begin{tikzcd}
K \arrow[r, rightsquigarrow, "\omega"] \arrow[d, dotted, "\overline{\omega}=\omega(\epsilon^{-1}A)"'] & A \arrow[d, "\epsilon^{-1}A"] \\
A_* & \bigvee A_*
\end{tikzcd}
$$

defines $\overline{(-)}:\widetilde{\mathscr{K}}(-,-)\to\mathscr{K}(-,-)$, equivalently $\delta:W\to Y$

LEMMA: For any wave $\omega:K\rightsquigarrow A$ in a total $\mathscr{K}$, any P in $\widehat{\mathscr{K}}$, and any $f:A\to\bigvee P$ in $\mathscr{K}$,

$$
\begin{tikzcd}
K \arrow[r, "\overline{\omega}"] \arrow[d, dotted, "\omega(f)"'] & A \arrow[d, dotted, "f"] \\
P \arrow[r, "\eta P"'] & (\bigvee P)_*
\end{tikzcd}
$$

Proof:

$$
\begin{tikzcd}
K \arrow[r, squiggly, "\omega"] \arrow[d, dotted, "\omega(f)"] \arrow[dd, dotted, bend right, "\omega(\bigvee \eta P . f)"'] & A \arrow[d, "f"'] \arrow[dr, "\epsilon^{-1} A"] & \\
P \arrow[d, "\eta P"] & \bigvee P \arrow[d, shift right, bend right, "\bigvee \eta P"'] \arrow[d, shift left, bend left, "\epsilon^{-1} \bigvee P"] & \bigvee A_* \arrow[dl, "\bigvee f_*"] \\
(\bigvee P)_* & \bigvee(\bigvee P)_* &
\end{tikzcd}
\qquad
\begin{tikzcd}
K \arrow[r, squiggly, "\omega"] \arrow[d, dotted, "\overline{\omega}"] \arrow[dd, dotted, bend right, "\omega(\bigvee f_* . \epsilon^{-1} A)"'] & A \arrow[d, "\epsilon^{-1} A"] \\
A_* \arrow[d, "f_*"] & \bigvee A_* \arrow[d, "\bigvee f_*"] \\
(\bigvee P)_* & \bigvee(\bigvee P)_*
\end{tikzcd}
$$

$\square$

LEMMA: For total $\mathscr{K}$, there are

$$\beta : W \bigvee \twoheadrightarrow 1_{\widehat{\mathscr{K}}} \quad \text{and} \quad \gamma : \bigvee W \twoheadrightarrow 1_{\mathscr{K}}$$

where $\beta P(K) = \beta(K, P) : \widetilde{\mathscr{K}}(K, \bigvee P) \twoheadrightarrow [K, P]$ is given by

$$\beta(K, P)(\omega : K \rightsquigarrow \bigvee P) = \omega(1_{\bigvee P}) : K \dashrightarrow P$$

and

$$\begin{tikzcd}
\bigvee W \arrow[rr, "\gamma"] \arrow[dr, "\bigvee \delta"'] & & 1_{\mathscr{K}} \\
& \bigvee Y \arrow[ur, "\epsilon"'] &
\end{tikzcd}$$

satisfying $\bigvee \beta = \gamma \bigvee$ and $\beta W = W \gamma$

THEOREM $\mathscr{K}$ is totally distributive iff γ is invertible iff $W \dashv \bigvee$

- For $\mathscr{K}$ totally distributive, write $\alpha = \gamma^{-1}$

PROPOSITION: For total $\mathscr{K}$,

$$
\begin{tikzcd}
 & L \arrow[dr, squiggly, "\psi"] & \\
K \arrow[ur, squiggly, "\omega"] \arrow[rr, squiggly, "\overline{\psi}\omega=\psi\overline{\omega}=:\psi\bullet\omega"'] & & A
\end{tikzcd}
$$

