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*Functoriality of modified realizability*

Recent work of the author [1], building on earlier work of Longley and of Hofstra & van Oosten, has completely solved the problem of describing the geometric morphisms between ordinary realizability toposes: up to equivalence of categories, they correspond to quasi-surjective (applicative) morphisms between the underlying Schönfinkel algebras. In addition, the identification of the ‘Herbrand realizability toposes’ of van den Berg with the Gleason covers of ordinary realizability toposes [2] shows that Herbrand realizability is also a 2-functor on the 2-category of Schönfinkel algebras and quasi-surjective morphisms, although we cannot assert that all geometric morphisms between such toposes arise in this way. Although the notion of modified realizability has been extensively studied by van Oosten and by Hyland and Ong, up to now little attention has been paid to its functoriality. In this talk I shall present a partial solution to the problem; it turns out to be complicated by the fact that a modified realizability topos depends, not only on the choice of a Schönfinkel algebra, but also on the choice of a ‘right ideal’ of distinguished elements which are always available as potential realizers.

References:

- [1] P.T. Johnstone, Geometric morphisms of realizability toposes, *TAC* 28 (2013) 241–249.
- [2] P.T. Johnstone, The Gleason cover of a realizability topos, *TAC* 28 (2013) 1138–1152.