2 The basic Solow model

"A thrifty society will, in the long run, be wealthier than an impatient one, but it will not grow faster" [Robert Lucas Jr.]

Learning Goals:

- Understand the extent to which capital accumulation can overcome the law of diminishing returns
- The mechanics of the Solow growth model.
- Evaluation of the Solow model through the lenses of the Kaldor facts.
- Distinguish the effects of a change in the saving rate from those of an exogenous change in technology
- Discuss the optimality of the saving rate in the context of the Solow model
- Growth accounting

2.1 Introduction

The neoclassical theory of economic growth was pioneered by two independent economists, the American Robert Solow and the Australian Trevor Swan[33]. The main innovation of the Neoclassical model in respect to the Malthusian Model is the replacement of land by "capital" in the production function. By "capital", we mean machinery, buildings, and other equipment. This modification is more than a mere change in form: contrary to land, which is available in finite supply, capital can be produced and accumulated. This opens an avenue to overcome diminishing returns on labour: by allowing the capital stock to expand over time, the Solow model avoids the tendency for productivity to decrease with the size of population that plagues the Malthus model. In the Solow model, the long run is not characterized by a low-

[33] Solow, R., 1956. "A contribution to the theory of economic growth", *Quarterly Journal of Economics* 50, 65-94. Swan, T., 1956. "Economic growth and capital accumulation", *Economic Record* 32, 334-61.

income equilibrium trap. Still, because capital itself faces diminishing returns, capital accumulation alone cannot generate long-run growth: in the Solow model, a higher investment rate translates into a higher level of per capita income, but the growth process thereby generated will be only temporary.

This chapter presents the Solow model in its simplest formulation. In Section 2.2, we describe the main assumptions of the model and we characterize the equilibrium. In Section 2.3, we confront the predictions of the model with the main stylized facts of Modern Economic Growth. In Section 2.4 we discuss how the main endogenous variables of the model respond to changes in the exogenous parameters. Section 2.5 discusses the optimality of the saving rate, from a social point of view. Section 2.6 extends the model to the case in which the saving rate is optimally decided by individual agents. Section 2.7 introduces a growth accounting exercise to illustrate the fundamental limitation of this simple version of the Solow model. Section 2.8 concludes.

2.2 The Solow model

2.2.1 The neo-classical production function

In the Malthus model, with all else constant, an increase in the size of the labour force leads to a decline in per capita income. This is a direct consequence of the Law of Diminishing Returns: as long as the availability of land is unchanged, its higher intensive use will translate into lower labour productivity. The Solow model retains the assumption of diminishing returns to labour but explores another property of the neoclassical production function: Constant Returns to Scale (see box). Under constant return to scale, it is possible for per capita income to remain constant over time, provided the capital stock expands at the same rate as the labour force.

In the following, we consider the following specification for the economy' aggregate production function[34]:

$$Y_t = A_t K_t^{\beta} N_t^{1-\beta} \quad , \quad 0 < \beta < 1 \qquad (2.1)$$

where K denotes for the economy' capital stock, *N* for the size of the labour force, Y for output and *A* is the level of technology. Throughout this chapter it will be assumed that the level of technology is constant over time:

$$A_t = A \quad . \qquad (2.2)$$

Dividing the production function (2.1) by the size of the workforce (*N*), one obtains an expression for the production function in the *intensive form,* relating per capita output to capital per worker:

$$y_t = A\left(\frac{K_t}{N_t}\right)^{\beta} = Ak_t^{\beta} \, , \qquad (2.3)$$

In (2.3), $y=Y/N$ denotes for per capita income and $k=K/N$ is the capital-labour ratio. Equation (2.3) stresses the role of the capital-labour ratio as a key driver of per capita income: what matters is not the levels of capital and labour but instead the proportion in which these two inputs are used.

From equation (2.3), we see that if capital and labour are set to expand at the same rate, output per capita will be constant overtime. The interesting feature of the Solow model is that we do not need to postulate K to grow at the same rate as the labour force to prevent per capita income to decrease. As we will see next, the properties of the model are such that the capital stock, even if endogenous, will end up growing at the same rate as the labour force in the long run, ensuring that per capita income remains constant over time.

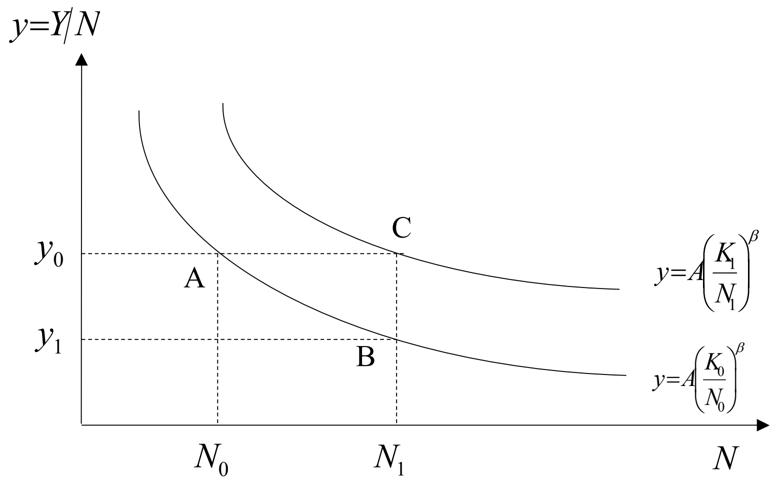
[34] The Solow model is consistent with more general specifications for the production function. We stick to the Cobb-Douglas production function for simplicity.

Box 2.1 Constant returns to scale

The key property of our production function is that it exhibits constant returns to scale (CRS). CRS means that if one increases the use of all inputs by a given proportion, output will rise in the same proportion. Formally, multiplying both inputs in (2.1) for any constant $q \neq 0$, we obtain $A(qK)^{\beta}(qN)^{1-\beta} = qY$. Note that CRS is not inconsistent with the LDR: the Law of Diminishing Returns states that output will expand less than proportionally than one varying input, when all other inputs are held constant. The Constant Returns to Scale property applies when *all* inputs vary in the same proportion at the same time.

The CRS property is illustrated in Figure 2.1. The figure displays the average product of labour as a function of the labour force. The schedule is negatively sloped because of the Law of Diminishing Returns: all else constant an increase in the use of labour from N_0 to N_1 implies a decline in the output per worker (from A to B). If however the capital stock expands exactly in the same proportion as labour, then the capital labour ratio remains constant (from, A to C)

Figure 2.1. Diminishing returns versus constant returns to scale



The Figure displays the average product of labour as a function of the size of the labour force. The curve is negatively sloped because of diminishing returns (from A to B). When the capital stock increases, the schedle shifts to the right. If the capital stock and population increase in the same propostion, the average product of labour remains constant (from A to C).

2.2.2 Main assumptions of the Solow model

Consider a closed economy with no government with a large number of small firms producing a single homogeneous good, Y, using two inputs: labour (N), and capital (K). Inputs are hired from households, who are also the owners of the firms and the consumers in this economy. Perfect competition and flexible prices are assumed, so full employment holds each moment in time. The labour force is the same as population and expands over time at an exogenous rate, n. Households consume a fraction (1-s) of their income and save the remaining fraction. The capital stock accumulates through investment and depreciates at a constant rate, δ[35].

At the micro level, the production function of the representative firm i takes the following form:

$$Y_{it} = AK_{it}^{\beta} N_{it}^{1-\beta} . \qquad (2.4)$$

where Y_i, K_i and N_i refer to output, capital and labour employed by each firm. Assuming that all firms are equal and face the same relative prices, the aggregate production function in this economy will be (2.1), where $Y = \sum_i Y_i$, $K = \sum_i K_i$ and $N = \sum_i N_i$.

Households use income to consume or to save. In the aggregate, this implies:

$$Y_t = C_t + S_t \quad , \qquad (2.5)$$

where, C and S denote for aggregate consumption and aggregate savings, respectively. Savings are in fixed proportion of income[36]:

$$S_t = sY_t \qquad (2.6)$$

[35] By investment, we mean the acquisition of real assets, such as machinery, buildings and other equipment with the aim to achieve higher production in the future. This shall be distinguished from purchases of financial assets, such as bonds or shares, with the aim to obtain future returns. A confusion may arise in that the later is often labelled as *financial investment.*

[36] Below, we argue that turning the savings rate a choice variable does not alter the properties of the equilibrium.

