5.3.3 A two-sector model of endogenous growth

Another incarnation of the AK model was proposed by Usawa, as early as in 1965, but the contribution remained basically obscured until it was popularized by Lucas (1988)[79]. The author extended the Solow model, considering two sectors of production: one producing final goods that can be either consumed or accumulated as capital stock (productive sector) and other pooling together various activities that contribute to the efficiency of labour, such as education, and research ("educational" or "research" sector). The production sector employs labour and capital. The educational sector employs labour only.

In the model, it is assumed that workers devote a fraction $1-\mu$ of their working time to production of goods and the remaining μ to the improve labour efficiency, λ. The production function for final goods is given by:

$$Y = AK^{\beta}\left[(1-\mu)\lambda N\right]^{1-\beta} \qquad (5.14)$$

The change in technology is a positive function of the *fraction* of labour allocated to the educational sector:

$$\dot{\lambda} = b\mu\lambda \qquad (5.15)$$

The parameter b captures the productivity *in the research sector*. In this model, the linearity that is needed to generate endogenous growth (the AK feature) arises from the fact that the production function for technology (5.15) depends linearly on the level of technology, through the *standing on shoulders effect*: a constant fraction of working time devoted to research produces a constant growth rate of technology. With such an assumption, a policy change that successfully increases the proportion of time allocated to research (μ) or that

[79] Usawa, H., 1965. Optimum technical change in an aggregative model of economic growth", International Economic Review, 6, 19-31. Lucas (1988), op. cit.

improves the productivity in that sector (b) will impact positively and permanently on the growth rate of per capita income.

As for physical capital accumulation, the assumption (5.9) is retained. This model can be solved in the same manner as the Solow model. For mathematical convenience, let's rewrite the production function (5.14) as follows:

$$\tilde{y} = A(1-\mu)^{1-\beta}\tilde{k}^{\beta} \qquad (5.16)$$

Where $\tilde{y}=Y/L$, $\tilde{k}=K/L$, and $L=\lambda N$. Proceeding as usual, the fundamental dynamic equation becomes:

$$\dot{\tilde{k}} = sA(1-\mu)^{1-\beta}\tilde{k}^{\beta} - [n+\delta+b\mu]\tilde{k} \qquad (5.17)$$

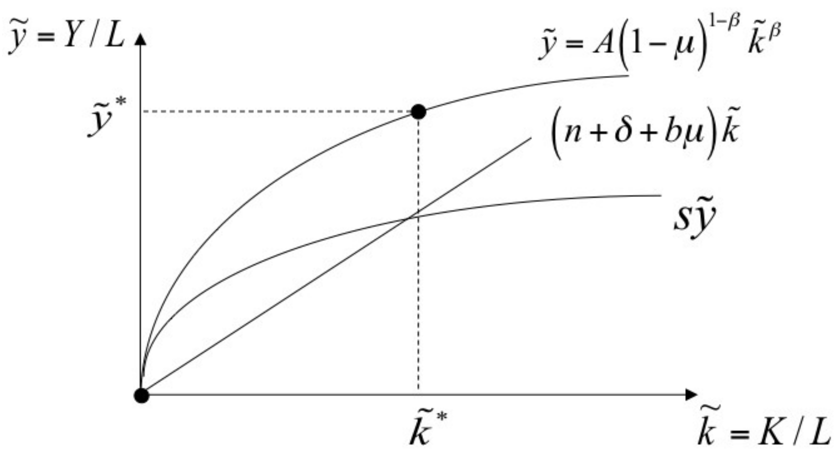
Comparing with (3.8) you can verify how similar this model is with the Solow model. The main difference is that the parameter determining the effectiveness of labour, rather than growing exogenously, is now dependent of other parameters in the model (ie, $\gamma = b\mu$). To find the steady state, we just need to solve for $\dot{\tilde{k}} = 0$, and use the definition $\tilde{y} = y/\lambda$ to obtain:

$$y_t^* = (1-\mu)A^{\frac{1}{1-\beta}}\left(\frac{s}{n+\delta+\gamma}\right)^{\frac{\beta}{1-\beta}}\lambda_t \qquad \text{with} \qquad \lambda_t = e^{\gamma t} \qquad (5.18)$$

Figure 5.5 illustrates the steady state of the model. Because both $\tilde{y}$ and $\tilde{k}$ are constant in the steady state, the output-capital ratio is constant and so will do the interest rate.

