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### *The calculus of modules for $\infty$ -categories*

Under very general conditions one may start with a theory of  $\infty$ -categories, such as  $n$ -fold Segal spaces (a model of  $(\infty, n)$ -categories), and derive an associated rich calculus of modules dubbed a *virtual equipment* by Geoff Cruttwell and Michael Shulman. This opens up the prospect of developing the foundations of  $\infty$ -category theory in a *model agnostic* way, which allows it to be applied equally to a very diverse range of higher  $\infty$ -categorical structures.

In this talk we examine how this calculus may be deployed to provide an entirely elementary development of a number of the key ingredients that should be present in any self respecting  $\infty$ -category theory. As exemplars, we develop the theory of exact squares and finality, weighted (co)limits and pointwise Kan extensions. Furthermore, we demonstrate that this theory coincides with established accounts in the special case of quasi-category theory to be found in the work of Joyal, Lurie and others.

While this gets us a long way, it is just a first step towards providing a full account of the foundations of  $\infty$ -category theory at this level of model agnostic generality. If time permits, we will briefly touch upon extensions of this work which encompasses questions of size and free co-completion.

#### References:

- [1] Riehl, E. and Verity, D., *Fibrations and Yoneda's lemma in a quasi-categorical context*, in preparation.
- [2] Riehl, E. and Verity, D., *Kan extensions and the calculus of modules for  $\infty$ -categories*, in preparation.

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