1 Population and economic development

"*The most decisive mark of the prosperity of any country is the increase in the number of its inhabitants*". [Adam Smith].

Learning Goals:

- Acknowledge the importance of the Malthus theory of population to explain historical facts in the ultra-long run.
- Understand the challenges posed by the Law of Diminishing Returns
- Understand the critical role of technology in overcoming diminishing returns to labour.
- Acknowledge that a larger population may also bring benefits, through faster technological change.
- Understand the factors that drive the changing attitude towards fertility along the process of demographic transition.
- Use the Malthusian theory and the theory of demographic transition to interpret the rising cross-country economic disparities that followed the industrial revolution.

1.1 Introduction

The world population has been expanding at impressive rates. Along the last two centuries, the World population increased from 1 billion to more than 7 billion. Although population growth is decelerating, population is still increasing and is expected to reach 9 billion in 2050.

A question that arises is whether the continuing population expansion will challenge our living standards, overwhelming the existing resources. Such question was first formulated by Thomas Malthus in its famous book "An essay on the Principle of Population", published in 1798. Malthus contended that a fixed endowment of natural resources could not feed a constantly increasing population. Thus, the population explosion that was already becoming evident in the 18th century could not be sustained forever. Malthus observed that societies throughout history had experienced at one time or another different types of "checks" that prevented population to overstretch their resource limitations. This includes epidemics, famines, and wars, but also abstinence and birth control.

This chapter addresses the Malthus theory of population and living standards, as well as its limitations. As we will see, the Malthus ideas provide a useful tool to interpret the almost constant living standards that characterized the pre-industrial era. The Malthusian model might also be thought to provide a reasonable narrative for some of the world's today poorest countries and regions. But fortunately, the Malthus pessimism regarding the future of humankind did not materialize: somehow ironically, at the time Malthus was writing his book, a set of countries in West Europe entered in a new phase of economic development, in which population and living standards were expanding together. The failure of the Malthus theory to describe modern growth is explained by three main limitations in the basic model. First, Malthus overlooked the potential of technology in overcoming the limitations imposed by resource constraints. Second, he ignored the potential benefits of a larger population in speeding up the rate of technological progress. Third, he ignored the possibility of technological change and increasing living standards impacting on the human attitude towards fertility.

This chapter reviews these questions, focusing on the relationship between the size of population, per capita income, and technology. The chapter is organized as follows. Section 1.2 introduces the Malthus theory in its basic formulation. In Section 1.3, we discuss the potential role of technology in overcoming the limitations imposed by diminishing returns. This chapter also introduces the possibility of technological change being a positive function of the size of population. Section 1.4 addresses the question of why the Malthus theory of population no longer applies in our days. The section reviews some theories that explain why the human attitude towards fertility has changed with economic development. In Section 1.5, we put all pieces together to offer an interpretation of why cross-country income disparities increased significantly in the two hundred years that followed the Industrial Revolution.

1.2 The basic Malthusian model

The original theory of Malthus was essentially descriptive, and much richer than the simplification adopted here. We turn, however, to a very simple model to sketch out the basic mechanics underlying his central argument.

Consider a closed economy (i.e. one with no international trade), with no government, and basically devoted to agriculture. In this economy, output takes the form of a single homogeneous good (Z), produced with labour (N) and land (T). In this simple formulation, population and the labour force are the same. Since we are focusing on the long run, prices are assumed flexible, ensuring full employment each moment in time.

The relationship between inputs and output is described by an aggregate production function of the form[1]:

$$Z_t = BT_t^{\beta} N_t^{1-\beta} . \qquad (1.1)$$

In equation (1.1) the subscript t is a time index. The term B is labelled "Total Factor Productivity" (TFP) and it measures the state of "technology", or the "efficiency" with which inputs are utilized in the production process. At moment, we consider it exogenous and equally available to all agents in the economy at no cost.

To capture the existence of physical limits to land expansion, we assume that the amount of land available to agriculture is fixed ($T = \overline{T}$). The labour force is endogenous, as explained below.

1.2.1 Diminishing Returns

The Law of Diminishing Returns (LDR) is one of the oldest and more important postulates in Economics. In short, it states that, increasing one ingredient of production keeping all other ingredients constant has a decreasing marginal impact on output. In the case at hand, if the availability of land is given and technology remains unchanged, the only way of expanding production is to increase the use of labour. The problem is that increasing the use of labour will come along with a lower level of output per worker, due to diminishing returns.