Given the constant saving rate, *s*, the flow equilibrium in this economy is given by:

$$sY_t = I_t \qquad (2.7)$$

where *I* denotes gross investment[37]. With the passage of time, capital wears out or becomes obsolete. This process is called *depreciation*. In this model, it is assumed that the depreciation rate (δ) is exogenous and constant over time. Hence:

$$\dot{K}_t = I_t - \delta K_t \qquad (2.8)$$

Equation (2.8) states that the change in the capital stock (net investment) is equal to gross investment minus depreciation. The fact that the capital stock depreciates over time implies that some minimum investment will be needed just to avoid the erosion of the existing capital stock.

Finally, the population growth rate is:

$$n = \dot{N}_t / N_t \qquad (2.9)$$

The above equations describe the basic Solow model.

2.2.3 Factor prices and factor income shares

Households are the owners of labour and capital, which they rent to firms. Labour services are paid at the wage rate *w*. Households acquire capital with their own savings or borrowing from each other at the interest rate *r*. Since the economy is closed the assets and liabilities emerging from this trade cancel out in the aggregate. Still, capital services must be

[37] Equation (2.7) implicitly postulates the price of the capital good to be the same as that of output. As an example, think that the only output in this economy was potatoes: potatoes can be either consumed or planted (invested) to grow more potatoes. If in alternative total investment included a plough, a given amount of saved output (potatoes) would translate into more or less capital accumulation (ploughs), depending on how many potatoes would be necessary to buy a plough. The implications of changing the relative price of capital will be examined later on.

paid the opportunity cost of holding capital, r, plus a compensation for the fact that capital erodes over time, δ . The sum $r+\delta$ is the "user cost of capital".

It is assumed that firms are price-takers both in the product market and in factor markets. The profit function of each individual firm is given by:

$$\pi_{it} = Y_{it} - (r_t + \delta)K_{it} - w_t N_{it} \qquad (2.10)$$

The first order conditions of profit maximization problem are:

$$\frac{\partial \pi_i}{\partial N_i} = (1-\beta)K_i^{\beta} N_i^{-\beta} - w_t = (1-\beta)\frac{Y_{it}}{N_{it}} - w_t = 0$$

$$\frac{\partial \pi_i}{\partial K_i} = \beta K_i^{\beta-1} N_i^{1-\beta} - (r_t + \delta) = \beta\frac{Y_{it}}{K_{it}} - (r_t + \delta) = 0$$

These conditions establish that the demands for labour and capital by each form are equal to the corresponding marginal products. Since all firms are alike, the aggregate demands for labour and capital are:

$$w_t = (1-\beta)\frac{Y_t}{N_t} \qquad (2.11)$$

$$r_t + \delta = \beta\frac{Y_t}{K_t} \qquad (2.12)$$

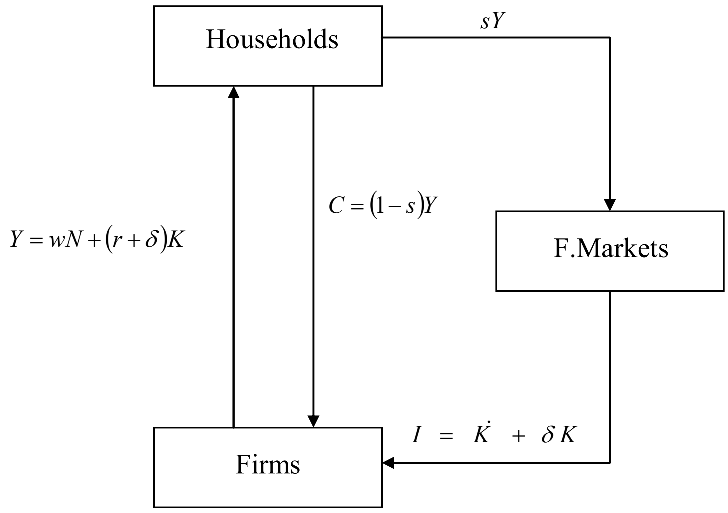
Equations (2.11) and (2.12) imply that the shares of capital and labour on national income, $(r+\delta)K/Y$ and wN/Y, are constant and equal to β and $1-\beta$, respectively. That is, even though the prices and quantities of capital and labour may vary over time, changes must be such that the shares of national income paid out to each factor of production remain constant. This is a direct implication of assuming perfect competition and a production function with constant returns to scale.

2.2.4 The flow income chart

The flow income chart of this economy is displayed in Figure 2.2. From the expenditure angle, the sum of investment and consumption demands equal to output. From the income angle, output is paid to labour and capital owners in the form of labour income and capital income. Households use their income to consume and to save. Savings are equal to investment.

In the figure, savings and investment are mediated by the "financial market". This financial market is assumed frictionless, in the sense that it involves no transactions costs[38].

Figure 2.2: The flow income chart in the basic Solow model



The figure displays the flow of payments in this economy. Factor incomes are paid to households, who consume and save, acquiring bonds. The resources raised by sales of bonds are invested by other households to acquire new capital (there are no transactions costs or frictions in the financial market).

2.2.5 The Fundamental Dynamic Equation

To understand how the model works, note that, in the absence of technological progress, the main determinant of per capita output (2.1) is the capital-labour ratio (i.e. the amount of capital available per worker). This variable is pre-determined each moment in time, given the existing stock of capital and the size of population. The capital-labour ratio may however

[38] If each household relied on own savings only to invest, a financial market would not be needed. But one may consider the possibility of different households having different propensities to save or to invest. You may think this model as with investors issuing bonds that pay an interest rate r to finance their purchases of capital. These bonds are acquired by savers. Each household can be simultaneously a saver and an investor, but not necessarily balanced. In the aggregate, net savings of the household sector will be equal to total investment.

change over time, depending on investment, depreciation, and population growth. Formally, let's take the time derivative on k, to obtain:

$$\dot{k} = \left(\frac{\dot{K}N - \dot{N}K}{N^2} \right) = \frac{\dot{K}}{N} - \frac{\dot{N}}{N}k \qquad (2.13)$$

After some substitutions using (2.3) and (2.7)-(2.9), we obtain the so-called Fundamental Dynamic Equation of the Solow model:

$$\dot{k}_t = sAk_t^{\beta} - (n + \delta)k_t \qquad (2.14)$$

This equation states that the capital-labour ratio increases with per capita saving ($sy = sAk_t^{\beta}$) and decreases with the depreciation rate (δ) and the population growth rate (n).

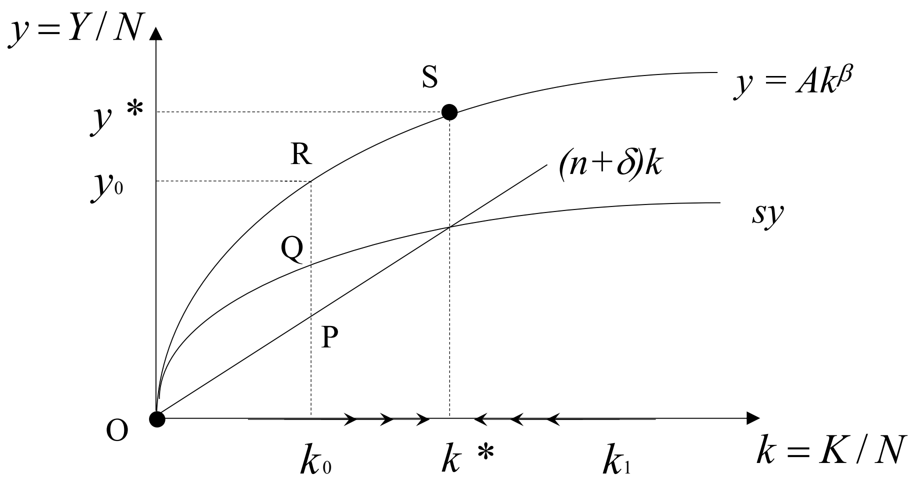
The term (n+δ) in (2.14) may be interpreted as the rate of depreciation of the capital-labour ratio: on one hand, the depreciation rate reduces k by causing the capital stock K to wear out over time; on the other hand, population growth results in less capital available to each worker (this negative effect of population growth on capital per worker is called *capital dilution*). According to equation (2.14), the change in the capital-labour ratio over time is positive whenever per capita savings (investment) exceed the depreciation of the capital-labour ratio, and conversely.