provides an associative composition for $\widetilde{\mathscr{K}}(-,-)$

Proof

$$
\begin{tikzcd}
K \arrow[r, squiggly, "\omega"] \arrow[rr, squiggly, bend left, "\overline{\psi}\omega"] & L \arrow[r, "\bar{\psi}"] \arrow[d, dotted, "\psi(f)"'] \arrow[dr, dotted, "e"] & A \arrow[d, dotted, "f"] \\
 & P \arrow[r, "\eta P"] & (\bigvee P)_*
\end{tikzcd}
\quad
\begin{tikzcd}
L_* \arrow[r, "\eta L_*=(\epsilon^{-1}L)_*"] \arrow[d, "\psi(f)_*"'] \arrow[dr, "e_*"] & (\bigvee L_*)_* \arrow[d, "(\bigvee \psi(f)_*)_*"] \\
P \arrow[r, "\eta P"'] & (\bigvee P)_*
\end{tikzcd}
\quad
\begin{tikzcd}
 & K \arrow[r, squiggly, "\omega"] \arrow[dl, "\bar{\omega}"] \arrow[d, dotted, "\bar{\omega}"] \arrow[dd, dotted, bend right=60, "\psi\overline{\omega}(f)"'] & L \arrow[d, "\epsilon^{-1}L"] \\
L \arrow[dr, dotted, "\psi(f)"] & L_* \arrow[d, "\psi(f)_*"] & \bigvee L_* \arrow[d, "\bigvee \psi(f)_*"] \\
 & P & \bigvee P
\end{tikzcd}
$$

$\square$

- For total $\mathscr{K}$, $\overline{(-)}\colon\widetilde{\mathscr{K}}\to\mathscr{K}$ is an identity-on-objects semifunctor

- For total $\mathscr{K}$, $\widetilde{\mathscr{K}} \circ_{\mathscr{K}} \widetilde{\mathscr{K}} \xrightarrow{\bullet} \widetilde{\mathscr{K}}$

(for $K \overset{\omega}{\rightsquigarrow} L \xrightarrow{f} M \overset{\psi}{\rightsquigarrow} A$, $\psi f \bullet \omega = \psi \bullet f\omega$)

THM For totally distributive $\mathscr{K}$, $\bullet\colon\widetilde{\mathscr{K}} \circ_{\mathscr{K}} \widetilde{\mathscr{K}}\to\widetilde{\mathscr{K}}$ is invertible

Proof
$$W \bigvee WA \xrightarrow[\simeq]{W\gamma A} WA$$

$$\widetilde{\mathscr{K}}(-, \bigvee WA) \xrightarrow[\simeq]{\widetilde{\mathscr{K}}(-,\gamma A)} \widetilde{\mathscr{K}}(-, A)$$

$$\widetilde{\mathscr{K}}(-, \int^L L \cdot \widetilde{\mathscr{K}}(L, A)) \xrightarrow[\simeq]{\widetilde{\mathscr{K}}(-,\gamma A)} \widetilde{\mathscr{K}}(-, A)$$

$$\int^L \widetilde{\mathscr{K}}(-, L) \cdot \widetilde{\mathscr{K}}(L, A) \xrightarrow{\simeq} \widetilde{\mathscr{K}}(-, \bigvee WA) \xrightarrow[\simeq]{\widetilde{\mathscr{K}}(-,\gamma A)} \widetilde{\mathscr{K}}(-, A)$$

$$\int^L \widetilde{\mathscr{K}}(K, L) \times \widetilde{\mathscr{K}}(L, A) \xrightarrow{\simeq} \widetilde{\mathscr{K}}(K, \bigvee WA) \xrightarrow[\simeq]{\widetilde{\mathscr{K}}(K,\epsilon A.\bigvee \delta A)} \widetilde{\mathscr{K}}(K, A)$$

$$\widetilde{\mathscr{K}} \circ_{\mathscr{K}} \widetilde{\mathscr{K}} \cong \widetilde{\mathscr{K}}$$

$$[K \overset{\psi}{\rightsquigarrow} L \overset{\chi}{\rightsquigarrow} A] \mapsto K \overset{\psi}{\rightsquigarrow} L \xrightarrow{i_\chi} \bigvee WA$$

$$\mapsto K \overset{\psi}{\rightsquigarrow} L \xrightarrow{i_{\overline{\chi}}} \bigvee YA \xrightarrow{\epsilon A} A = K \overset{\psi}{\rightsquigarrow} L \xrightarrow{\overline{\chi}} A = K \overset{\chi \bullet \psi}{\rightsquigarrow} A$$