This model is hybrid, in the sense that it shares characteristics with the Solow model and with the AK model. It shares with the neoclassical model the feature that it displays level effects and transition dynamics: like in the Solow model, changes in *s* produce "level effects", causing output per unit of efficiency to increase from one steady state to the other. However, it borrows from the AK model the linearity that is needed to obtain unceasing growth: technically, sustained growth is obtained in this model because the production function of technology is free from diminishing returns[80]. Thus, for example, a policy that induces an increase in the proportion of time that people allocate to research (μ) or in the effectiveness of that time, b, will permanently tilt the growth rate of per capita income ("growth effect"). The model can generate sustained growth of per capita income without the need to assume exogenous shifts in the production function.

Figure 5.5: the steady state in the two-sector model



The dynamics of the Usawa model is similar to that of the Solow model. There is a stable steady state when the break-even investment line crosses the savings locus. The main difference is that both the production function and the break-even investment line are tilted when the fraction of time devoted to research changes.

Figure 5.6 describes the path of per capita income in this economy following an increase in the proportion of time allocated to research: at the impact, there is a negative effect on per capita income, because less time is devoted to production. As the times go by, however, the

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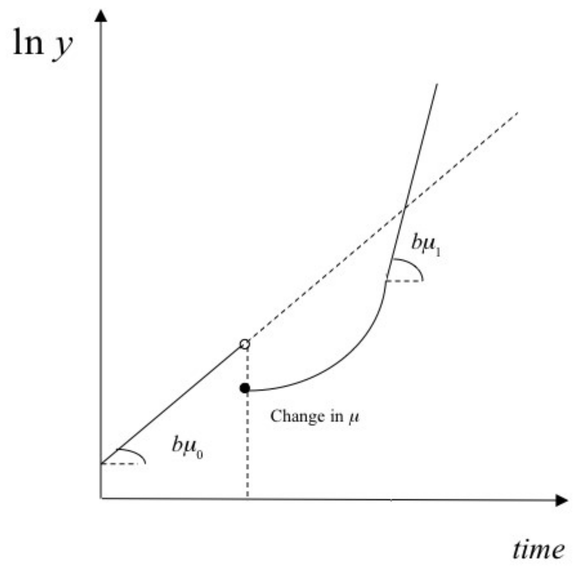
[80] This assumption is not free of controversy. It could be argued that new knowledge become more difficult to achieve as the stock of knowledge increases ("fishing out effect"). In that case, the production function of knowledge would exhibit diminishing returns (ie, $\dot{\lambda} = b\mu\lambda^{\varepsilon}$ with $\varepsilon < 1$) and the growth rate of technology would fall down to zero, no matter how much effort was allocated to the educational sector. Sustained growth could not be achieved. On the other hand, the production function (5.15) is ignoring the possibility that technological progress depends on the number of researchers (rather than on the fraction of population engaged in research). Equation (5.1) is therefore just a convenient specification that prevents the growth rate of knowledge from decreasing or expanding over time.

growth rate of per capita output accelerates, due to the faster rate of technological expansion. Sooner or later output per capita will pass the level it would have reached had there be no change in the research effort. Referring to Figure 5.5 the production function and the break-even investment line shift in opposite directions. Hence, $\tilde{k}$ starts decreasing until meeting the new steady state. This implies that, during the transition period the growth rate of per capita output is less than the corresponding level in the steady state. As the economy approaches the new steady state, the decline in $\tilde{k}$ decelerates, implying a convergence of per capita income growth to the new steady state growth rate, $b\mu_1$.

Figure 5.6 – The path of output per capita following an increase in μ



When the fraction of time employed in research increases, output in the production sector declines, causing per capita income to fall. However, there is a growth effect that, in the long run, more than offsets the negative level effect. Immediately after the shock. The growth rate of per capita income is low, reflecting the fact that the capital per unit of efficiency labour (horizontal axes in figure 5.5) is approaching a new – lower – level.

A question that naturally arises is how people decide the optimal level of μ. Intuitively, the time allocated to the research sector versus final good production shall be determined by non-arbitrage conditions, stating that the payoff of the two activities at the margin should be the same. On the other hand, the payoff to R&D shall depend on how future production is valued today. Thus, the optimal allocation of time shall balance a variety of factors, such as the productivity of research (b), the opportunity cost of research time (the wage rate), the level of impatience of people, and so on.