3 Exogenous Growth

"The Solow model did not assume that technical progress was exogenous—that is, determined outside the model. Rather, the model made the assumptions necessary to produce a model of an economy with a dynamic equilibrium, a path to which, in the long run, the economy would settle down. The implication of those assumptions was that technical progress had to be exogenous to the model". [Lant Pritchett]

Learning Goals:

- Understand why the Solow model cannot explain technological change
- Solve the model with exogenous technological progress
- Acknowledge the extent to which the modified model helps explain real world facts
- Explain the model' implications regarding convergence
- Understand the implications for growth accounting of using alternative definitions of technological progress

3.1 Introduction

As shown in Chapter 2, the basic Solow model does not account for the essential stylized fact of Modern Economic Growth: that output per capita tends to grow over time. This limitation was noted by Robert Solow itself in its original article, where he also provided a brief indication of how technological progress could be incorporated into the model.

This chapter shows how the Solow model can be adapted to account for the possibility of technological progress. As we will see, this modification rescues the model from its main limitation and renders it capable of describing most stylized facts of economic growth. The Chapter is organized as follows: in Section 3.2, we explain why the Solow model cannot account for endogenous technological progress. Section 3.3 presents the extended version of the Solow model with exogenous technological progress. Section 3.4 discusses how the main variables of the model adjust to changes in exogenous parameters. In Section 3.5, we show how this extended version is helpful to understand many facts of modern economic growth. Section 3.6 discusses the implications for growth accounting. Section 3.7 concludes.

3.2 Perfect technological diffusion

Technology is different from most other goods, in that it is composed by *ideas*, rather than by *objects*. One implication of technology's non-physical nature is that its use is *nonrival*: it can be used by more than one person at the same time without loosing its effectiveness. This is in sharp contrast to physical capital: if someone is using an equipment, no one else can use that equipment at the same time. In other words, equipment is "rival" in its use. This is not true for ideas and knowledge, even if they do come packaged up in bits of capital equipment: the fact that a given company uses some software to manage its operations does not preclude other firms from using the same software[48].

Technology may vary, however, in its degree of *excludability*. Excludability is the degree to which an owner of something can prevent others from using it without consent. Much of the knowledge, because of its nature, is non-excludable: it is difficult to prevent an agent from using a good idea, once he or she becomes aware of it. Still, there are ways of preventing others from using particular pieces of knowledge: for instance, trade secrets, patents and copyrights, are mechanisms though which agents try to keep competitors away from their inventions.

In this chapter, we stick to the assumption of *perfect technological diffusion*: that is, once a new technology becomes available, it becomes equally available to all agents at the same time. The reason to assume that technology spills over at no cost is that we are dealing with a model with perfect competition. Under perfect competition, all information is freely available to all agents in the economy.

The implication of assuming perfect technological diffusion is that technology becomes a *pure* public good: no user will be willing to pay for it and no self-interested agent will engage in a deliberate effort to produce it. In terms of our model, this implication is

[48] On the no rivalrous nature of knowledge, there is a famous quote of Thomas Jefferson, in a letter to Isaac McPherson, in 1813: "Its peculiar character…is that no one possesses the less, because every other possesses the whole of it. He who receives an idea from me, receives instruction himself without lessening mine; as he who lights his taper at mine, receives light without darkening me".

rather convenient: we don't need to worry with returns to innovation or to model the research activity. The other face of the coin is that technological progress is doomed to enter in the model exogenously: since there is no reward to innovation, any technological progress must take place for non-economic reasons (such as unintended discoveries that come out through the passage of time).

3.3 The extended Solow model

3.3.1 Labour augmenting technological progress

In the model of Chapter 2, the state of technology, A, was assumed constant over time. In this chapter, it is assumed instead that technology expands over time at the constant rate, g:

$$A_t = Ae^{gt} \qquad (3.1)$$

With such specification, technological progress has the effect of "renumbering" the isoquants of the production function: as time goes by, each isoquant corresponds to a higher level of output than before. The "shape" of the isoquants remains unchanged: for each relative factor price, the optimal proportion in which inputs are used remains unchanged. Technological progress specified this way is labelled "Hicks Neutral"[49].

3.3.2 The two components of "technology"

Not all productivity changes are related to technology in the narrow sense: two economies may have access to the same body of technical knowledge but may follow different models of social organization. These differences will show up as differences in TFP in a production function with capital and labour. To account for these different aspects of

[49] Technically, technological progress is said to be "Hicks Neutral" if the Marginal Rate of Technical Substitution remains unchanged, for each given capital-labour ratio.

"technology", equation (3.1), includes two components: a constant parameter (A), and a term that expands over time.

The first component, A, shall be interpreted as capturing the influence of factors that affect the *level* of productivity. For instance, a country climate may influence the overall relationship between inputs and output, for each level of technology. We may also interpret this component as capturing aspects of economic "efficiency", such as those related to taxation, externalities, bureaucracy, and the quality of institutions. These factors influence the effectiveness with which inputs to production *and* a given state of technology are combined to produce output. The second component – which grows continuously over time – is thought to capture the role of technological progress, in the engineering sense.

3.3.3 Other assumptions

The remaining assumptions of the model are the same as in the basic Solow model. For your convenience, we reproduce the main equations here:

$$Y_{it} = A_t K_{it}^{\beta} N_{it}^{1-\beta} . \qquad (2.3)$$

$$sY_t = I_t \qquad (2.7)$$

$$\dot{K}_t = I_t - \delta K_t \qquad (2.8)$$

$$n = \dot{N}_t / N_t \qquad (2.9)$$

3.3.4 Labour in efficiency units

Since technological progress (3.1) causes the production function (2.3) to shift upwards continuously over time, solving the model in this new version is not as straightforward as it was in the basic formulation. But with the help of a small trick, we can turn the new formulation similar to the previous one. The trick is to re-write the model in terms of a new variable, L, defined as "labour in efficiency units".

Substituting (3.1) in (2.3) and aggregating across firms, we can rewrite the aggregate production function in the following convenient form:

$$Y_t = AK_t^{\beta} L_t^{1-\beta} , \qquad (3.2)$$

where:

$$L_t = N_t \lambda_t \qquad (3.3)$$

$$\lambda_t = e^{\gamma t}\text{, with } \gamma = g/(1-\beta)\,. \qquad (3.4)$$

In (3.2), the term L measures labour in "efficiency" units (i.e, the number of workers adjusted for their – time varying - efficiency level). The term λ refers to the "effective labour input per worker".

Under the assumptions above, the "effective labour input per worker" grows at an exogenous rate, γ. That is, as time goes by, the typical worker becomes more efficient because new abilities are costlessly bestowed upon him at the rate γ. The rate γ is labelled the "Harrod neutral" or "Labour Augmenting rate of technological progress". It is called *Labour Augmenting* because, analytically, it produces the same effect in production of an increase in raw labour, N[50].

Note that, with the transformation above, the production function (3.2) gets a form similar to that of (2.1): the main difference is that we replaced N by L. This similarity is the trick we needed to return to the "previous problem", which we already know how to solve.