So $\quad \widetilde{\mathscr{K}} \circ_{\mathscr{K}} \widetilde{\mathscr{K}} \xrightarrow{\overset{\bullet}{\simeq}} \widetilde{\mathscr{K}} \quad$ and $\quad (\widetilde{\mathscr{K}}, \bullet^{-1}, \overline{(-)}) \quad$ idempotent comonad

$$\begin{tikzcd}
\widetilde{\mathscr{K}}\,\mathscr{K}\,\widetilde{\mathscr{K}} \arrow[r, shift left, "\rho\widetilde{\mathscr{K}}"] \arrow[r, shift right, "\widetilde{\mathscr{K}}\lambda"'] & \widetilde{\mathscr{K}}\,\widetilde{\mathscr{K}} \arrow[r] \arrow[dr, "\bullet"'] & \widetilde{\mathscr{K}} \circ_{\mathscr{K}} \widetilde{\mathscr{K}} \arrow[d, "\bullet"] \\
& & \widetilde{\mathscr{K}}
\end{tikzcd}$$

$$\begin{tikzcd}
\widetilde{\mathscr{K}}\,\mathscr{K}\,\widetilde{\mathscr{K}} \arrow[r, shift left, "\rho\widetilde{\mathscr{K}}"] \arrow[r, shift right, "\widetilde{\mathscr{K}}\lambda"'] & \widetilde{\mathscr{K}}\,\widetilde{\mathscr{K}} \arrow[r, "\bullet" description] & \widetilde{\mathscr{K}}
\end{tikzcd}$$

$$
\begin{tikzcd}
 & M \arrow[dr, squiggly, "\psi"] \arrow[dd, "f" description] & \\
K \arrow[ur, squiggly, "\omega"] \arrow[dr, squiggly, "\tau"'] & & A \\
 & L \arrow[ur, squiggly, "\sigma"'] &
\end{tikzcd}
\qquad
\begin{tikzcd}
 & M \arrow[dr, squiggly, "\psi"] \arrow[dd, squiggly, "\phi" description] & \\
K \arrow[ur, squiggly, "\omega"] \arrow[dr, squiggly, "\tau"'] & & A \\
 & L \arrow[ur, squiggly, "\sigma"'] &
\end{tikzcd}
$$

$$\psi \otimes_{\mathscr{K}} \omega = \psi \otimes_{\widetilde{\mathscr{K}}} \omega$$

"$\supseteq$" holds for total $\mathscr{K}$, "$=$" holds for totally distributive $\mathscr{K}$

$$
\begin{tikzcd}
\widetilde{\mathscr{K}}\,\widetilde{\mathscr{K}}\,\widetilde{\mathscr{K}} \arrow[r, shift left, "\bullet\widetilde{\mathscr{K}}"] \arrow[r, shift right, "\widetilde{\mathscr{K}}\bullet"'] & \widetilde{\mathscr{K}}\,\widetilde{\mathscr{K}} \arrow[r] & \widetilde{\mathscr{K}} \circ_{\widetilde{\mathscr{K}}} \widetilde{\mathscr{K}} = \widetilde{\mathscr{K}} \circ_{\mathscr{K}} \widetilde{\mathscr{K}}
\end{tikzcd}
$$

$$
\begin{tikzcd}
\widetilde{\mathscr{K}}\,\widetilde{\mathscr{K}}\,\widetilde{\mathscr{K}} \arrow[r, shift left, "\bullet\widetilde{\mathscr{K}}"] \arrow[r, shift right, "\widetilde{\mathscr{K}}\bullet"'] & \widetilde{\mathscr{K}}\,\widetilde{\mathscr{K}} \arrow[r, "\bullet" description] & \widetilde{\mathscr{K}}
\end{tikzcd}
$$

So $(|\mathscr{K}|, \widetilde{\mathscr{K}}, \bullet)$ taxon [KOS]

An *i-module* $P:\mathbf{1}\to\widetilde{\mathscr{K}}$ is a **set**-valued matrix (array) $P:1\to|\mathscr{K}|$ together with an associative action $\lambda:\widetilde{\mathscr{K}}P\to P$ for which

$$\begin{tikzcd} \widetilde{\mathscr{K}}\widetilde{\mathscr{K}}P \arrow[r, shift left, "\bullet P"] \arrow[r, shift right, "\widetilde{\mathscr{K}}\lambda"'] & \widetilde{\mathscr{K}}P \arrow[r, "\lambda"] & P \end{tikzcd}$$

is a coequalizer

• Write $\mathbf{iMod}(\mathbf{1},\widetilde{\mathscr{K}})$ for the category of such i-modules and equivariant (matrix) arrows