2.2.6 Graphical illustration

Figure 2.3 describes the dynamics and the equilibria of the model, as implied by equation (2.14). The uppermost curve is the production function in per capita terms (2.3). The figure also depicts the two terms at the right-hand side of (2.14): per capita savings (sy), and the depreciation term, $(n+\delta)k$. The later is represented by the *break-even investment line*: it depicts, for each level of capital per worker, the exact amount of gross savings that will be necessary to offset the depreciation of the capital labour ratio.

To see how the capital-labour ratio evolves over time, consider an initial situation where the capital-labour ratio is equal to k_0. In that case, per capita income will be y_0, of which QR devoted to consumption and k_0Q devoted to savings. Since per capita savings exceed the "break even investment" (given by k_0P), from equation (2.14) it follows that the change in the capital-labour ratio will be positive. In words, since the economy generates more savings (and hence more investment) than the needed to keep the amount of capital-labour ratio constant, the capital-labour ratio will increase. Then, as k increases, savings per capita increase less than proportionally. Because of diminishing returns, income per worker increases less than proportionally than capital per worker, implying that savings cannot increase as fast as depreciation. And a moment will come when the two schedules cross each other: at $k=k^*$ per capita savings are just the needed (but no more) to equip the new entrants into the labour force and to replace the depreciating capital, implying that the capital labour ratio remains constant. This is the *steady state* (equilibrium) of the model[39].

Figure 2.3. Dynamics and equilibria in the Solow Model



The figure displays the production function in the intensive form, per capita savings and the break-even investment line. The steady state occurs when the break-even investment line crosses the schedule describing per capita savings. If the economy starts out on the left (right) of the steady state, per capita savings will be greater than (less than) the required to keep the capital labour ratio constant.

2.2.7 The steady state

[39] We invite the reader to use a similar reasoning to explain why the capital-labour ratio converges to the steady state departing from k_1.

Formally, the equilibria of the model are obtained solving (2.14) for $\dot{k}=0$. This equation has only two solutions, the trivial one (*k=0*) and:

$$k^* = \left(\frac{sA}{n+\delta}\right)^{\frac{1}{1-\beta}}. \qquad (2.16)$$

Because the model predicts that the economy converges to the steady state (2.16) from any departing point in its neighbourhood, this equilibrium is said to be *stable*[40].

Substituting (2.16) in (2.3), one obtains the steady state level of per capita income:

$$y^* = A^{\frac{1}{1-\beta}}\left(\frac{s}{n+\delta}\right)^{\frac{\beta}{1-\beta}} \qquad (2.17)$$

Since parameters *A, s, n* and δ are all constant, equations (2.16) and (2.17) imply that, in the steady state, capital per worker and per capita income are also constant.

Note that this outcome is in full conformity with the CRS property: if labour and capital are set to grow at rate *n,* then output will also grow at rate *n* (implying a constant level of per capita income). This is why the CRS assumption plays a key role in this model.

2.3 The Solow model and the facts of economic growth

2.3.1 The Solow model and the Kaldor acts

Robert Solow developed his famous model with the main purpose being a better understanding of the growth performance of the US economy in the twentieth century. He was particularly interested in explaining the long-run tendency for output and capital to grow at the

[40] Formally, the equilibrium described by k^* is *locally* stable because the condition $\partial\dot{k}/\partial k < 0$ holds for any point in its neighbourhood. The reader may verify that the same condition does not hold in the neighbourhood of the trivial steady state, *k=0*. The later is an *unstable* equilibrium.

same rates – a statistical regularity first documented for the U.S. economy by the Russian-American economist, Simon Kuznets. Solow also wrote the model with the so-called "Kaldor's facts" in mind. These are six "remarkable historical constancies" (empirical regularities) that the British economist Nicholas Kaldor identified as characterizing modern economic growth.

In particular, the Kaldor stylized facts are[41]:

1. Output per worker grows over time at a sustained rate
2. The capital stock per worker grows over time at a sustained rate
3. The capital-output ratio exhibits no clear trend over time;
4. The real return to capital is relatively constant over time;
5. The shares of labour and of capital on national income are roughly constant over time;
6. There are wide differences in the growth rate of productivity across countries.

Kaldor did not claim that these facts hold each moment in time. For instance, per capita output falls during recessions and the real interest rate fluctuates significantly over the business cycle. Over long periods of time, however, these facts tend to show up in the statistical data. Hence, they shall provide a natural benchmark for models focusing on long run growth to be confronted with.

As we just saw, equations (2.16) and (2.17) imply that, in the long run, per capita income and the capital labour ratio do not grow at all. Hence, the basic Solow model fails to capture the Kaldor Stylized facts 1, 2, and 6.

To check whether Fact 3 is met, let's divide (2.16) by (2.17), to obtain the output-capital ratio in the steady state:

[41] Kaldor, N, 1961. *Capital Accumulation and Economic Growth*. In F.A. Lutz and D.C. Hague (eds.), The theory of Capital. New York: St Martin Press. Kaldor, N., 1957. A model of Economic Growth. The *Economic Journal* 67 (268), 591-624.

$$\left(\frac{Y}{K}\right)^{*} = \frac{n+\delta}{s} \qquad (2.18)$$

This ratio is constant because the three parameters on the right-hand side are all constant (in Figure 2.3, this ratio is measured by the slope of the ray OS). We conclude that the model captures the stylized fact number 3: the long run tendency for capital (K) and output (Y) to grow at the same rate. If the average product of capital is constant in the steady state, then the interest rate will also be constant in the steady state (from equation 2.12). Hence, the Kaldor's stylized fact 4 is also implied by the model. Regarding fact number 5, we already saw that it holds in this model (equations 2.11 and 2.12).

Summing up, the basic Solow model captures facts 3, 4,5, but it fails to capture facts 1, 2, and 6.

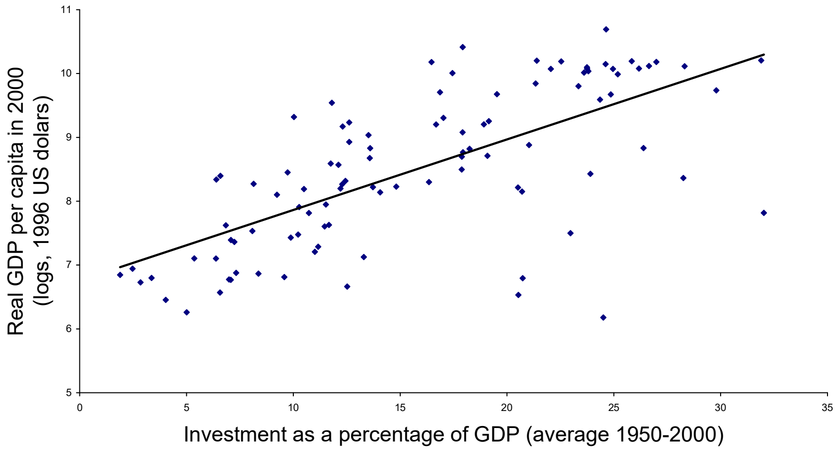
2.3.2 Savings, population and per capita income in the real world

Other implication of the Solow model concerns the relationship between per capita income and saving rates and population growth rates. According to equation (2.17), countries with higher saving rates and slower population growth should enjoy higher standards of living than countries with lower saving rates and fast population expansion.

In figures 2.4 and 2.5, we check how these two predictions of the model go along with the real-world facts. The figures plot the level of GDP per capita in the year 2000 with, respectively (i) gross investment as a share of GDP and (ii) population growth rates. Figure 2.4 reveals a positive correlation between investment rates and per capita incomes. Figure 2.5 reveal a negative correlation between population growth and per capita incomes. Both figures are in broad accordance to the Solow model.