Formally, let's divide both terms in (1.1) by the number of workers, N, obtaining:

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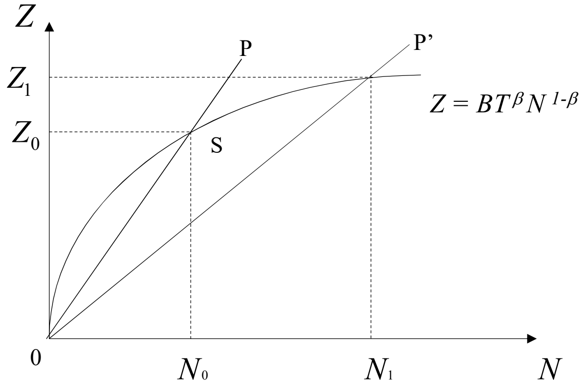
[1] Specification (1.1) corresponds to a well-known class of production functions, named Cobb-Douglas. The main properties of the Cobb-Douglas production function are constant returns to scale, diminishing returns and a unit elasticity of substitution between inputs. The Malthus theory only requires diminishing returns, but we stick with this formulation, for simplicity.

$$y = B\left(\frac{\overline{T}}{N}\right)^{\beta} \quad , \qquad (1.2)$$

where $y = Z/N$ denotes for output per capita (or per worker: remember that in this model the population and the work force are the same).

The LDR is illustrated in Figure 1.1. In the figure, the vertical axis measures the level of output (Z) and the horizontal axis measures the size of the labour force, N. The average product of labour is measured by the slope of the ray that departs from the origin and crosses the production function in each point. For instance, when the workforce is equal to N_0, output will be Z_0. The corresponding average product of labour is Z_0/N_0 (i.e, the slope of OP). Given the shape of the production function, when the number of workers rises to N_1, output expands less than proportionally, and hence the average product of labour declines to OP′.

Figure 1.1: Output, population, and productivity



The figure illustrates the Law of Diminishing Returns, referring to the production function (1.1). The slopes of the rays OP and OP' measure output per worker, as given by equation (1.2). All else equal, when the number of workers increases from N_0 to N_1, output per worker declines (the slope of OP' is lower than that of OP).

1.2.2 The Malthus theory of population

Malthus formulated his theory of population observing first the wild. He noted that animals and plants are "impelled by a powerful instinct to the increase of their species". He also pointed out that “superabundant effects” are repressed by “want of room and nourishment”[2].

Malthus then argued that a similar mechanism holds for human beings: the “passion between sexes” compels humans to proliferate. However, resources are limited. Hence, a point will come when a human population expanding without control reaches the limit up to which food sources can support it. Beyond this point, any further increase in population would result in food shortage and henceforth in death caused by starvation, diseases, and violence. To these *natural* barriers, Malthus added more *conscientious* prevention mechanisms, which operate via birth control: the Humankind differs from other specimens in that it may deliberately reduce fertility to avoid resource shortages. In his terminology, Malthus distinguished two types of checks that prevent population to expand beyond the land carrying capacity: “*positive checks*”, like epidemics, famines, and wars, which cause death; and “p*reventive* checks”, like abortion, birth control and postponement of marriage, which refrain birth.

In terms of our simple model, the Malthus theory of population is accounted for introducing a “subsistence level of per capita income” ($\bar{y}$) defined as the minimum income necessary to sustain the life of a human being. Thus, whenever per capita income rises above that subsistence level, population expands; when per capita income falls below the subsistence level, population declines.

1.2.3 Dynamics and equilibrium

Summing up, the model has two basic ingredients: the Law of Diminishing Returns (LDR) and the Malthusian theory of population. With these two ingredients, the model solves in an intuitive manner: in the absence of technological progress, the LDR implies that a growing labour force will lead to a more intensive use of land and thereby to a decline in output per