• Define $J:\mathbf{iMod}(\mathbf{1},\widetilde{\mathscr{K}})\to\mathbf{Prof}(\mathbf{1},\mathscr{K})=\widehat{\mathscr{K}}$ by $J(P,\lambda)=(P,\lambda_1)$ where

$$\begin{tikzcd} \mathscr{K}\widetilde{\mathscr{K}}\widetilde{\mathscr{K}}P \arrow[r, shift left, "\mathscr{K}\bullet P"] \arrow[r, shift right, "\mathscr{K}\widetilde{\mathscr{K}}\lambda"'] \arrow[d, "@\widetilde{\mathscr{K}}P"'] & \mathscr{K}\widetilde{\mathscr{K}}P \arrow[r, "\mathscr{K}\lambda"] \arrow[d, "@P"] & \mathscr{K}P \arrow[d, "\lambda_1"] \\ \widetilde{\mathscr{K}}\widetilde{\mathscr{K}}P \arrow[r, shift left, "\bullet P"] \arrow[r, shift right, "\widetilde{\mathscr{K}}\lambda"'] & \widetilde{\mathscr{K}}P \arrow[r, "\lambda"] & P \end{tikzcd}$$

THEOREM [MRW] For $\mathscr{T}$ a small taxon, $\mathbf{iMod}(\mathbf{1},\mathscr{T})$ is a totally distributive category $\square$

THEOREM For $\mathscr{K}$ totally distributive, the adjunction $\langle\alpha,\beta; W \dashv \bigvee : \mathbf{Prof}(\mathbf{1},\mathscr{K}) \to \mathscr{K}\rangle$ gives rise to an adjoint equivalence $\langle\widetilde{\alpha},\widetilde{\beta}; \widetilde{W} \dashv \widetilde{\bigvee} : \mathbf{iMod}(\mathbf{1},\widetilde{\mathscr{K}}) \to \mathscr{K}\rangle$:

$$\begin{tikzcd}
 & \mathbf{iMod}(\mathbf{1},\widetilde{\mathscr{K}}) \arrow[dl, shift left, "\widetilde{\bigvee}"] \arrow[dd, "J"] \\
\mathscr{K} \arrow[ur, shift left, "\widetilde{W}"] \arrow[ur, phantom, "\overset{\dashv}{\cong}"] \arrow[dr, shift right, "W"'] \arrow[dr, phantom, "\dashv"] & \\
 & \mathbf{Prof}(\mathbf{1},\mathscr{K}) = \widehat{\mathscr{K}} \arrow[ul, shift right, "\bigvee"']
\end{tikzcd}$$

Proof: It is straightforward that W factors as $J\widetilde{W}$ since each $WA = \widetilde{\mathscr{K}}(-,A)$ is an i-module. We define $\widetilde{\bigvee} = \bigvee J$. Clearly, $\widetilde{\alpha} = \alpha{:}1_{\mathscr{K}} \to \bigvee W = \bigvee J\widetilde{W} = \widetilde{\bigvee}\widetilde{W}$ is an isomorphism. Next, we require a $\widetilde{\beta}{:}\widetilde{W}\widetilde{\bigvee} \to 1_{\mathbf{iMod}(\mathbf{1},\widetilde{\mathscr{K}})}$ for which we require, for each i-module (P,λ), a $\widetilde{\mathscr{K}}$-equivariant

$\widetilde{\beta}(K,(P,\lambda))\colon\widetilde{\mathscr{K}}(K,\bigvee(P,\lambda_1))\twoheadrightarrow[K,(P,\lambda)]$. The requisite *functions* are provided by the $\beta(K,(P,\lambda_1))\colon\widetilde{\mathscr{K}}(K,\bigvee(P,\lambda_1))\twoheadrightarrow[K,(P,\lambda_1)]$ but these must be shown to be $\widetilde{\mathscr{K}}$-equivariant:

$$\beta(\omega\bullet\psi)=\beta(\omega\bar{\psi})=(\omega\bar{\psi})(1)=\omega(1)\bar{\psi}=\beta(\omega)\bar{\psi}=(p\chi)\bar{\psi}$$

$$=p(\chi\bar{\psi})=p(\chi\bullet\psi)=(p\chi)\psi=\beta(\omega)\psi$$

where we have used that (P,λ) is an i-module to express $\beta(\omega)$ as $p\chi$. A similar expression for a typical K-element p of P is used to show $\widetilde{\beta}$ invertible. $\square$

REMARK $\widetilde{\mathscr{K}}$ does not have a small set of objects, however it is locally small and by the last theorem $\mathbf{iMod}(\mathbf{1},\widetilde{\mathscr{K}})$, a power object for $\widetilde{\mathscr{K}}$ as a taxon, is locally small. By the [S&W] *definition* of smallness based on the [F&S] Theorem, it is reasonable to say that $\widetilde{\mathscr{K}}$ is *small as a taxon* and attempt to clarify this set-theoretically.

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