3.3.5 The Fundamental Dynamic Equation revisited

To solve the model, we make use of a new variable, $\tilde{y} = Y/L$, which will be labelled as output "per unit of efficiency". Remember, however, that our variable of interest – the one that measures economic progress – is per capita income, $y=Y/N$. Using (3.3), the relationship between the two variables is:

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[50] Technically, technological progress is said to be "Harrod Neutral" if does not alter the shares of labour and capital on income, for each given capital-labour ratio. When the production function is a Cobb-Douglas, the shares of labour and capital are constant and equal to their elasticities in production, 1-β and β, respectively. Hence, any Hicks neutral technological progress will also be Harrod neutral. For a given rate of Hicks neutral technological progress (g), the equivalent rate of Harrod neutral technological progress (γ) is larger. The reason is that, in the later case the burden of technological progress is carried by one factor, only..

$$y_t = \tilde{y}_t \lambda_t = \tilde{y}_t e^{\gamma t} \qquad (3.6)$$

Dividing all terms in (3.2) by L, one obtains the production function in the *intensive* form:

$$\tilde{y}_t = A\tilde{k}_t^{\beta}, \qquad (3.7)$$

where and $\tilde{k} = K/L$ denotes physical capital "per unit of efficiency".

Taking time derivatives in $\tilde{k}$ and using (2.7), (2.8), (2.9), and $\dot{L}/L = \dot{\lambda}/\lambda + \dot{N}/N = \gamma + n$, the modified version of the Fundamental Dynamic Equation results as follows:

$$\dot{\tilde{k}}_t = sA\tilde{k}_t^{\beta} - (n+\delta+\gamma)\tilde{k}_t \quad . \qquad (3.8)$$

The two terms in the right-hand side of (3.8) are depicted in Figure 3.1, together with equation (3.7). The first term measures gross investment per unit of efficiency labour. The second term gives the "break-even investment", that is, the one that would be necessary to compensate for the "depreciation" of $\tilde{k}$. Note that the later includes the depreciation of physical capital and the growth rate of "effective" labour, $n+\gamma$.

Apart from the way the endogenous variable is defined, the interpretation of equation (3.8) is the same as that of the corresponding equation in the basic Solow model, (2.14): in brief, $\tilde{k}$ rises whenever gross investment per unit of efficiency is higher than the break-even investment - as in $\tilde{k}_1$ of Figure 3.1 - and conversely. The model of Section 2 is a particular case of the model developed in this section, with $\gamma=0$.

3.3.6 The steady state

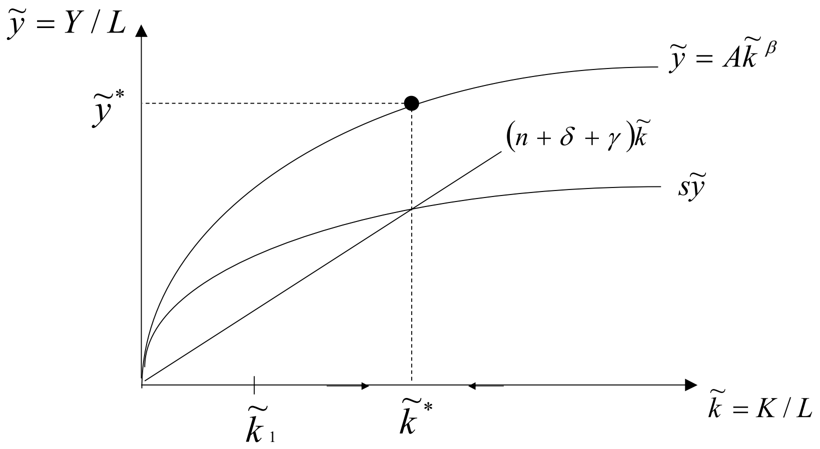
The steady state of the model is obtained setting $\dot{\tilde{k}} = 0$ in (3.8). Using $k_t = \tilde{k}_t e^{\gamma t}$, $y_t = \tilde{y}_t e^{\gamma t}$, we find the paths of capital per worker and per capita income in the steady state:

$$k^* = \left(\frac{sA}{n+\delta+\gamma}\right)^{\frac{1}{1-\beta}} e^{\gamma t} \qquad (3.9)$$

$$y_t^* = A^{\frac{1}{1-\beta}} \left(\frac{s}{n+\delta+\gamma} \right)^{\frac{\beta}{1-\beta}} e^{\gamma t} \quad . \tag{3.10}$$

Equation (3.10) states that, in the steady state, per capita income grows continuously at the rate γ. How can that be? The labour augmenting technological progress has the effect of neutralising the diminishing returns to capital: by economising progressively the input whose supply cannot be changed – labour – technological progress allows affective labour to increase at the same rate as the number of machines (capital) and at the same the number of machines per worker increases at rate γ.

Figure 3.1. The Solow model with technological progress



The figure depicts the equilibrium of the Solow model with technological progress. The vertical and horizontal axes measure output and capital per labour in efficiency units. The equilibrium is found when savings per unit of efficiency labour is equal to break-even investment.

3.4 Transitional Dynamics

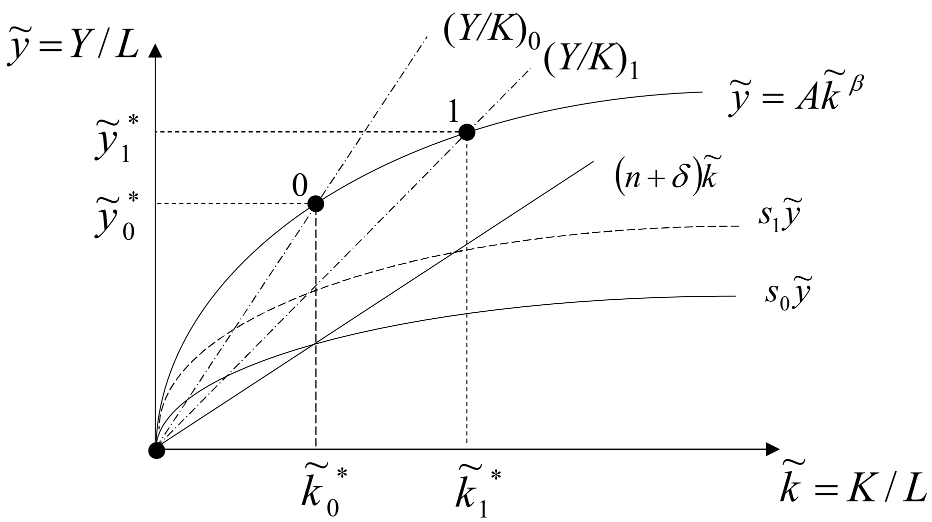
3.4.1 What happens if the Savings Rate increases?

In figure 3.2 we examine the implications of a once-and-for-all increase in the saving rate. The rise in the saving rate shifts the steady state to the right, from point 0 to point 1. In the new steady state the average product of capital, *Y/K*, is lower than in the initial steady state. This means that the interest rate has declined from one steady state to the other (equation 2.12).

