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Symmetrizing categories of lenses

So called *lenses* arise in applications where data needs to be synchronised between two otherwise independent systems. A variety of kinds of lenses have been introduced (examples include [1], [2] and [4]), but in each case lenses compose and form a category, typically with sets or categories as objects. When the categories of lenses have sets as their objects the lenses are called *set-based*, and when they have categories as their objects they are called *category-based*.

Commonly lenses are asymmetric in that the data synchronisation is simple in one direction, but requires some kind of lifting property in the other direction. An example of a lens familiar to category theorists is a split fibration — the total space is fibred over a base space with the functor giving synchronisation in one direction and the chosen cartesian liftings representing synchronisation in the other direction. In system terms such lenses are master-slave systems — the total space is the complete system with full information, and the base space is some kind of view, that is a partial representation of the data.

More generally interactions between systems are symmetric in that synchronisation in each direction requires a mixture of some simple data manipulation and some liftings. Researchers have therefore repeatedly taken some category $\mathcal{C}$ of asymmetric lenses and, largely in an ad-hoc way, constructed corresponding categories of symmetric lenses. The authors have argued previously that these constructions should all be made via a bicategory $\mathrm{Sp}\,\mathcal{C}$ of spans in $\mathcal{C}$, and we have studied appropriate equivalences among spans which reduce the bicategory to the expected category of symmetric lenses. The goal has been a unified category theoretic process for symmetrizing categories of lenses.

Naturally an appropriate equivalence of spans would include the usual isomorphisms of spans, but the isomorphisms themselves are too fine an equivalence relation for the applications since the same symmetric lens — the same bidirectional synchronisation of data — can be achieved through non-isomorphic spans. A natural and appealing generalisation of the isomorphism equivalence relation arises by using certain non-trivial lenses themselves to generate the equivalence relation. This suggestion has been very well-received in the bidirectional transformation community, and corresponds to the intuition that lenses are themselves a desirable generalisation of isomorphism of systems, providing as they do bidirectional data synchronisation. Indeed we had proposed [6] a unified process for symmetrizing categories of lenses by constructing $\mathrm{Sp}\,\mathcal{C}$ and reducing the hom sets by the equivalence relation generated by those non-trivial lenses of $\mathcal{C}$. We demonstrated that for a variety of kinds of set-based lenses the process yielded the known categories of symmetric lenses, and showed how it could be applied to other categories of set-based lenses to provide an appropriate symmetrization, avoiding further ad-hoc definitions of symmetric set-based lenses.

*Joint work with Robert Rosebrugh.

We present here a study of category-based asymmetric and symmetric lenses and provide a counter-example that demonstrates that the equivalence referred to in the previous paragraph is still too fine an equivalence relation for the category-based lens applications. Our analysis leads to a coarser equivalence relation $\mathcal{E}$ for the category-based symmetrizing process. When $\mathcal{C}$ is the category of Diskin et al's asymmetric delta lenses [2] our main theorem provides an isomorphism between Diskin et al's symmetric delta lenses [3] and

$$(\mathrm{Sp}\,\mathcal{C})/\mathcal{E}.$$

Further we illustrate the equivalence relation in the special case of spans of split fibrations.

References:

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[6] Michael Johnson and Robert Rosebrugh (2014) Spans of lenses, *CEUR Proceedings* 1133 (2014) 112–118.