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Categories of "lax fractions"

The concept of orthogonality is closely linked to full reflective subcategories (and, thus, to idempotent monads). The concept of Kan-injectivity plays for KZ-monads (or lax-idempotent monads) the same role as orthogonality for the idempotent ones, as demonstrated in the papers [2, 1] in the case of order-enriched categories (i.e., categories enriched in **Pos**).

In an order-enriched category $\mathcal{X}$, an object A is said to be (left) Kan-injective with respect to a morphism $f : X \to Y$, if the **Pos**-morphism $\mathcal{X}(f, A) : \mathcal{X}(Y, A) \to \mathcal{X}(X, A)$ is a right adjoint retraction. And a morphism $k : A \to B$ is Kan-injective with respect to f if it satisfies the equality $(\mathcal{X}(f, B))^* \cdot \mathcal{X}(X, k) = \mathcal{X}(Y, k) \cdot (\mathcal{X}(f, A))^*$, where $(\mathcal{X}(f, A))^*$ denotes the left adjoint of $\mathcal{X}(f, A)$. For a given subcategory $\mathcal{A}$, we study the class $\mathcal{A}^{KInj}$ of all morphisms with respect to which all objects and morphisms of $\mathcal{A}$ are Kan-injective. In particular, we show that $\mathcal{A}^{KInj}$ is, in a certain sense, closed under weighted colimits. In the case of $\mathcal{A}$ being a KZ-monadic subcategory of $\mathcal{X}$, we construct a category of "lax fractions" for $\mathcal{A}^{KInj}$. This construction resembles the one of a category of fractions for the class of morphisms inverted by a reflector into a full reflective subcategory.

References:

[1] J. Adámek, L. Sousa and J. Velebil, Kan-injectivity in order-enriched categories, *Math. Structures Comput. Sci.* 25 (2015) 6–45.

[2] M. Carvalho and L. Sousa, Order-preserving reflectors and injectivity, *Topology and its Applications* 158 (2011) 2408-2422.