Because capital accumulation is bounded each moment in time by the availability of savings, the economy does not jump immediately from point 0 to point 1: it slowly converges to the new steady state. Figure 3.3 describes the time paths of the main variables of the model, following a rise in the saving rate. The top panel depicts the evolution of output per unit of efficiency labour ($\tilde{y} = Y/L$) and capital per unit of efficiency labour ($\tilde{k} = K/L$). The Middle panel depicts the paths of output per capita ($y=Y/N$) and of per capita consumption ($c=C/N$) – these are displayed in logs, so as to stick with linearity. The bottom panel depicts the paths of the interest rate (r) and of the growth rate of per capita consumption (γ_t).

Let t_0 be the moment at which the saving rate raises to the new level. Assume that in the moment just before, the economy was in a steady state, with a constant level of output per unit of efficiency and with per capita income growing at the exogenous rate γ (like in point 0 in figure 3.2). Since capital and labour are both pre-determined, at the time of the shock *(t_0,)* per capita income remains initially unchanged. Per capita consumption, however, falls at the impact, because a higher proportion of income is devoted to savings.

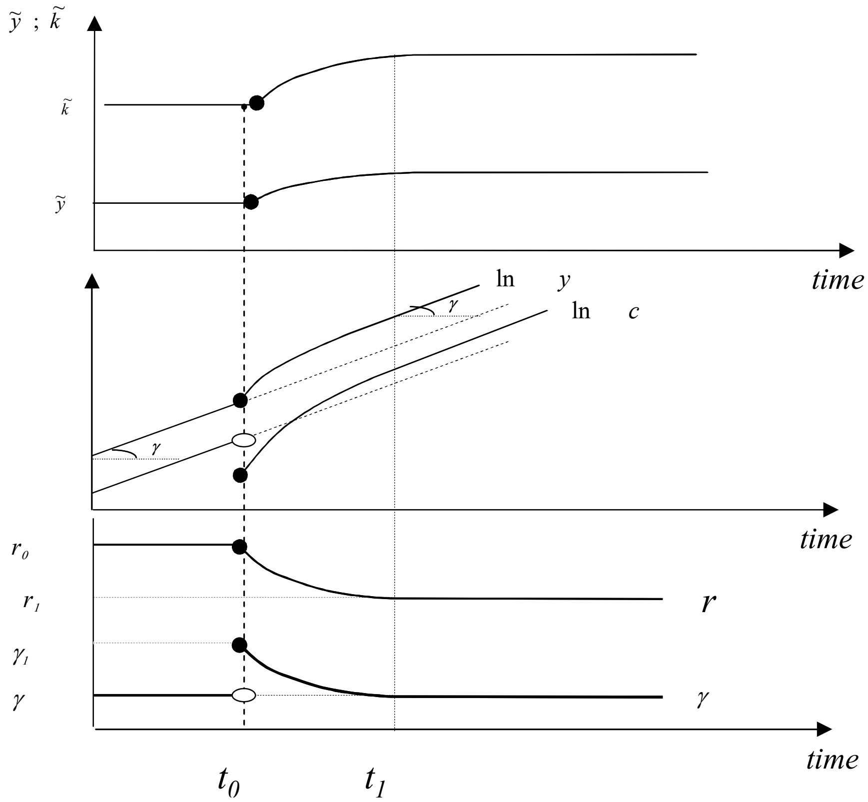
Figure 3.2. A higher saving rate raises the steady state level of output per unit of efficiency units



The figure shows how the the equilibrium of the model changes with an increase in the saving rate. In the new steady state (point 1), both capital and output per unit of efficiency labour are higher than before, and the average product of capital (Y/K) is lower, due to diminishing returns.

The adjustment process takes place between t_0 and t_1. Since savings become temporarily greater than the break-even investment, both *K/L* and *Y/L* start increasing. This means that the growth rates of output per capita and of per capita consumption jump temporarily ahead of the rate of technological expansion, γ. As the economy approaches the new steady state, diminishing returns show up, implying that the growth rate of output per capita falls back, approaching the exogenous rate, γ. The new steady state occurs when output per capita grows ate the same rate as technology, $\gamma_t = \gamma$.

Figure 3.3 – Implications of a rise in the saving rate



The figure describes the time paths of output and capital per efficiency units, per capita income and consumption, the interest rate and the growth rate of per capita output, following a once-and-for all increase in the saving rate. At the impact per capita consumption declines, reflecting the increased savings. After the shock, per capita income accelerates, expanding temporarily faster than technology. As time goes by, the growth rate of per capita income decreases until meeting the rate of technological expansion in the new steady state. From one steady state to the other, the interest rate declined, reflecting the higher abundance of capital relative to the previous path.

As in the case without technological progress, in the new steady state (after t_1), the consumption path may be above or below the original consumption path. Referring to our discussion in Section 2.5, this will depend on how the new and the old saving rates compare to the Golden Rule saving rate. Figure 3.4 depicts the special case, in which the rise in the saving rate leads to a higher level of per capita consumption in the steady state. You may easily verify that the Golden Rule saving rate in this version of the model is the same as before: it corresponds to the share of capital in total income, β. Also remember that an increase in the savings rate towards the golden rule is not necessarily welfare improving.

Box 3.1. Growth effects and level effects

At this stage, it is important to introduce the distinction between *level effects* and *growth effects*:

- A level effect occurs when changing a model' parameter changes the steady state without affecting the growth rate of the economy in the steady state.
- A growth effects occurs when a change in a parameter alters the growth rate of the economy in the steady state.

In the Solow model, changes in parameters like the saving rate and the population growth rate produce level effects, only. A growth effect could only occur if the exogenous rate of technological progress was changed.

3.5 The extended Solow model meeting the real-world facts

3.5.1 Revisiting the Kaldor' facts

As discussed in Chapter 2, the main limitation of the simpler version of the Solow model is that it cannot account for two stylized facts of economic growth, namely that per capita output and capital per worker tend to grow over time (Kaldor's facts 1 and 2). As show in equations (3.9) and (3.10), the assumption that technology expands continuously brings the model into compliance with these two facts.

As before, the model is consistent with the evidence that the shares of labour and capital on national income tend to be constant over time (Kaldor' fact 5). Note that firms take technology as given, so they maximize profits with respect to K and N, as in the simple Solow model. Hence, conditions (2.11-2.12) also hold in this more sophisticated version of the model. The stylized fact 5 is a direct consequence of assuming CRS and perfect competition.

Dividing (3.10) by (3.9) we obtain the expression for the average product of capital in the steady state:

$$\left(\frac{Y}{K}\right)^{*}=\frac{n+\delta+\gamma}{s} \tag{3.11}$$

Since all parameters at the right-hand side of (3.11) are constant, this means that the equilibrium average product of capital is constant over time. As a by-product – and using equation (2.12) - it follows that the interest rate is constant in the steady state. This is the Kaldor fact number 4.

