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Fibrations of polynomial and analytic functors and monads

It is well known that many notions like monad, Kleisli and Eilenebrg-Moore category, mnonidal category, category of monoids, action of a monoidal category and category of actions along an action introduced in the context of the 2-category **Cat**, when suitably internalized, make sense in any 2-category $\mathcal{A}$ with finite (2-dimensional) products. Moreover, we can say that a 0-cell in $calA$ has some limits and/or colimits if the category $\mathcal{A}(\mathcal{X}, \mathcal{C})$ has such limits for any 0-cell $\mathcal{X}$ in $\mathcal{A}$ and/or colimits and the precomposing functors preserve them. This permits to internalize to any 2-category with finite products several arguments that holds true in **Cat** and use them directly in different contexts.

We have shown, among other things, that if we have a monoidal monad $(\mathcal{R}, \phi, \eta, \mu)$ on a monoidal object $(\mathcal{C}, \otimes, I, \alpha, \lambda, \rho)$ so that $\mathcal{C}$ has reflexive coequalizers, both structures respect them, and $(\mathcal{R}, \eta, \mu)$ admits an Eilenberg-Moore object in $\mathcal{A}$, then $(\mathcal{R}, \phi, \eta, \mu)$ admits an Eilenberg-Moore object in the 2-category $Mon(\mathbf{Cat})$ of monoidal objects, (lax) monoidal functors, and monoidal transformations in $\mathcal{A}$. This extends earlier results due to F. Linton, R. Guitar, and G. Seal, as well as some of our earlier results. A similar result for the Kleisli object is also true and it is of a much simpler nature. Both Eilenberg-Moore and Kleisli objects lift along 2-fibration to a 2-category of lax slices, and similar results also hold for the 2-category of (lax) actions of monoidal objects on a given 0-cell.

The development of this abstract theory was inspired by the theory of polynomial and analytic functors and monads. When applied to a symmetrization monad on a fibration of signatures in a (suitable) 2-category of fibrations, it shows in a natural way how these notions arise and how they are related to each other. The theory applies to some other natural cases of monoidal monads, as well. For example, it gives rise to the semi-analytic and all finitary functors and monads on slices of Set.

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