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Alexander Grothendieck and the modern conception of Space

Already before 1960, the profound innovation by Eilenberg and Mac Lane had inspired further work that still plays a basic role in our present advances. I refer to the results of Kan, Isbell, Yoneda, and Grothendieck. Grothendieck's Tohoku article introduced the notion of subobject (still not grasped by many writers) and an emphasis on functor categories as a key method of construction. Grothendieck's elaborate construction of algebraic geometry via local ringed spaces, was rejected by himself already in 1973 in his lecture in Buffalo; efforts to take that qualitative leap into account have so far been incomplete.

Although in the 1950's Grothendieck was considered to be one of the leaders of functional analysis, recent journalistic accounts of his career seem to view that as a youthful deviation from his path to algebraic geometry. However, closer attention reveals that among his several advances in functional analysis was his calculation of the dual space of a space of analytic functions, specifically revealing it concretely as another space of analytic functions on a domain complementary to the original domain of definition. The study of the shapes of these domains led to a concentration on analytic geometry, where some of the first toposes emerged (but NOT as 'generalized spaces'). The close connection between compact analytic spaces and algebraic spaces emphasized the contrast between the two realms with respect to an implicit function theorem, leading to the other original class of toposes, namely the étale, which indeed constitutes a kind of generalized space, but not a localic one.

A serious re-elaboration of the history of Grothendieck's career will be a necessary part of the program to re-establish the foundations of geometry in a way that is in accord with Grothendieck's basic insights, but that makes maximum use of recent clarifying advances. Also helpful will be a more responsible use of the undefined term 'generalization'.

I propose to continue the following dialog: 'What is a Space?' 'A space is an object in a category of spaces.' 'So what is a category of spaces?'