A novelty with the augmented model is that real wages increase over time: since the wage rate is proportional to per capita output (equation 2.11), in the steady state wages are increasing at rate γ. This is another feature of the augmented model that makes it more compliant with the facts of Modern Growth.

Finally, we turn to fact number 6 ("There are wide differences in the growth rate of productivity across countries"). As long as we stick to the assumption of perfect technological diffusion, the model predicts that *in the long run all countries should be growing at the same rate*, γ. That is, the steady states of the different countries should be characterized by per capita incomes evolving in *parallel* over time.

Note however that such conclusion only holds in the long run: equation (3.10) refers to the steady state, it is expected to hold only for countries that already adjusted fully to changes in their exogenous parameters. For countries that are engaged in a *transitional dynamics*, the current growth rate of per capita income may be higher or lower than γ, depending on whether the starting point is below or above the corresponding steady state: countries that start out below (at the left of) their respective steady states are expected to grow faster than countries that start out above their steady states. Hence, growth rates may differ considerably across countries each moment in time. This is consistent with the Kaldor' fact number 6.

3.5.2 Absolute convergence

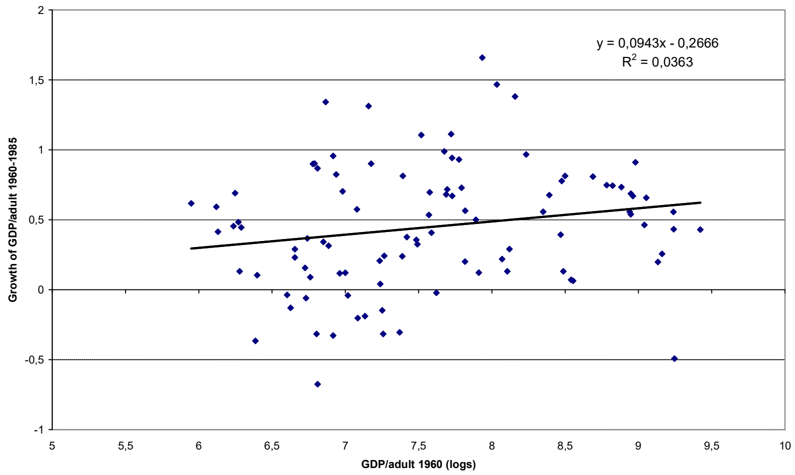
A question that has received considerable attention in the economic profession is whether there is a general tendency for poor countries to grow faster than rich countries. At least since David Hume[51], economists have been arguing that technological spillovers, diminishing returns and capital mobility provide poor countries with an impetus to "catch up". The hypothesis that poor economies tend to grow faster than rich economies is known as *absolute convergence*.

A simple way to investigate the convergence hypothesis using cross-sectional data is by plotting growth rates of per capita incomes against the initial levels of per capita incomes, for different countries along a period of time: if there was a general tendency for per capita incomes to approach each other, then poorer countries should grow faster than richer countries. Figure 3.5 illustrates such an exercise, using a sample of 98 non-oil countries. The figure relates the growth rate of GDP per working age person from 1965 to 1985 (vertical axis) with the corresponding 1965 level. If there was a general tendency for poor countries to grow faster than rich countries, the slope of the regression line should be negative. However, this is not the case. The conclusion is that "absolute convergence" does not hold as a general rule in this sample of countries during this time period.

Note however that this evidence does not contradict the Solow model: *the Solow model does not imply that countries should converge to same level of per capita output*. As stated in equation (3.10), countries differing in terms of the fundamental parameters (those that determine the steady state, such as the saving rate and the population growth rate) are expected to reach different steady states.

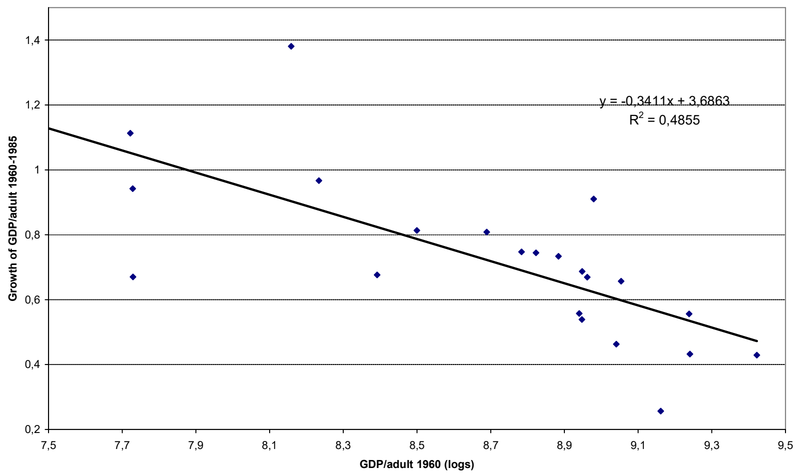
Figure 3.5: Evidence of Non-Convergence in 98 Countries

[51] Hume, D., 1758. Essays and Treaties' on Several Subjects. London: A. Millar.



The figure crosses growth rates of per capita incomes with initial levels of per capita incomes. The fact that no negative correlation is found implies that there is no systematic tendency for initially poorer countries to grow faster than rich countries. Source: Mankiw, G., D. Romer and D. Weil, 1992. "A contribution to the empirics of economic growth", Quarterly Journal of Economics, 107 (2), 407-38

Figure 3.6: Evidence of Absolute Convergence among 22 OECD Countries



The figure crosses growth rates of per capita incomes with initial levels of per capita incomes, restricting the sample to OECD economies. The negative correlation obtained implies that in this particular sub-sample there has been a tendency for initially poorer countries to grow faster than richer countries. Source: Same as figure 3.5

Of course, economies that are similar in terms of the fundamental parameters, such as the saving rate and the population growth rate are expected to approach steady states that are close to each other. Thus, if one restricts the sample to countries that are similar, one may well observe that those with initially lower levels of per capita income grow faster than those with higher per capita incomes. An example of this is displayed in Figure 3.6. This figure restricts the sample of Figure 3.5 to OECD countries, only. In this particular sub-sample, we are able to identify a negative correlation between growth rates and initial per capita incomes. This is not to say that all these countries are converging exactly to the *same* steady state: what happens is that, in this particular sub-sample, departures from steady state (i.e, transitional dynamics) account for a larger share of the cross-country variation of per capita incomes than differences in the steady states, which are arguably small. Note that the data refers to the post-WWII period, during which many European countries were rebuilding their capital stocks[52].

Summing up, in light of the Solow model, it will be impossible to predict whether a country will grow faster or slower by observing its initial income relative to other countries. In light of the Solow model, it is not the initial income that determines a country' growth rate, but instead its *distance relative to the steady state*: economies with per capita incomes that fall behind their steady states should grow faster than economies with per capita incomes that are above the respective steady states. This property of the model, is known as *conditional convergence* and will be subject to further scrutiny in the next chapter.