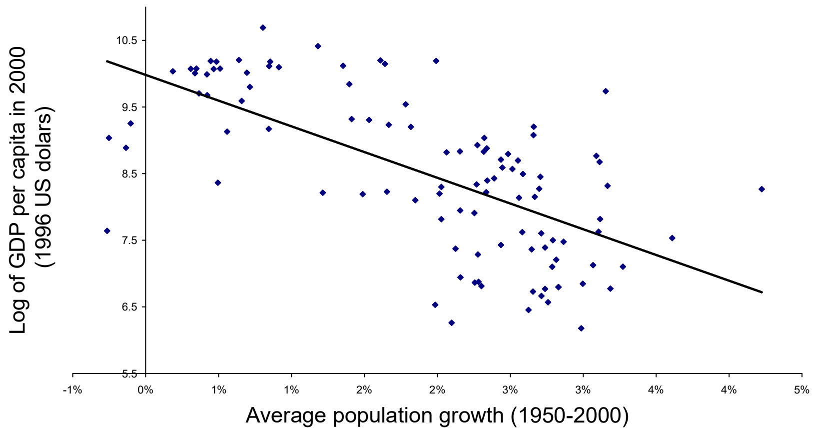
Note, however, that this evidence does not prove that the Solow model is correct. For example, it could be that the low saving rates in the poorest countries were explained by the fact that people living at the margin of subsistence cannot afford to save. On the other hand, poorest countries may exhibit faster population expansions simply because they still didn't make their demographic transition (see Box 2.2). Thus, while this evidence is in accordance to the Solow model, it does not prove that the Solow model is the right one to capture this evidence. Correlation is not causality.

Figure 2.4: Per capita GDP and gross Investment 1950-2000



Source: Summers, Robert and Heston, Alan. (1991). The Penn World Table (Mark 5): an expanded set of international comparisons, 1950-1988, *Quarterly Journal of Economics*, 106(2), May, 327-68.The sample includes 169 countries and average data over the 50 year period from 1950 to 2000.

Figure 2.5: Per capita GDP and population growth rates, 1950-2000



Source: same as Figure 2.4.

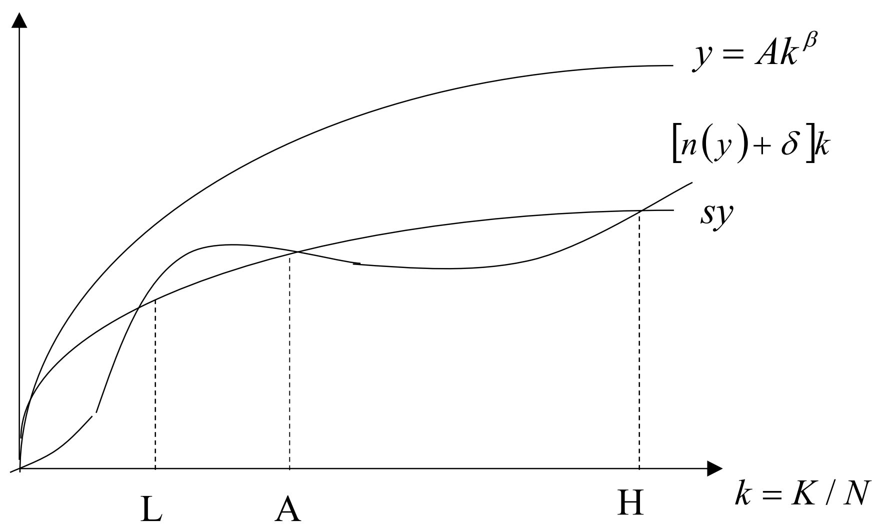
Box 2.2 Demographic transitions and poverty traps

The model outlined above postulates a constant rate of population expansion, n. The departure from the Malthusian assumption looks sensible to address the growth problem of economies that already made their demographic transitions. However, it may be interesting to investigate how the model predictions may change when one allows the rate of population growth to become a function of per capita income.

The stylized fact of demographic transitions (figure 1.5) is that a country' population is expected to expand at moderate rates both when per capita income is low (high birth rates and high death rates) and when per capita income is high (low birth rates and low death rates). In intermediate stages, the growth rate of population is expected to be high, because death rates are low and birth rates are still high. Adding a non-linear relationship between population growth and per capita income to the Solow model, one obtains a break–even investment curve that is non-linear, as depicted in Figure 2.6. In this case, the model may display multiple equilibria.

In Figure 2.6, there are four possible equilibria: origin, L, A, and H. The equilibrium represented by H corresponds to a low population grow rate and a high level of per capita income. Equilibrium A is an intermediate equilibrium, characterised by fast population growth. Equilibrium L is characterised by a low rate of population growth and a low level of per capita income.

Figure 2.6. A poverty trap in the context of the Solow model



The figure extends the basic Solow model to allow the population growth rate to depend on per capita income, in light with the stylized facts described in figure 1.5. In this case, there are three non-trivial equilibria, L and H (stable) and A (unstable). Because equilibrium L is inferior to H, it is called a poverty trap.

The equilibria described by H and L are both stable: like in the baseline model, departing from a neighbour point at their left (right), per capita savings are greater than (are less than) the break even investment, and hence the capital labour ratio will increase (decrease) until reaching the steady state. The equilibrium described by A is unstable: if, departing from this equilibrium, the capital stock decreases (raises) by a small amount, then the saving rate becomes lower (higher) than the break even investment and the capital labour ratio will decrease (increase), until the low (high) income steady state is reached. Because the equilibrium L is stable and dominated by another possible outcome (equilibrium H), it is called a "poverty trap".

A key question in models with multiple equilibria concerns what equilibrium will prevail. In the case at hand, the stability properties of the model imply that *history* plays a critical role in equilibrium selection: if an economy starts out in the bad equilibrium L, it will remain in the bad equilibrium. If the economy starts out in the good equilibrium, H, it will remain in the good equilibrium.

Moreover, a policy designed to move the economy out of the poverty trap L may fail to do so, unless it is of magnitude enough to push the economy to the right side of threshold point A. Suppose that the economy was initially at point L and then it received some external aid to improve its capital stock. Unless the amount of aid was large enough to tilt the economy to the right of point A, the impact would be merely temporary: the initial rise in output per capita would lead to an acceleration of population expansion, which in turn would require a higher saving rate just to stand still. Since the saving rate is constant, the capital-labour ratio would start falling back, driving the economy again to the poverty trap. In the end, the extra capital obtained was "spent" in a larger population, rather than in improving living standards. If, however, the donation was large enough for the capital-labour ratio to expand ahead the critical level A, then the economy would have been able to escape the trap, engaging in the demographic transition and moving towards the high-income steady state, H. This reasoning suggests that that international assistance to a poor country trapped in a Post-Malthusian regime will be useless unless it is large enough for the economy to overcome the trap.

Note however that a large investment in physical capital is not the only avenue for this economy to escape the trap: a policy of birth control and family planning, if successful in reducing the gap between the birth rate and the death rate in the intermediate stage, could eventually smooth the break-even investment line the enough for the multiple equilibria to disappear.

2.4 Transitional Dynamics

An important feature of the Solow model is that, if the economy is not in the steady state, it will converge to the steady state. But the economy does not jump instantaneously from one steady state to the other: since capital accumulation is bounded by the availability of savings, there will be an adjustment period, during which the economy *approaches* the new steady state.

This point is very important because in the real-world economies may be found in adjustment processes, rather than in the steady state: e.g. perhaps because the savings rate has risen recently but not yet been reflected fully in higher per capita income. In the light of the Solow model, that economy will be experiencing a transitory growth, until reaching the new steady state. The following sections address specifically the issue of transition dynamics.

2.4.1 What happens if the savings rate increases?

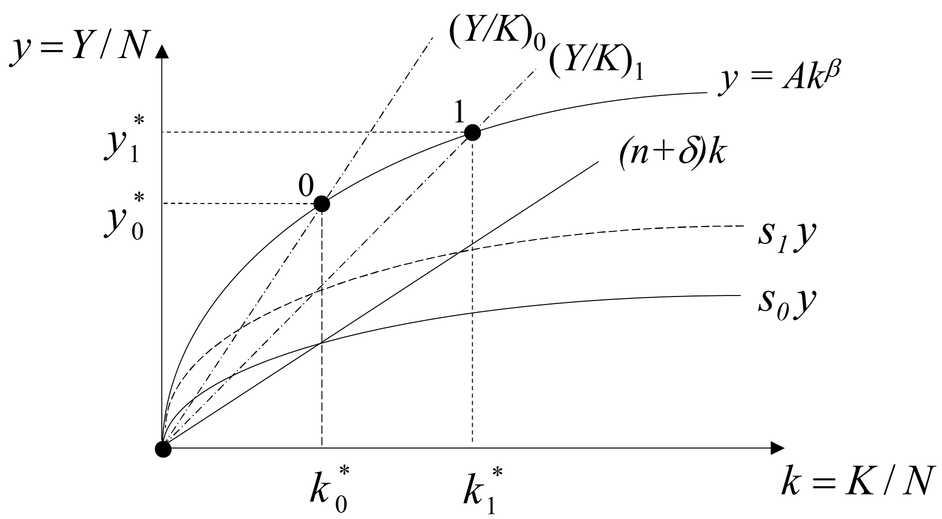
Figure 2.7 shows how the model adjusts to a rise in the saving rate. When the saving rate rises from s_0 to s_1, the curve measuring per capita savings shifts upwards, moving *the* steady state from k_0^* to k_1^*. Thus, the economy engages in an expansionary process until the new equilibrium is met. In the long run, the rise in the saving rate has produced a *level effect*: that is, in the new steady state, the economy will enjoy a higher *level* of output per worker than in the steady state before. During the transition from one steady state to the other, per capita output had increasing. But this growth surge was a merely *temporary*.

