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On the “Smith is Huq” condition in S -protomodular categories

The “Smith is Huq” condition, which says that two equivalence relations on the same object centralize each other (in the Smith-Pedicchio sense [1, 2]) as soon as their normalizations commute (in the Huq sense [3]), has proved to have several important algebraic consequences. For example, it allows to simplify the description of internal categories, internal groupoids and crossed modules: see [4] for a detailed account of these properties in semi-abelian categories, and [5] for an extension to the case of pointed Mal’tsev categories.

In this talk we consider a relative version of the “Smith is Huq” condition in the context of pointed S -protomodular categories [6]: two S -equivalence relations centralize each other as soon as their normalizations commute. As proved in [6], in a S -protomodular category the fact that two S -equivalence relations centralize each other is a property, like for equivalence relations in Mal’tsev categories. We show that the categories of monoids, of semirings, and, more generally, all categories of monoids with operations (in the sense of [7]) are S -protomodular categories satisfying the “Smith is Huq” condition. We explore then some consequences dealing with the description of internal structures.

References:

- [1] J.D.H. Smith, Mal’cev varieties, *Lecture Notes in Mathematics*, vol. 554 (1976), Springer.
- [2] M.C. Pedicchio, A categorical approach to commutator theory, *J. Algebra* 177 (1995), 647–657.
- [3] S.A. Huq, Commutator, nilpotency and solvability in categories, *Q. J. Math.* 19 (1968) n.2, 363–389.
- [4] N. Martins-Ferreira, T. Van der Linden, A note on the “Smith is Huq” condition, *Appl. Categorical Structures* 20 (2012), 175–187.
- [5] D. Bourn, N. Martins-Ferreira, T. Van der Linden, A characterisation of the “Smith is Huq” condition in the pointed Mal’tsev setting, *Cah. Topol. Géom. Différ. Catég.* **LIV** (2013), n. 4, 243–263.
- [6] D. Bourn, N. Martins-Ferreira, A. Montoli, M. Sobral, Monoids and pointed S -protomodular categories, submitted, preprint DMUC 14-04.
- [7] N. Martins-Ferreira, A. Montoli, M. Sobral, Semidirect products and crossed modules in monoids with operations, *J. Pure Appl. Algebra* 217 (2013), 334–347.

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