Box 3.2. Explaining the convergence test

In figure 3.5, the convergence hypothesis was investigated plotting the growth rates of per capita income against in a period, against the initial levels of per capita incomes. Formally, the hypothesis of "absolute convergence" can be tested empirically by estimating the following regression equation:

$$\ln y_{it} - \ln y_{i0} = a + b \ln y_{i0} + \varepsilon_i \qquad (3.13)$$

[52] Evidence of a negative relationship between per capita income growth and the initial level of per capita income is often found in samples restricted to industrial countries or their regions (for instance, Baumol, 1986, Barro e Sala-i-Martin, 1991), but this evidence reveals fragile to small sample modifications (De Long, 1988). Baumol, W., 1986. "Productivity Growth, Convergence and Welfare: What the Long Run Data Show", American Economic Review 76 (5), 1072-1085. De Long, J., 1988. "Productivity Growth, Convergence and Welfare: Comment". American Economic Review 78 (5), 1138-1154. Barro, R. and Sala-i-Martin, X., 1991. "Convergence across states and regions". Brooking Papers on economic activity 1, 107-158.

where the dependent variable is the growth rate of per capita GDP between period zero and period t in country i and the regressors are: a constant (a) and the initial level of per capita GDP in country i ($\ln y_{i0}$). The term ε_i is a random disturbance. In (3.13), the "absolute" convergence hypothesis is assessed by investigating the sign and significance of b: If $b<0$, this means there is a general tendency for initially poor economies to grow faster than rich economies.

To see how this test relates to the Solow model, consider the following equation, describing the dynamics of per capita income towards the steady state (see Appendix 3.1 for details):

$$\ln y_t - \ln y_0 = \gamma t + \left(1 - e^{-\nu t}\right)\left(\ln \tilde{y}^* - \ln y_0\right), \qquad (3.14)$$

with $\nu = (1-\beta)[n+\delta+\gamma] > 0$.

Equation (3.14) states that the growth rate of an economy depends on its distance relative to the steady state: if the economy starts out in the steady state, the expected growth rate is γ; if the economy is below (above) the steady state, its grow rate will be higher (lower) than γ. In general, this equation states that *per capita income converges to a steady state and the speed at which it does so relates inversely to the initial distance to the steady state*. This is what we mean by *conditional convergence.*

The relationship between the parameters of the regression equation (3.13) and those of the structural relationship (3.14) is straightforward:

$$a = \gamma t + \left(1 - e^{-\nu t}\right)\ln \tilde{y}^* \qquad (3.15)$$

$$b = -\left(1 - e^{-\nu t}\right)$$

Thus, a regression equation of the form (3.13), by postulating the same intercept to all countries, implicitly imposes that steady states are the same. With no surprise, tests for *absolute* convergence perform very poorly in World-wide samples, where dramatically different countries are pooled together in a regression equation.

To overcome this limitation, many studies have allowed the intercept (the steady state) to differ across countries. This is done adding to the regression model variables that are thought to determine the steady state ($\ln \tilde{y}^*$), such as the saving rate, the population growth rate, and efficiency, A (equation 3.10). We will discuss these tests of *conditional* convergence in Chapter 4.

Box 3.3. How Long?

The term $v = (1-\beta)[n+\delta+\gamma] > 0$ in equation (3.14), measures the *speed* of adjustment of per capita income to the steady state. To quantify this, let's calibrate thos equation with reasonable parameters:

- According to the Solow model, the elasticity of labour in the production function can be assessed by the (observable) share of labour on national incomes. National accounts data for different countries reveal that the labour share in national income varies from 60% to 70%. Thus, a reasonable assumption for the elasticity of capital in the production function is $\beta=1/3$.
- In the Solow model, the rate of technological progress, γ, is equal to the growth rate of per capita output in the steady state. The long-run evidence for industrialised countries reveals that per capita incomes have evolved at an average rate around γ=2%. A popular assumption in the literature is to consider $\delta+\gamma=0.05$.
- The population growth rate varies considerably across countries. As an example, consider a population expanding at 1% per year.

Under the above assumptions, the speed of convergence towards the steady state will be $v=0.04$. With such a value, how long it will take for that country to get "halfway" to its balanced growth path? Using equation (3.14), the answer is $e^{-0.04t} = 0.5$, which solves for $t=17$ years.

3.5.3 Interpreting growth patterns using the Solow model

By now, we have been confronting the Solow model with stylized facts referring to samples of countries. A different question is whether the model is helpful to interpret specific growth patterns of individual countries. To address this question, let's consider Figure 3.4, that depicts the evolution of per capita incomes in some *advanced* economies, namely the United States, France, Japan and Germany, over the period from 1871 to 2001.

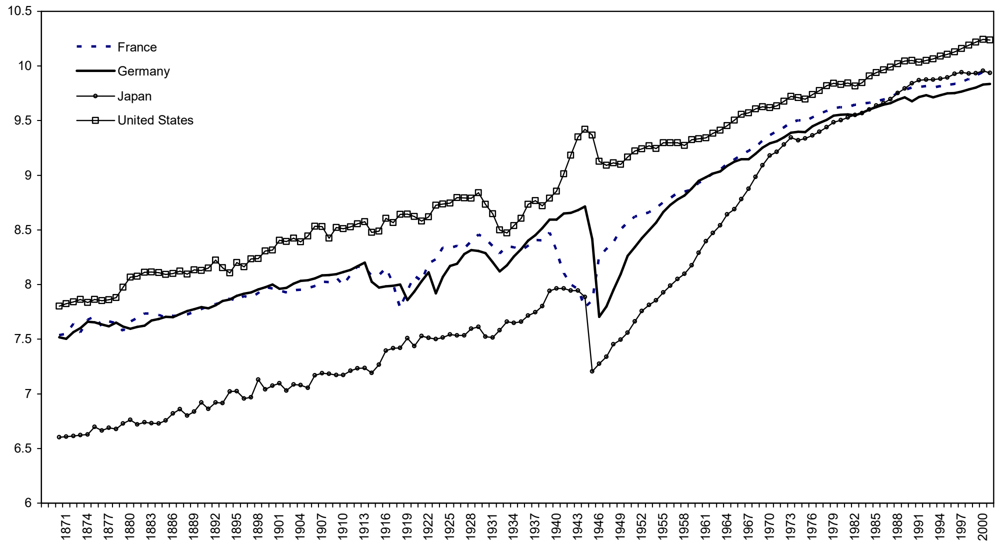
We observe that:

- In all cases, per capita GDP exhibits an upward trend. This is accounted for by the Solow model with technological progress.
- The growth rates of per capita output appear to be stable in the long run and quite similar across countries. This suggests that the draconian assumptions of technology evolving at a constant rate g and with all countries drawing from a *common technological pool* may not be at odds with reality in this very *particular sample*, composed by a group of advanced countries[53].
- The long-term paths of per capita incomes are parallel but not coincident. The Solow model does not imply that steady states should be the same for all countries: according to equation (3.10), the level of per capita income in each country depends on the saving rate (s), the population growth rate (n) and efficiency (A).
- During the long period from 1870 to 2011, some major disruptions pushed the US, France and Germany away from their respective long-run paths. These events included the Great Depression in the 1930s, the First World War (1914-1918) and the Second World War (1939-1945). As time went by, per capita incomes look like having return to the earlier paths. According to the Solow model, the steady state levels of per capita output are independent of a country' initial capital endowments. So, if some disaster destroys part of the capital stock, per capita GDP will fall initially, but then it will recover until returning to the initial steady state. In Figure 3.4, we see that this prediction of the model fits quite well the cases of US, Germany and France. For instance, during WWII, per capita output in Germany and France dropped significantly, but this was followed by a fast recovery that brought these economies back to the earlier path.