The implication of what we just learned is that, in low-income countries with low savings (say s=10%), a growth surge could eventually be achieved by raising the saving rate. However, once the economy reached the new steady state, per capita income would stagnate again. Thus, in order to achieve further increases in output per worker, one would need to raise the saving rate again, and again. And clearly, there are limits in exploring this avenue: the maximum level of the savings rate observed in countries in the real world is around 30-35%. Rates much higher than this obviously eat into available consumption and so the current standard of living.

Also important for our deliberation is the behaviour of the interest rate: note that the average product of capital (Y/K) in the new steady state is lower than in the old steady state. Visually, in Figure 2.7 you see that the slope of the ray that departs from the origin and crosses the production function at point *1* is lower than the one corresponding to the ray that crosses the production function at point *0*. This can be checked by reference to equation (2.18), where the saving rate enters in the denominator. Referring to (2.12), this means that *the interest rate has declined*: intuitively, one consequence of a higher availability of capital per worker in the economy is that the marginal product of capital has declined.

The implication of this finding is straightforward: for continuous growth of per capita income to be obtained through successive increases in saving rates, real interest rates should be decreasing over time. Since this is not a real-world fact, the conclusion is that sustained growth of per capita incomes as we observe in the real world cannot be accounted by successive raises in the saving rates. Savings are a bad candidate to explain economic growth.

Figure 2.7. A higher saving rate raises the steady state level of per capita income but only boosts growth rates temporarily



The figure shows how the steady state in the Solow model changes with an exogenous increase in the saving rate. In the new steady state (point 1), the capital labour ratio and per capita output are higher than before, but the average product of capital (Y/K) is lower, implying that the real interest rate has declined. In the long term, per capita income is again constant (the increase in the saving rate produced a "level effect").

2.4.2 What happens if population growth or the depreciation rate decline?

From equation (2.17) we see that a fall in the population growth rate has a similar effect to that of a rise in the savings rate. In terms of the Figure 2.3, the difference is that the change in the steady state will be caused by a downward shift of the break-even investment line. Thus, both output per worker and capital per worker will increase, but this will happen only during the transition from one steady state to the other. In the new steady state, the average product of capital - and interest rate - will be lower than in the initial steady state.

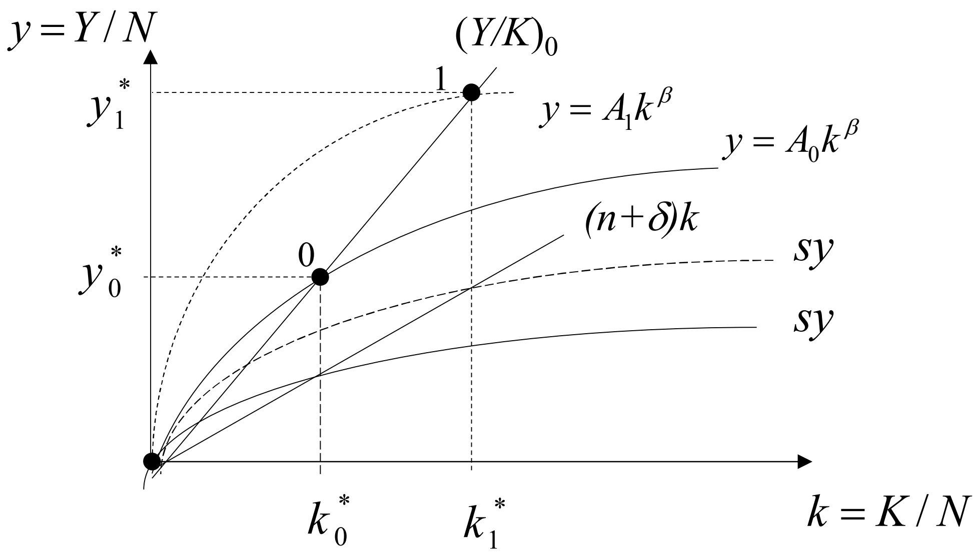
2.4.3 What happens if the level of technology improves?

Figures 2.8 describes how the model adjusts to a once-and-for-all technological change. In the figure, when TFP changes from A_0 to A_1, both the production function and the curve measuring per capita savings shift upwards, tilting the steady state from k_0^* to k_1^*.

In contrast to the case of an increase in the saving rate, in this case there is an initial jump in per capita output: the productivity increase means that more production is achieved out of the initial capital labour ratio. The paths of output per worker, the average product of capital, and of the interest rate are described in Figure 2.9. As shown in the figure, at the time of the shock (t_0), all the three variables jump upwards. During the adjustment to the new steady state, the average product of capital declines again and so will do the interest rate. In the new steady state (after t_1), *the average product of capital and the interest rate are the same as before the shock* (you may confirm this by observing that the *long run* level of Y/K (equation 2.18), does not depend on the level of technology, A). In the long run, the technological improvement materialized with higher levels of per capita income and capital per worker, but with the same interest rate.

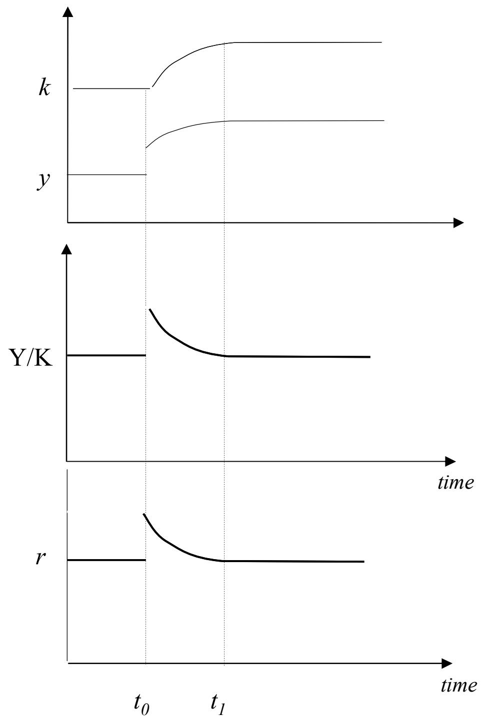
This finding has a very important implication: if we allowed the level of technology to expand continuously over time, it would be able to achieve continuous growth in per capita income without any tendency for the interest rate to decrease over time. This conformity with the real-world facts makes technology a better candidate than savings for explaining economic growth.

Figure 2.8. An increase in technology raises the steady state level of per capita income but leaves the output-capital ratio unchanged



The figure shows how the steady state in the Solow model changes with an exogenous increase in the technology parameter A. In the new steady state (point 1), the capital labour ratio and per capita output are higher than before, but the average product of capital (Y/K) is the same, implying that the real interest rate returned to the previous level. In the long term, per capita income is again constant (the technological improvement produced a "level effect").

Figure 2.9. The time paths of per capita output, the capital –labour ratio, the output capital-ratio and the interest rate, following an improvement in technology



Following a technological improvement, per capita income first jumps (at a constant k), and then starts growing along with the increase in k, until the new steady state is reached. The average product of capital jumps at the impact, and returns to the initial level in the long run.

2.5 The Golden Rule

2.5.1 The Golden Rule of capital accumulation

In the Solow model, a higher saving rate delivers a higher level of per capita income in the steady state. Does this mean that any increase in the saving rate is desirable? From the household point of view, the answer to this question is negative. The reason is that households care with *consumption*, not with income. While a higher saving rate brings a higher per capita income in the steady state, a higher *proportion* of the later will be foregone consumption. The final impact on consumption will depend on the balance between these two opposing effects.