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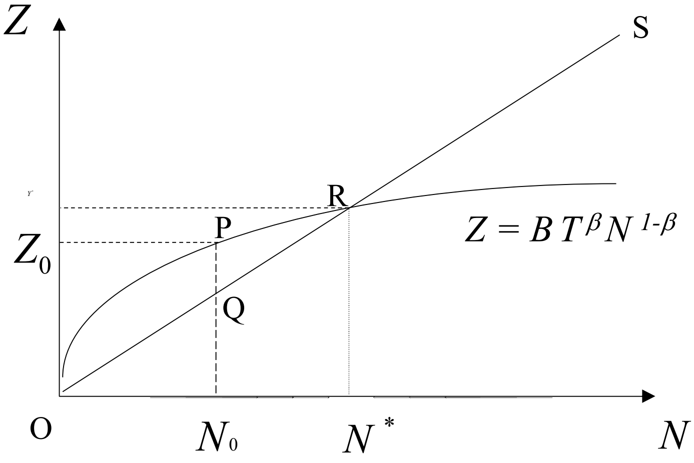
[2] Malthus, T., 1798. An Essay on the Principle of Population as it Affects the Future Improvement of the Society. London: J. Johnson.

worker. As output per worker declines, population expansion decelerates. At the time output per worker equals the "subsistence" level, both population and output stop expanding. This is the equilibrium of the model. Thus, given the land availability and the level of technology, the size of the population in this model is self-equilibrating.

Figure 1.2: Dynamics and equilibrium in the Malthus Model



The exogenous subsistence level of per capita income ($\bar{y}$) is represented by the slope of the line OS. Since at point P per capita income Z_0/N_0 is higher that the subsistence level, there will be a tendency for population to expand (move to the right). This process ends up at point R, where output per worker is equal to the subsistence level. This is the steady state of the model.

Figure 1.2 illustrates the dynamics of the model. In the figure, the exogenous level of subsistence income ($\bar{y}$) is represented by the slope of the line OS. Suppose that initially there are N_0 workers producing Z_0. Since in this point per capita income Z_0/N_0 is higher that the subsistence level, there will be a tendency for population to expand. As population expands, land will be used more intensively implying a decline in output per worker. This process ends up at point R, where output per worker is again equal to the subsistence level. At this point, there is no tendency for population to expand: any further increase in population would result in famine, disease, and birth control. This is the equilibrium of the model.

Box 1.1. Key concept: Stable Steady State

Technically, point R in Figure 1.2 is an equilibrium, or *steady state*, because once it is reached, there will be no tendency for the economy to move away from it.

An important property of this equilibrium is that the dynamic forces of the model are such that this equilibrium will be met, irrespectively of the departing point: for instance, if population is initially smaller than N*, per capita income will be above the subsistence level and population will expand. Conversely, if population is initially larger than N*, then the level of per capita income will fall below the subsistence level and population will decline. Because the economy will approach the equilibrium irrespectively of the departing point, this equilibrium is said to be *stable*.

Box 1.2. Historical episode: The Black-Death

The Black Death was one of the most devastating pandemics in human history. It peaked in Europe between 1346 and 1353, killing almost one half of the population at that time. In the case of England, for instance, the Black Death led to a decline in population from about 6 million to 3.5 million people in two years only (1348-1349).

Since population takes time to recover, for a long period of time after the Back Death, the European populations remained below trend. If the diminishing returns hypothesis was right, the higher availability of land per worker after the Black Death should have resulted in higher per capita incomes (such as from R to P in Figure 1.2). The historical data actually confirms this prediction: Along the 150 years that followed the black death, GDP per capita jumped ahead of its previous trend, reflecting a lower population and diminishing returns, in a context of sluggish technological change[3].

1.2.4 What happens when technology improves?

We now turn to the question of how the equilibrium of the model changes when the level of technology improves. As an example, suppose that an ingenious farmer started using waterpower, becoming more productive. As you may guess, the other farmers in the region, observing their fellow success, would rapidly copy the idea. An important characteristic of knowledge is that it is non-rival: the same invention can be shared by many people without

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[3] Clark, G. , 2001, The Secret History of the Industrial Revolution". UC Davis.

losing its effectiveness. Since knowledge transmits easily across the society, each agent can benefit from the discoveries of all the others.