[53] Note that this set of countries is very specific: they share a set of characteristics that make them permeable to technological innovations discovered by each other. Many other countries in the world will hardly draw so easily from this "technological pool".

- In the case of Japan, a "level effect" is likely to have occurred after the Second World War. The path of Japan points indeed to a distinct case from those of, Germany and France: after WWII this country seems to have moved from a lower steady state to a higher one, closer to that of United States. In light of the Solow model, such move could be explained by a change in a fundamental parameter, such as the saving rate, the population growth rate or other country-specific effects, as captured by the efficiency parameter, A. Any change in one of these exogenous parameters implies a change in a country steady state.

Figure 3.4- Per Capita GDP in Japan, France, Germany and US, 1871-2001



The figure displays the evolution of per capita incomes (in logs) in a sample of industrial countries. The fact that per capita incomes evolve basically in parallel is consistent with the idea that these countries share the same body of technological knowledge. This figures also suggest a tendency for these countries to return to respective balanced growth paths after major disruptions. The exception is Japan, that managed to achieve a "level effect" after WWII. Source: Maddison, A., 2001. The World Economy: a Millenial Perspective. Development Centre, Paris.

3.6 Growth accounting revisited

3.6.1 Autonomous versus induced contributions

Having in mind the extended version of the Solow model, we now revisit the growth accounting exercise introduced in Section 2.7. As for an illustration, consider the figures of US economic growth. According to the World bank (1991)[54], during the first half of the twentieth century, the growth rate of GDP was about 3% per annum, on average. Estimates also indicate that in the same period, the capital stock expanded at about 3% per annum, whereas labour (hours worked) expanded at 1% per annum, only. Assuming a labour share in national income of two thirds, the implied Solow residual (using 2.29) is:

$$\hat{A} = 0.03 - \left(\frac{1}{3}\right)0.03 - \left(\frac{2}{3}\right)0.01 = 0.013.$$

This means that labour and capital jointly accounted to about 1.7 p.p. per annum to GDP growth. The residual balance of 1.3 percentage points per annum is accounted for by "technological change". Using the intensive form (2.30), the conclusion is even more starling: 1/3 of the change in per capita GDP is accounted for the increase in the capital labour ratio, and the remaining 2/3 of the change is accounted for TFP:

$$\hat{A} = 0.02 - \left(\frac{1}{3}\right)0.02 = 0.013.$$

The fact that both capital and output expanded at around 3% is however consistent with the view that this economy evolved along a balanced growth path (with Y/K constant, and hence with a constant saving rate). But if the US economy was indeed evolving along a balanced growth path, why should a growth accounting exercise like (2.29) indicate a contribution of capital to GDP growth equal to 1%?

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[54] World Bank, World Development Report 1991, Washington..

To answer this question, note that the Solow model with exogenous growth predicts the capital stock to growth over time in the steady state at the rate $\dot{K}/K = \gamma + n$. This growth, however, is not autonomous (i.e, it is not implied by a change in the saving rate); it is instead an endogenous response to the expansion of the effective labour force: if there was no population growth or technological progress ($\gamma=n=0$), then the capital stock would be constant, unless the saving rate had recently increased.

Taking this into account, we conclude that growth accounting exercises based on equation (2.29) measures the total contribution of capital to growth, without disentangling whether the observed growth in capital is induced by an increase in the saving rate (transitional dynamics), or is a mere response to a growing effective labour. When the economy is in the steady state, for instance, such an exercise will attribute $\beta(\gamma+n)$ to the growth of capital, $(1-\beta)n$ to the growth of population and $g=\gamma(1-\beta)$ to technological progress. And yet, these parameters are zero if there is no population growth or technological progress.

3.6.2 An alternative approach

To disentangle whether a country growth process is mainly driven by transitional dynamics or by steady technological change, there is a growth accounting exercises based on the following re-arrangement of the production function, (3.7)[55]:

$$y_t = A_t^{\frac{1}{1-\beta}} \left(\frac{K_t}{Y_t} \right)^{\frac{\beta}{1-\beta}} \qquad (3.16)$$

[55] David, P., 1977. Invention and Accumulation in America's Economic Growth: a Nineteen century parable, Journal of Monetary Economics, Special Supplement VI, 176-228.

This rearrangement emphasizes the relationship between the capital output ratio and per capita income. In light of the Solow model, in a steady state, this ratio changes when the saving rate changes, but is independent of the technological level, A (equation 3.11).

Log-differentiating this, we get

$$\hat{y} = \frac{1}{1-\beta}\hat{A} + \frac{\beta}{1-\beta}\left(\hat{K} - \hat{Y}\right). \qquad (3.17)$$

In (3.17) the contribution of capital to the growth rate of per capita GDP is now evaluated by the extent to which its growth rate exceeds that of output growth. This decomposition actually *expurgates* the growth rate of the capital stock from the part that is induced by the exogenous parameters. Thus, growth accounting based on equation (3.17) will capture the contribution of capital, *only* to the extent that the country is involved in a process of transition dynamics.

Using this new approach and the same figures for the US economy, one obtains:

$$2\% = 2\% + \frac{1}{2}\left(3\% - 3\%\right) = \gamma$$

That is, along the first half of the last century, capital accumulation by itself did not account for the US per capita income growth: the only exogenous source of per capita income growth along this period was (Harrod neutral) technological change, which expanded at 2%, on average. The capital stock evolved at a rate of 3% per year, but this was merely a response to the increasing population and TFP growth.

Box 3.4. The TFP controversy in East Asia

There are many examples in the literature of practical applications of growth accounting. A particularly controversial application refers to the so-called Asian Tigers (Hong Kong, Singapore, South Korea and Taiwan). Economists have for many years turned to these successful countries for clues that can explain sustained development and so provide examples to the rest of the world. In particular, in 1991, the World Bank came down strongly in favour of the idea that the East Asian economies, by operating sounder domestic policies than other developing countries, relied mostly on TFP growth: almost 2 percentage points of its overall growth of almost 7 per cent per annum[56]. By contrast, TFP growth rates in both Africa and Latin America were almost zero in that same period.