Mathematically, the saving rate that maximizes the level of per capita consumption in the steady state can be found in the following manner:

$$\max_k c = y_t - sy_t \text{, subject to } \dot{k} = 0 .$$

where $c=C/N$ denotes for per capita consumption. In the steady state, this is equivalent to chose k to maximise[42]:

$$c = Ak_t^{\beta} - (n+\delta)k . \qquad (2.19)$$

The first order condition of this problem leads to:

$$\frac{\partial y}{\partial k} = n + \delta \qquad (2.20)$$

[42] The symbol "*" referring to the steady state is suppressed to simplify the algebra.

This condition is called the "Golden Rule of Capital Accumulation"[43]. Geometrically, it implies that the slope of the production function (i.e. the marginal product of capital) must equal to the slope of the break-even investment line (equation 2-20). This is represented by point G in figure 2.10, where the vertical distance between the production function and the breakeven investment line (ie, per capita consumption) is maximized. Figure 2.11 offers an alternative representation of the golden rule, showing only the steady state per capita consumption as a function of the capital labour ratio. Whenever the steady state level of capital per worker is below the golden rule (that is, when $k^* < k^G$), the rise in income that results from a possible increase in k^* more than offsets the rise in the amount of savings that is necessary to sustain such equilibrium, implying that more resources are available for consumption. Conversely, whenever the steady-state capital-labour ratio is higher ($k^* > k^G$) the rise in income that would result from any further raise in k^* is less than the required increase in savings, implying that less resources become available for consumption.

Solving (2.20), the golden rule level of k^* is given by:

$$k^G = \left(\frac{\beta A}{n+\delta}\right)^{\frac{1}{1-\beta}} \quad . \qquad (2.21)$$

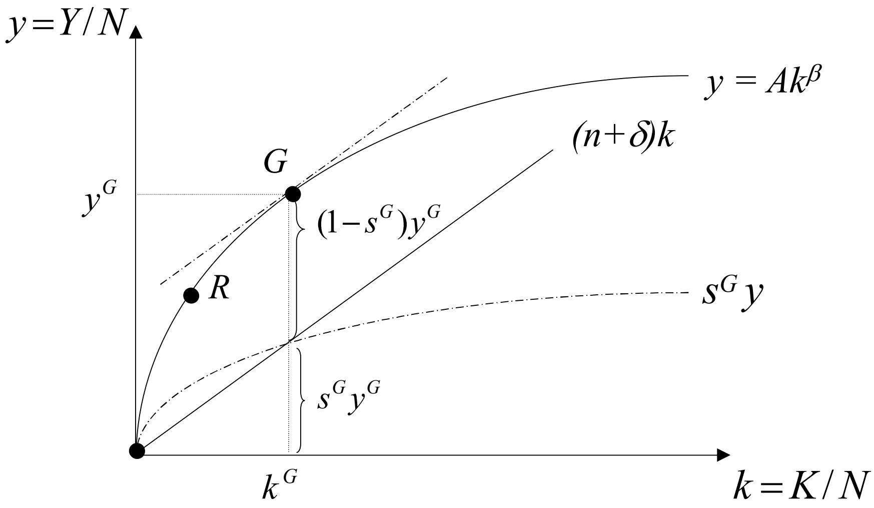
The value of s that turns k^G into a steady state is called the “golden rule” saving rate. Comparing (2.21) with the general solution for steady states (2.16), we conclude that the golden rule saving rate is:

$$s^G = \beta \qquad (2.22)$$

[43] Phelps, E., 1961. "The Golden Rule of Accumulation: a Fable of Growth man". *American Economic Review*, 51, 638-643.

That is, the golden rule saving rate is equal to the share of capital in income. In other words, if households spend all labour income in consumption and save all capital income, they will be maximizing per capita consumption in the steady state[44].

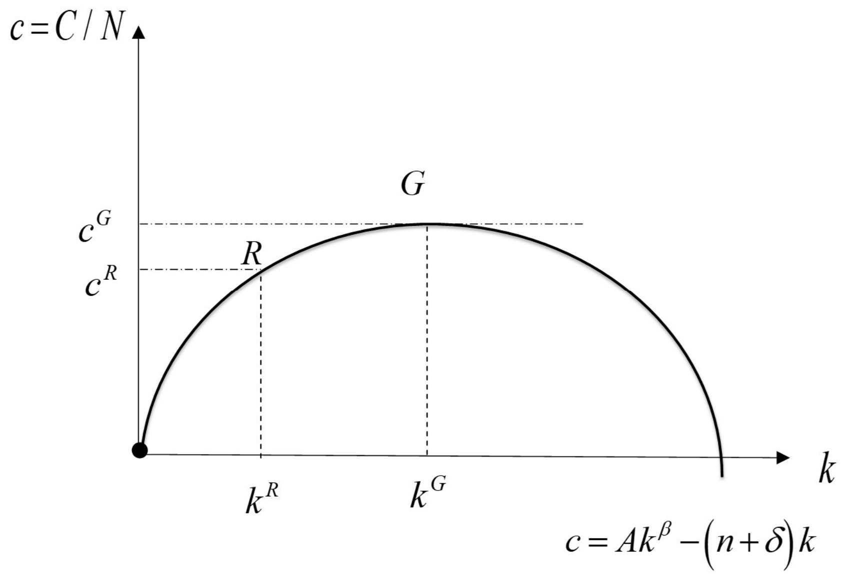
Figure 2.10: Illustration of the Golden Rule



A steady state in the Solow model must be such that the savings locus crosses the breakeven investment line. Given an economy' breakeven investment line, among all possible saving rates, the golden rule saving rate is the one that determines the larger distance between the breakeven investment line and per capita output. That will be the one where the slopes of the two schedules are the same.

Figure 2.11. Per capita consumption in the steady state

[44] An alternative avenue to obtain (22) is replacing (2.17) in the maximization problem (2.19) and take the derivative in order to *s*.



The figure describes the steady state level of per capita consumption as a function of the capital-labour ratio. The golden rule corresponds to the maximum possible consumption level in the steady state C. Allocation R corresponds to the "modified golden rule" (see Section 2.6).

2.5.2 Dynamic efficiency

Suppose that the saving rate in a given economy was lower than the golden rule saving rate, delivering a steady state at the left of the golden rule (say in point R in figure 2.11). Would that mean that the government should step in, trying to increase the economy' saving rate? Eventually, you would be tempted to subsidize savings to achieve a steady state with a higher level of per capita consumption. A problem however is that this move would imply a sacrifice of current consumption: forcing consumers to save more today to achieve higher consumption in the future involves a trade off between todays' utility and tomorrow' utility that the policymaker may not be able to evaluate. Because of this, any equilibrium at the left of the golden rule is said to be *dynamically efficient*. An equilibrium is said to be dynamic efficient if the path of savings cannot be changed to strictly increase consumption at some point in time without lowering it at any point in time[45].

A different case occurs when the initial steady state lies at the right-hand side of the golden rule (that is, if $k^* > k^G$). In that case, by reducing savings today, consumption would increase *both* today and tomorrow. Since a "free lunch" is readily available, this case is labelled as *dynamically inefficient*.

At the first sight, a case of dynamic inefficiency looks at odds with the principle that agents are optimizers: after all, if households were saving too much, they should realize that by reducing savings today, they would be able to consume more, today and in the future. As a general principle, if savings are decided by optimizing agents, any policy altering their choices should make people worse off. So, unless there are good reasons to believe that some impediment prevents consumers from optimally decide their savings, or that any market failure is turning individual decisions socially unacceptable, there will be no case for intervention.

There are, however, situations where excess savings may occur. First, individuals may not be given the choice. A textbook example is the case of the old Soviet Union: during the communist regime, most consumption goods were rationed, implying that individuals had income to spend but no goods to buy, being forced to save more than desired. Second, individual optimization tends to ignore social impacts, giving rise to externalities. For instance, the current generation, concerned with its income after retirement, may engage in a saving effort that reveals excessive from the social point of view. In that case, a policy aiming to reduce current savings today such as a social security mechanism based on inter-generational transfers (pay as you go), could help mitigate the concerns of young individuals, preventing excessive savings and hence improving the welfare of both current and future generations. Whether these examples are realistic or not is a different question. In practice, we observe that the shares of capital in national income vary from 0.3 and 0.4. According to the model

[45] Phelps, E., 1965. Second Essay on the Golden Rule of Accumulation, *American Economic Review* 55, 793-814.

formulation, this corresponds to the contribution of capital to output, β. Since real world saving rates are, in general, lower than 30%, one may conclude that "dynamic inefficiency" is not at all a general case.