Figures 1.3 describes the impact of a technological change in the Malthus model. In the figure, the vertical axis measures per capita income, instead of total production[4]. Due to diminishing returns, per capita income is a negative function of the population size. Suppose that the economy starts out in Q, with per capita income equal to the subsistence level, $\bar{y}$. Then, a technological improvement from B_0 to $B_1 > B_0$ causes the schedule describing output per capita to shift upwards. At the impact (with the size of population unchanged), per capita output increase to y'. Since at point P per capita income stands above the subsistence level, population in this economy starts expanding (this is the Malthus theory of population). Then, as population slowly expands, diminishing returns show up, driving per capita income back to the subsistence level. At the time the new equilibrium, R, is reached, all gains from technological progress had been channelled to the increase in the size of population, to N_1^*. In the long run, the initial improvement in living standards has been completely offset by population expansion.

Similar results hold when the availability of land increases. If, for instance, a swampy stretch of land was drained, that would be accounted for a rise in parameter T. In terms of Figure 1.3, the change is equivalent to when B increase: after all, what matters is that the "carrying capacity" of the economy (in terms of number of inhabitant has increased.

Figure 1.3: Productivity shift in the Malthus Model

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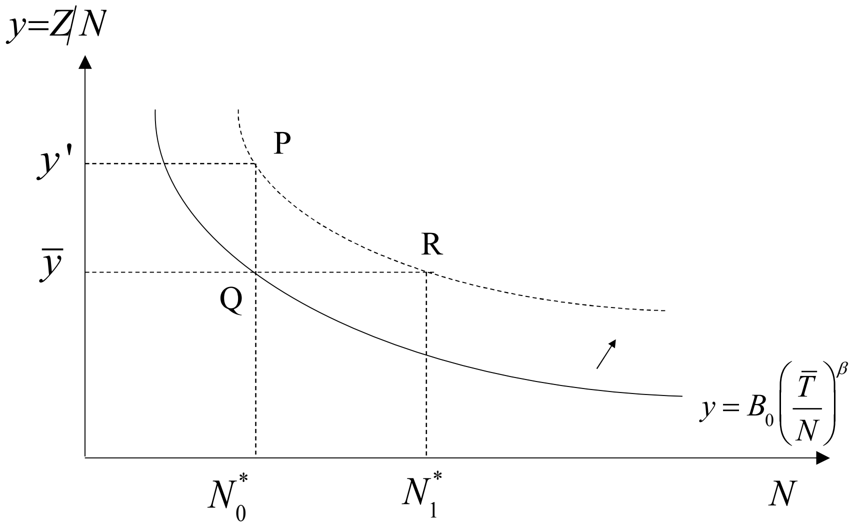
[4] In the economic jargon, the figure is said to describe the production function in the *intensive form* (equation 1.2), rather than in the *extensive form* (equation 1.1).



Departing from Q, suppose that a technological improvement (or an increase in the availability of land) moves the per capita output schedule to the right (dashed curve). With a constant population, per capita income jumps to y', ahead of the subsistence level. Thus, population starts expanding until the new equilibrium, R, is reached. In the long run, the gains from technological progress are totally channelled to the population size.

Box 1.3. Key concept: Transition dynamics vs. change in the steady state

Both Figure 1.2 and Figure 1.3 describe how the Malthusian economy evolves along time until reaching the steady state. The adjustment process is called *transition dynamics*. There is however a critical distinction between the cases described in Figure 1.2 and in Figure 1.3: in Figure 1.2, the economy is initially out of the steady state (point P) and approaches the steady state (point R). In Figure 1.3, the steady state of the model has changed: hence, the economy engages in a transition dynamic from the former steady state (Q) to the new steady state (R).

1.2.5 Smith' mark of prosperity

The model just described reveals, in a simple manner, the dramatic implications of the LDR: for any given state of technology, the growth of the economy settles at a point where income per person is constant at the very low subsistence level. Formally, the equilibrium level of population is found by setting the level of per capita output (1.2) equal to the subsistence income, $\bar{y}$. Solving for *N*, one obtains:

$$N^* = \left(\frac{B}{\bar{y}}\right)^{\frac{1}{\beta}}\bar{T} \qquad (1.3)$$

This equation states that the equilibrium level of population increases with the availability of land and with the level of the technology. In other words, *population densities*, defined as the number of inhabitants per unit of available land (*N/T*), should be higher in countries with superior technology, B - note how well this prediction of the model fits with the Adam Smith claim quoted at the beginning of this chapter!

The prediction that cross-country differences in technology should give rise to differences in population density rather than to differences in living standards was investigated empirically by some authors. Among these, Robert Lucas Jr., from the University of Chicago, found that, prior to the Industrial Revolution, differences in living standards across regions in the world were indeed much smaller than differences in technology[5].