In an influential 1995 article, Alwyn Young dissented from this view. The author found that the bulk of the impressive growth achieved in the four East Asia miracle countries in the period 1966-1990 could be attributed much more narrowly to their fast rates of factor accumulation, and not to their exceptional levels of TFP growth. Columns (1), (2) and (3) of Table 3.1 present a summary of the Young estimates. These are obtained using a decomposition similar to (2.29), with the difference that the author aggregated raw labour and education levels into a single measure of labour. The data in column (3) indicate that TFP growth accounts for only a small proportion of GDP growth in these countries. Singapore for example was a particularly bad performer, with TFP growing at a rate close to zero. Young concluded that these growth miracles have achieved very high growth rates because of their ability to achieve high investment rates (in physical capital and in human capital), and a drastic increase in the fraction of population at work (largely via the increased labour force participation of women). Paul Krugman popularise these results contending that the key for success in the Asian Tigers was "transpiration" (factor accumulation) rather than "inspiration" (technological change)[57].

The implication of this finding for the economic profession was obviously disappointing: if technological catch up played only a minor role in East Asia's growth, this means that traditional factor accumulation was relatively more important than the quest for policies that encourage economic efficiency and innovation.

[56] World Development Report 1991. Oxford University Press, New York.

[57] Krugman, P. , 1994. "The myth of Asia's Miracle", Foreign Affairs 73(6), Nov-Dec, pp.62-78.

With no surprise, such controversial conclusion became subject to further scrutiny in the years that followed. One avenue that was explored relates to our discussion in Section 3.9: traditional growth accounting tends to overstate the role of factor accumulation because it does not control for the component of factor accumulation that is merely induced by technological change. Peter Klenow and Rodriguez Clare stressed this point and computed the Harrod neutral rates of technological progress *implied by the Young estimates* of Total Factor Productivity. Their results are reproduced in column (4) of Table 3.1. Clearly, the adjusted measures of TFO growth are higher than those in Column (3).

Table 3.1. - Productivity growth in East Asia

	Young (1995)				Klenow and Rodríguez-Clare (1997)		
	Y (1)	Y/N (2)	TFP (3)	Harrod Neutral (4)	Y/N (5)	Harrod Neutral (6)	Rank (98 countries) (7)
Hong Kong	7.3	4.7	2.3	3.7	5.5	4.4	4
Korea	10.3	4.9	1.7	2.5	5.4	2.5	17
Singapore	8.7	4.2	0.2	0.3	5.1	3.3	6
Taiwan	9.4	4.8	2.6	3.5	5.3	3.0	7

Notes: Columns (1) - (3) display average growth rates for the period 1966-1990 (1966-1991 in Singapore). Columns (4)-(6) are from Klenow and Rodriguez Clare (1997) refer to the period 1960-85. Column (7) displays the rank order of estimates in Column (6) in a sample of 98 countries. Sources: Young, A ., 1995. "The tyranny of numbers: confronting the statistical realities of the East Asian growth experience", Quarterly Journal of Economics CX (3), 641-680. Klenow, P. and Rodriguez-Clare, "The Neo-Classical Revival in Growth Economics: Has it Gone Too Far?", NBER Macroeconomic Annual, 1997.

A second problem relates to data measurement. In general, growth accounting exercises are very sensitive to key assumptions, such as the weights assumed in the production function, and the treatment of human capital. To illustrate how sensitive estimates are to changes in methodology, we display in columns (5) and (6) of Table 3.1 the estimates proposed by Klenow and Rodriguez Clare for output per capita and for (Harrod neutral) productivity growth in these countries. Although the estimates for South Korea and Taiwan roughly match those of Young, estimates for Singapore and Hong Kong are much higher. In the case of Singapore, the difference is qualitatively important[58].

The third caveat to the Young results is that the contribution of TFP growth should be evaluated *per se* and not as a proportion of output growth. In light of the neoclassical model, for any two countries enjoying the same rate of technological progress, the one that starts with a lower capital labour ratio should exhibit faster growth. Thus, a lower *proportion* of output growth will be accounted for by improvements in TFP, even though both countries share the same rate of technological progress. Column (7) displays the rank order of the Harrod neutral rate of technological progress estimated by Klenow and Rodrigues-Clare in their 98 countries sample. Clearly, the TFP growth rates in the four East Asian economies are quite respectable. In general, estimates of TFP growth using comparable data for a large set of countries reveal that East Asian productivity growth is relatively high when compared to other regions.

All in all, despite the initial controversy, the evidence points to the case that TFP growth (technology and aggregate efficiency) has indeed played a much greater role in the economic transformation of East Asia than the original Young estimates suggest.

3.7 Discussion: What we have achieved?

In Chapter 2 we saw that, because of diminishing returns, the basic Solow model is not capable of describing the real-world fact that per capita income tend to increase over time in economies engaged in modern growth.

In this chapter, we saw that assuming an exogenous rate of technological progress we turn the model capable of describing most stylised facts of economic growth. In particular, the model conciliates a sustained growth of per capita income with an interest rate that

[58] The authors attributed the bulk of the difference to different assumptions regarding the capital income share (they used 0.30, instead of 0.48 in Young, 1995) and also to the different data set used for employment growth.

remains constant over time. With this refinement, the model becomes much more compliant with real world facts. In plus, we saw that the Solow model is consistent with the evidence that poor countries do not tend to grow faster than rich countries. In a word, the Solow model can be fairly said to provide the correct answers to the set of questions it was intended to address.

The main drawback of the model is that it cannot address the causes of technological progress. The model describes how economies evolve over time, and can be extended to account for the role of technological progress in delivering long-run growth. However, the model fails to explain why technological progress takes place at all. The reason for this was already discussed at the beginning of this chapter: by assuming perfect competition, the model implicitly assumes that technology is freely available to everybody, so no profit-making economic agent will find incentives to invent new technologies. Without accounting for the incentives to invent new technologies, the model is forced to assume that technology grows exogenously. Questions such as: "who produces technological progress and why" cannot be addressed by the Solow growth model.

Despite its limitations, the Solow model provides a framework that shall be seen as the centrepiece to describe the mechanics of per capita income growth over time. As such, it became the workhorse of growth models and it provides the basis for many advanced models in macroeconomics.

3.8 Key ideas of Chapter 3

- Because it assumes perfect competition and rules out market failures, the Solow model can only account for exogenous technological progress.
- The exogenous rate of technological progress allows effective labour to expand faster than population. If – as implied by the model – the capital stock expands proportionally to effective labour, then output will also expand at the same rate as effective labour. The implication is that capital per worker and per capita income will both increase at the rate given by the labour augmenting rate of technological progress.
- With this small amendment, the Solow model can account for the fact that most countries see output per capita increasing over time.
- As before, changes in the saving rates can only produce “level effects”: that is, the growth rate of per capita income increases temporarily, but in the long run it falls back to the level given by the rate of technological progress.

- The Solow model does not imply that in the long run all countries should have the same level of per capita income ("absolute convergence"). According to the Solow model, per capita incomes differ in the steady state depending on country characteristics, such as the saving rate and the population growth rate. The Solow model implies that countries that start out behind their respective steady states should grow faster than countries that are close to their steady states. This property of the model is known as "conditional convergence".
- The Solow model is capable of describing many stylized facts of economic growth. If fails however to explain economic growth, because the parameter that ultimately determines the rate at which per capita incomes are expanding is exogenous to the model.
- Growth accounting exercises have different interpretation depending whether the technological parameter is specified as Harrod neutral or as Hicks neutral. If one wants to capture the contribution of capital after expurgating the capital accumulation that is induced by technological change, the Harrod neutral measure should be used.

