Richard Garner
Macquarie University

The Campbell–Baker–Hausdorff adjunction

The Lie algebra associated to a Lie group G encodes the first-order infinitesimal structure of G near the identity; on the other hand, the *formal group law* associated to G is a collection of formal power series which encode all finite-order infinitesimal behaviour. One obtains the formal group law from the Lie group by Taylor expanding the multiplication with respect to some chart around the identity; alternatively, one may obtain the formal group law from the Lie algebra by applying the *Campbell–Baker–Hausdorff* (CBH) formula, which expresses the group multiplication power series near the identity purely in terms of iterated Lie brackets.

There are a number of ways of deriving the CBH formula, some geometric and some algebraic in nature. The aim of this talk is to describe a categorical approach drawing on synthetic differential geometry. We consider a category $\mathcal{E}$ of microlinear spaces wherein formal group laws may be construed as genuine internal groups; we then construct an adjunction, the Campbell–Baker–Hausdorff adjunction of the title, between internal groups and internal Lie algebras in $\mathcal{E}$. Applying the left adjoint to a finite dimensional Lie algebra yields its associated formal group law; applying it to the free Lie algebra on two generators yields the free group on two non-commuting tangent vectors, whose multiplication may be seen as a pure combinatorial manifestation of the CBH formula.