2.6 The case with endogenous savings

2.6.1 An optimal consumption rule

In the Solow model, the saving rate is assumed exogenous. The neoclassical model can however be extended, to account for the case in which individuals optimally decide their saving rates. It is not in the scope of this book to solve complicated dynamic optimization models. So, in the following – and throughout the book – we will refer to a very simple formula (for details on how this formula is obtained, see Appendix 2.1).

$$\gamma_t = r_t - \rho \qquad (2.26)$$

In (2.26). $\gamma_t = \dot{c}/c$ denotes for the growth rate of per capita consumption, and ρ is the rate of time preference (that is, the rate at which individuals are willing to trade one unit of utility today for utility in the future). The rate of time preference measures the impatience of individuals. In the competitive equilibrium the interest rate is determined by the marginal product of capital (eq. 2.12).

According to (2.26), as long as the interest rate is higher than the rate of time preference, individuals will optimally increase their consumption over time. If however, the interest rate falls below the rate of time preference, individuals will optimally reduce consumption over time. When $r_t = \rho$, the optimal level of consumption will be constant over time. That will be the steady state of the model.

2.6.2 What happens when the rate of time preference decreases?

To see how the economy adjusts to the long run in the model with endogenous savings, suppose that, departing from an initial steady state, the degree of impatience of individuals, as reflected in the parameter ρ, decreases. At an unchanged interest rate, savings will increase, and consumption will fall instantaneously. Since more savings translate into more investment, the capital labour ratio will then start increasing, inducing a temporary growth of per capita income and of consumption.

Then, as the stock of capital per worker increases, the marginal product of capital will decrease and so will do the interest rate (eq. 2.12). At the time the interest rate equals the new rate of time preference, the desired consumption becomes constant over time (eq. 2.26) and the process of capital accumulation stops. In the new steady state, both consumption and per capita income are higher than before, but constant over time.

The main conclusion is that making savings endogenous does not alter the main property of the model: if, for any reason, people become more patient and save more, there will be a permanent increase in the steady state level of per capita income, but no permanent effect on economic growth.

2.6.3 The modified golden rule

In the steady state the growth rate of consumption is zero, and the capital output ratio is given by (2.18). Using these conditions together with the equation for the interest rate (2.12), and solving for the saving rate, one obtains:

$$s = \beta \frac{n+\delta}{\rho+\delta} \quad (2.27)$$

This saving rate corresponds to the "modified golden rule". Intuitively, the impatience reflected in the rate of time preference means that an infinitely lived consumer will, in general, prefer a steady state consumption lower than the maximum feasible (golden rule)[46]. Condition (2.27) also rules out the possibility of a balanced growth path above the golden-rule: if the

[46] Formally, the condition $\rho > n$ is needed for the solution of the maximization problem to be well defined (for an explanation, see for instance, Romer 20011, chapter 2.1 [Romer, D., 2001, Advanced Macroeconomics, McGraw-Hill].

economy was in such a path, households would optimally reduce their saving and take advantage of this opportunity.

The steady state level for capital per worker implied by (2.27) can be obtained directly substituting that expression in (2.16). This gives:

$$k^R = \left[\frac{\beta A}{\rho+\delta}\right]^{\frac{1}{1-\beta}}$$

This optimized capital-labour ratio is represented in Figure 2.11 by point R.

2.6.4 Exogenous or endogenous savings?

The consumption rule (2.26) was obtained assuming that all individuals have access to a frictionless financial market, where they can borrow or lend any amount of income at a given interest rate r. Acknowledging this assumption is very important, when thinking how the model with endogenous savings applies to the real World. In the real world, financial markets are far from perfect. Many poor households have no access to borrowing, and this is especially true in developing countries. In that case, households are more likely to consume based on current income, rather than on future incomes. This means that a consumption function depending only on current income and with an exogenous saving rate, as assumed in the basic Solow model, may actually be more sensible to describe emerging economies with underdeveloped financial markets.

2.7 The Solow Residual

In the sections above, we explore the contributions of capital accumulation and of technological change to the growth rate of per capita income. Empirically, a technique to disentangle the contribution of these two factors using observable data is "growth accounting".

Basically, the technique departs from a log-differentiation of the production function (2.1):

$$\frac{\dot{Y}}{Y}=\frac{\dot{A}}{A}+\beta\frac{\dot{K}}{K}+(1-\beta)\frac{\dot{N}}{N}=\hat{A}+\beta\hat{K}+(1-\beta)n \qquad (2.28)$$

This equation states that the growth rate of output equals the growth rate of total factor productivity ("A") plus a weighted average of the growth rates of physical capital and labour, where the weights are the corresponding elasticities in the production function. In the intensive form, the equivalent is:

$$\hat{y} = \hat{A} + \beta\hat{k} \qquad (2.28a)$$

If factor markets are competitive, as assumed in the Solow model, the parameter β shall be equal to the share of capital in national income (equations 2.11 and 2.12). In the real world, it has been observed that the share of capital in national income ranges from 0.3 to 0.4. Thus, based on observed variables one can estimate the unobservable growth rate of the technological parameter, as the difference between the actual growth of output and the growth implied by factor accumulation (hence the label *Solow residual*)[47]:

$$\hat{A} = \hat{Y} - \beta\hat{K} - (1-\beta)n \qquad (2.29)$$

In the intensive form (2.28a), the Solow residual is obtained as:

$$\hat{A} = \hat{y} - \beta\hat{k} \quad , \qquad (2.29a)$$

with $\hat{y} = \hat{Y} - n$ and $\hat{k} = \hat{K} - n$.

By construction, the Solow residual measures the part of actual output growth that is not accounted for by factor accumulation. This is more than simple "technological progress". Technological progress, in narrow sense, refers to changes in the "efficiency" with which the existing technology and inputs are combined. The Solow residual also accounts for unmeasured changes in the quality of inputs (skill increments in the labour force, in the quality of land) and statistical errors.

[47] Solow, R. 1957. "Technical change and the aggregate production function", *Review of Economics and Statistics* 39, 79-82.

In his original paper, Solow (1957, op. cit) found out that the fraction of per capita income growth in The US along 1909-1949 accounted for capital accumulation was very small: only one eighth, comparing to seven-eighths attributed to "technical change". This finding was corroborated by many other studies for many other economies.

Table 2 displays the results of a recent growth accounting exercise focusing on Britain since the 13th century (references in the table). The figures indicate that capital deepening played little role in economic growth in the pre-industrial era: until 1830, most economic growth was explained by the growth rate of TFP. During the transition to modern economic growth (1830-1860) capital accumulation by-passed technological change as the main source of growth. Still, TFP contributed with 0.43p.p. to the 1.11 growth rate of per capita income growth in that period.

This evidence points to a fundamental limitation of the Solow model in its basic formulation: by assuming that A is constant, this model ignores a critical ingredient of economic growth: technological progress.

Table 2.1: Accounting for the growth of British GDP per head, 1340s to 1860s (% per annum)

	Output Growth	Population growth	Output per worker	Contributions:	
				Capital Deepening	TFP Growth
1340s-1400s	-0.73	-1.27	0.54	0.34	0.20
1400s-1450s	-0.21	-0.13	-0.08	-0.13	0.05
1450s-1640s	0.5	0.53	-0.03	-0.01	-0.02
1640s-1690s	0.84	-0.04	0.88	0.22	0.67
1690s-1830s	1.08	0.74	0.34	0.01	0.26
1830s-1860s	2.28	1.17	1.11	0.68	0.43

Notes: Output and capital stock are in constant 1700 prices. Wights for labour and capital are 0,5 and 0,4 respectively. Source: Broadberry, S and A M de Pleijt (2021), "Capital and Economic Growth in Britain, 1270-1870: Preliminary Findings", CEPR Discussion Paper 15889.

2.8 Key ideas in Chapter 2

- The Solow model rightly accounts for the role of capital in production and stresses the key role of saving in generating the resources that are necessary to invest in new capital. This, in turn, exactly offsets the diminishing returns due to the growing labour force.
- The model also provides a sensible story about why historical ratios of capital to output and real interest rates appear to be relatively stable in the long run. Finally, it offers the credible suggestion that countries with high savings rates and low population growth

rates should expect higher levels of per capita income in the long run than countries with low saving rates and rapid population expansion.