Appendix 3.1 Transition dynamics in the Solow model

The stability properties of the neoclassical growth model ensure that the economy converges to its steady state and that the speed of adjustment depends on how far the economy is from the steady state.

Formally, the speed of convergence may be assessed using a first-order Taylor-series approximation of (3.8) around the steady state (3.9). This gives:

$$\dot{\tilde{k}} = \left.\frac{\partial \dot{\tilde{k}}}{\partial \tilde{k}}\right|_{\tilde{k}=\tilde{k}^*}\left(\tilde{k}-\tilde{k}^*\right) = \left[\beta s\frac{Y}{K} - \left(n+\delta+\gamma\right)\right]\left(\tilde{k}-\tilde{k}^*\right) = -(1-\beta)(n+\delta+\gamma)\left(\tilde{k}-\tilde{k}^*\right)$$

$$= -\nu\left(\tilde{k}-\tilde{k}^*\right) \text{ with } \nu = (1-\beta)(n+\delta+\gamma) > 0\,,$$

where we used equation (3.11) to eliminate *Y/K*. The last equation is a first-order, non-homogeneous differential equation, whose solution is given by:

$$\tilde{k}_t - \tilde{k}^* = e^{-\nu t}\left(\tilde{k}_0 - \tilde{k}^*\right)$$

This equation states that the change in $\tilde{k}$ each moment in time declines as the economy approaches its steady state. When the economy is in the steady state, the second term on the right-hand side is zero, implying a constant level of $\tilde{k}$. When the economy is below the steady state, the second term on the right-hand side is positive, implying that $\tilde{k}$ is rising. This means the model exhibits *local stability*.

Since y is a continuous function of k, as a linear approximation, y approaches the steady state at the same rate as k. Then, it can also be shown that the dynamics of y in natural logarithms is given by[59]:

$$\ln \tilde{y}_t - \ln \tilde{y}^* = e^{-\nu t}\left[\ln \tilde{y}_0 - \ln \tilde{y}^*\right]$$

Thus, the speed of adjustment of per capita income to the steady state is given by $\nu = (1-\beta)[n+\delta+\gamma] > 0$. This equation suggests a natural regression to study the rate of convergence in the context of the Solow model: subtracting $\tilde{y}_0$ in both sides, rearranging and using the identity $\ln y_t = \ln \tilde{y}_t + \gamma t$ to eliminate $\tilde{y}_t$ and $\tilde{y}_0$, one obtains (3.14).

Problems and Exercises

Key concepts

- *Perfect technological diffusion. Labour in efficiency units. Harrod neutral vs Hicks neutral technological progress. Level effect vs. growth Effect. Absolute Convergence. Conditional Convergence.*

Essay questions:

- Comment: "In the Solow model, technology *has to* be assumed exogenous".
- Comment: "The Solow model does not imply that poor countries should grow faster than rich countries"

Exercises

3.1. Consider an economy where the production function is given by: $Y_t = A_t K_t^{1/3} N^{2/3}$, where $A_t = 16e^{0,02t}$ describes the technology and N is the (constant) number of workers. In this economy, 25% of income is saved the capital depreciation rate is 1%. (a) Describe the main equations of the model and find out the fundamental dynamic

[59] The student will thank us for skipping the tedious mathematical derivation.

equation for K/L, where L is labour in efficiency units. (b) Find out the equilibrium values of K/L, Y/L and K/Y. (c) Describe the time-paths of per capita income (Y/N), the wage rate, the interest rate and the factor income shares in the steady state. Are these paths in accordance to the real-world facts? (d) Suppose that a war destroyed part of the stock of capital of that economy. Describe the subsequent evolution of per capita income (Y/N). (e) How did the growth rate of per capita income and the interest rate evolved during the transition path? Explain.

3.2. In economy W, the aggregate production function is given by $Y_t = A_t K_t^{1/4} N_t^{3/4}$, where N refers to population. In this economy, s=24%, n=1%, δ=0, and $A_t = 8e^{0.015t}$. (a) Find out the steady state values of K/L and Y/L, where L is labour in efficiency nits. Represent the equilibrium in a graph and explain why it is stable. (b) Compute the interest rate, the capital and labour income shares, and explain the steady state patterns of wages and per capita income. Are these results in accordance to the Kaldor facts? (c) Assume that a benevolent planner managed to increase the saving rate in this economy to s=27.78%. (f1) Would the growth rate of per capita income change? (f2) What about the interest rate? (f3) Consumers would be better off? Explain, quantifying when possible.

3.3. Consider an economy (Oldland) where the production function is given by $Y = A_t K_t^{1/3} N_t^{2/3}$, where N measures the number of workers. It is known that, in this country 25% of income is saved, population is expanding at 0.5% per year, the capital stock depreciates at 3% and $A_t = 20e^{0.01t}$. (a) Find out the equilibrium levels of K/L, Y/L and K/Y of this economy, where L represents labour in efficiency units. Discuss the stability of the equilibrium and represent it in a graph.(b) Describe the short and long run effects of a rise of the saving rate in the following variables: per-capita income, growth rate of per-capita income, per-capita consumption and interest rate. (c) Admit that in Oldland per-capita income was ten times higher than in Newland. In what conditions could you state that Newland was growing faster than Oldland? (d) Knowing that technology was the same in both countries, find out what the interest rate in Newland should be. Would the two economies converge? Discuss.

3.4. Consider an economy composed by a large number of small and identical firms. The available technology for each of them is given by $Y_i = 0.5 K_i^{0.5} L_t^{0.5}$, where L=λN measures labour in efficiency units. Population doesn't growth, the depreciation rate is 3.5% and the saving rate is 20%. Admit also that $\lambda = e^{0.015t}$. (a) Find out the equilibrium values of K/L, Y/L and K/Y of this economy. (b) Describe the long run behaviour of per-capita output, wages and the interest rate. (c) Assume now that saving rate was determined according to the following rule $\gamma = \dot{c}/c = r_t - \rho$, where ρ is the rate of time preference and r the interest rate. Explain this rule. Find out the value of ρ that is consistent with s=20%. (d) Describe the adjustment of the model following a decrease in ρ to 0.05. What will be the implied saving rate in the steady state?

3.5. Consider an economy where capital and output are growing at 3% per year and the labour force is expanding at 1%. Assuming that the elasticity of capital in the production function (β) was one third, compute the contribution of capital to output growth, using: (a) conventional growth accounting; (b) growth accounting based on the following re-parameterisation of the production function: $\hat{y} = A^{1/(1-\beta)} (K/Y)^{\beta/(1-\beta)}$. Interpret the differences. Repeat the exercise assuming that output growth was 4,5%.