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On monoidal (co)nuclei and their applications

Quantic (co)nuclei provide a convenient technique for constructing quotients and subquantales of quantales [3]. This talk shows its analogue for the monoidal approach to topology of [2] in the form of the categories $(\mathbf{T}, V)\text{-}\mathbf{Cat}$, the latter based in a monad \mathbf{T} on \mathbf{Set} and a unital quantale V . As a consequence, we get a machinery for constructing quotient categories and subcategories of the categories $(\mathbf{T}, V)\text{-}\mathbf{Cat}$, thereby providing a common framework for several of the already defined ones in the literature (for example, sets, preordered sets, metric spaces, topological spaces, V -closure spaces, V -weighted H -labelled graphs, and V -enriched multi-ordered sets). We also get a representation theorem for the categories $(\mathbf{T}, V)\text{-}\mathbf{Cat}$, which arises as an analogue of the quantale representation theorem of [3]. We then apply our (co)nuclei technique to the (op-)canonical extensions of monads of G. Seal [4, 5] and the topological theories of D. Hofmann [1], thereby providing quotient categories and subcategories of $(\mathbf{T}, V)\text{-}\mathbf{Cat}$ in the form of $(\mathbf{T}, 2)\text{-}\mathbf{Cat}$ for the two-element quantale 2 .

References:

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