- In its current formulation the model fails, however, to explain the most basic fact of modern economic growth: that per capita income tends to increase over time. In the Solow model, any growth in per capita income must be transitory in nature, reflecting the adjustment of the economy from one steady state to the other. The reason underlying reason is that there are diminishing returns on physical capital.
- Of course, continuous growth of per capita income can be generated in the context of the Solow model if saving rates are set to increase continuously over time. In this case, however, the real interest rate should decrease over time. Given that this is not a real-world fact, the conclusion is that the sustained growth of per capita incomes as we observe in the real world, cannot be explained by successive increases in saving rates.
- In this model, an exogenous improvement in technology leads to a higher level of per capita income, without causing any decline in the interest rate. This suggests that the key to overcome the main limitation of the Solow model is to allow for continuous improvement in technology.
- In the context of the Solow model, there is a saving rate that maximizes the level of per capita consumption in the steady state. This is not to say that a change in the saving rate towards that level will be welfare improving: this will only be the case when the starting situation is of dynamic inefficiency.
- Making saving endogenous does not rescue the model from its main limitation, that the long run growth rate of per capita output is zero.
- Growth accounting is the technique of measuring a country' productivity change through the difference between output growth and the "contribution" of inputs. In general, growth accounting exercises reveal that total factor productivity expands over time. This evidence stresses the need to enrich our model, allowing the technology to expand over time.

Appendix 2.1: The optimal consumption

The problem of how much an individual should save was first addressed in an inter-temporal optimizing framework by a mathematician from Cambridge UK called Frank Ramsey. Ramsey died at the age 26 and his seminal contribution remained obscure for long time by the economics profession, because at that time most economists were not familiar to dynamic optimization. His work was re-discovered four decades later, by Cass and Koopmans, who used it to characterize the optimal saving paths in the context of the neoclassical growth model[48].

In this appendix, we illustrate the optimal consumption rule in a dynamic context referring to a discrete time framework. Consider an individual whose life-time utility function is given by

$$U=\sum_{t=1}^{\infty}\frac{u(c_t)}{(1+\rho)^{t-1}}\ , \qquad \text{(a2.1)}$$

where $u(c_t)$ is a concave function, c_t is real consumption in period t and ρ is a given rate of time preference.

Assume that this individual has full access to financial markets, being therefore able to borrow or lend any amount of income at the interest rate *r*. His problem is to maximize the lifetime utility function, subject to $\sum_{t=1}^{\infty}\frac{c_t}{(1+r)^{t-1}}=\Omega$, where Ω denotes for lifetime wealth.

From the first order conditions of the maximization problem, one obtains the so-called *Euler equation* for each two consecutive periods:

$$u'(c_{t+1})=u'(c_{t+1})(1+\rho)/(1+r).$$

This equation states that the marginal utility of consumption in the next period must be equal to the marginal utility of consumption in the current period, weighted by the ratio of the rate of time preference to the market discount rate. In other words, this rule implies that the consumption level each period must be such that an extra unit of consumption would make the same contribution to lifetime utility no matter to what period is allocated.

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[48] Ramsey, F., 1928. A mathematical theory of savings. *Economic Journal*, 38, Nº 152, 543-559. Cass, D. , 1965. Optimum growth in an aggregative model of capital accumulation. *Review of Economic Studies* 32 (3), 233-240. Koopmans, T., 1965. On the concept of optimal growth. In *The Econometric Approach to Development Planning*. Rand-McNally, Chicago.

In the main text, we stick with the convenient assumption of logarithmic preferences, that is $u(c_t) = \ln c_t$. In this case, the Euler equation simplifies to:

$$c_{t+1}/c_t = (1+r)/(1+\rho). \qquad \text{(a2.2)}$$

Denoting by γ the growth rate of per capita consumption, the later expression becomes equal to:

$$1+\gamma = (1+r)/(1+\rho),$$

Which, by approximation gives (2.26).

Problems and Exercises

Key concepts

- *Gross investment vs. net investment. Break even investment line. The Kaldor facts. Poverty trap. Golden rule. Dynamic inefficiency (a la Phelps). The Solow residual*

Essay questions:

- To which extent the basic Solow model is capable of describing real world facts?
- Why can't the Solow model generate a sustained growth of per capita income?
- Is the Golden Rule saving rate an optimal saving rate?

Exercises

2.1. Consider an economy where the aggregate production function Y=AF(K,N) exhibits Constant Return to Scale, positive and decreasing marginal productivity and unit elasticity of substitution between factors. Admitting that the saving rate, the population growth rate, technology and the rate of capital depreciation are all constant and exogenous: (a) Describe in a graph the steady state of this economy. Is it a stationary steady state? Why? (b) Describe in a graph the effects of the following changes on the long run level of per capita output: An increase in the population growth rate; an earthquake that destroys part of the capital stock. (c) Describe the effects of a rise in the saving rate in the time paths of the following variables: capital per worker; per capita income; per capita consumption. (d) Describe the effects of a rise in the level of technology on the time paths of the following variables: per capita output; capital per worker; interest rate. (e) In light of the Solow model, is there a tendency for per capita output levels in different countries to approach each other in the long term? Why?

2.2. Consider an economy where the production function is given by: $Y_t = 20K_t^{1/3}N_t^{2/3}$, where N_t is the number of workers in period *t*. In this economy, 25% of income is saved, the labour force grows at 2.5% and capital depreciates at 2.5%. We also know that in this economy there is perfect competition, and wages and prices are fully flexible. (a) Compute the steady state values of capital and output per worker. Represent in a graph and describe the stability of the equilibrium. (b) Suppose this economy was affected by a hurricane, which reduced its capital stock. Discuss the subsequent dynamic adjustment of this economy with the help of a graph.

2.3. Consider two economies, A and B, sharing the same technology, given by $Y = K^{0.5}N^{0.5}$. Assume that the saving rates in A and B are, respectively 10% and 20% and that the sum $n+\delta$ is equal to 10% in both countries. (a) Suppose that initially the capital-labour ratio was equal to 2 in both countries. What will be the corresponding initial levels of per capita consumption and per capita income? (b) Starting from the position described in a), compare the evolution of per capita income in both economies as time goes by. Discuss.

2.4. Consider an economy where the production function is given by $Y_t = 0.2K_t^{1/3}N_t^{2/3}$. In that economy, 25% of income is saved, capital depreciation is 5% and population is constant and equal to 1000 inhabitants. (a) Find out the steady state values of per capita income, per capita consumption, real wages and the interest rate. (b) Find out the saving rate that would maximize C/N in *steady state,* where C is consumption. Illustrate with the help of a graph the adjustment dynamics of Y/N and C/N admitting that the saving rate actually changed to that level. (c) Suppose you were a benevolent planner with power to set a tax on production, which revenues were distributed lump sum to consumers. What would be the level of τ if you wanted to maximize the steady state level of per capita consumption? Would such policy be welfare improving?

2.5. Consider an economy where the aggregate production function is given by $Y_t = A_t K_t^{1/2} N_t^{1/2}$. In this economy, the saving rate is 20%, capital depreciates at 5% per year, and population is constant and equal to N=100. (a) Assume that $A_t = 1$. (Find out the steady state values for: (a1) per capita income; (a2) interest rate; (a3) capital and labor income shares; (a4) wage rate. (a5) To what extent does this model comply with the Kaldor stylized facts? (a6) Represent the equilibrium in a graph and discuss its stability. (b) Sticking with $A_t = 1$, analyze graphically and quantify, when possible, the short term and long-run effects of a fall in the saving rate to s=2.5% on: (b1) per capita income; (b2) per capita consumption; (b3) the interest rate. Considering your findings, would the saving rate be a good candidate to explain: (b4) why some countries are much richer than others? (b5) long term growth? Elaborate. (c) Departing from s=20%, analyze graphically and quantify, when possible, the implications of a decrease in A to $A_t = 0.125^{0.5}$, namely on: (c1) per capita income; (c2) the interest rate. Based on your findings, could A be a good candidate to explain: (c3) why some countries are much richer than others? (c4) long term growth? (c5) What would be the theoretical problems with this explanation?

2.6. Consider an economy where the labour income share is 75%. What would be the Solow residual, if both output and capital were growing at 3% per year and the labour force was expanding at 1.5%?