1.3 Population and technology

1.3.1 Population size and technological change

The Malthus theory implies that population expansion exerts a *negative* influence on living standards: given the extent of arable land and the level of technology, a larger population implies less output per person. One may argue, however, that the size of population also exerts a *positive* effect on living standards, through its influence on technological progress[6].

The reasoning is simple: If each person has a given probability of inventing something, all else equal, a larger and more *diverse* population should, in principle, be able to generate more inventions per unit of time than a society with fewer members. As William Petty, a 17th century economist and philosopher, once stated: "It is more likely that one ingenious curious

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[5] Lucas, R., 2004, The industrial revolution: past and future, 2003 Annual Report Essay, Federal Reserve Bank of Minneapolis.

[6] Pioneers on this idea include Kuznets, S., 1960, Population change and aggregate output, in A.J. Coale (ed.), *Demographic and economic change in developed countries*, Princeton University Press. Simon, J., 1977, *The Economics of Population Growth*, Princeton University Press. Simon, J., 1981, *The Ultimate Resource*, Princeton University Press.

man may rather be found among 4 million than 400 persons". On the other hand, unlike most other goods, technology is non-rivalrous: that is, an invention can be shared by many people without losing its effectiveness. If technology is free to spill over across the society, each agent will benefit from the discoveries of all others. Thus, a society with a larger population should enjoy faster technological change, just because a larger population has the potential to contain a greater number of potential inventors.

Arguably, the relationship between technology and the size of population is too complex to be described by a simple equation. But often economists need to rely on simple formulations just to make the point. A specification that became widely adopted in endogenous growth models is[7]:

$$\hat{B} = \frac{\dot{B}}{\dot{B}} = bN_t \,. \quad (1.4)$$

In this equation, the exogenous parameter *b* represents the likelihood of somebody inventing something. This formulation captures the idea that a larger population has the potential to generate a larger number of "ingenious curious man", and therefore to meet faster technological progress. It also assumes that technological change is a linear function of the existing stock of knowledge or "the standing on shoulders effect" (Box 1.4).

Assuming that technology evolves over time according to (1.4) has a natural implication for our model: *population and technological progress will reinforce each other*. A larger population will bring faster technological progress, and faster technological progress will cause population to expand faster.

Box 1.4. The "Standing on shoulders" effect

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[7] Romer, P., 1990. "Endogenous technological change". Journal of Political Economy 98, s71-s102. Grossman, G. and Helpman, 1991, H. Innovation and growth in the global economy. Cambridge, MA: MIT Press. Aghion, P. and Howitt, P., 1992, "A model of growth through creative destruction". Econometrica, 323-51.

An important property of knowledge is its cumulative nature: inventors develop new ideas learning from old ideas. As an example, consider James Watt's discovery of the steam engine in 1769. The idea was not inspired by the observation of steam rising from the spout of a kettle, as they say. The idea came about while James Watt was repairing an earlier steam engine invented 57 years earlier by Thomas Newcomen. The later, in turn, was an improvement of a steam engine patented in 1698 by the Englishman Thomas Savery, which followed another designed by the Frenchman Denis Papin around 1680, which in turn had precursors in the ideas of the Dutchman Christiaan Huygens, and so on[8].

The cumulative nature of knowledge was coined the "standing on shoulders" effect, owing this name to a famous quotation from Isaac Newton ("If I have been able to see further, it was only because I stood on the shoulders of giants"). In term of equation (1.4) it is implied by the assumption that technological change $\dot{B}$ is a linear function of the technological level. B.

1.3.2 Path dependence and initial conditions

The possibility of population and technology reinforcing each other opens a channel for *path dependence* in economic development: the state of technology in given territory at a given moment in time is determined by the size of population in the period before, which in turn was determined by past technology, and so on, until the beginning of times. That being so, the initial conditions (e.g. natural resources, climate) should have a played a key role in explaining cross-regional disparities of per capita income along time.

To illustrate this idea, consider two regions, say Q and R, completely isolated from each other, so that any technological improvements in one region (change in B) could only spill over within that region' borders. Further assume that these two regions differed only in terms of the

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[8] Diamond, J., 1998. *Guns, Germs and Steel: a short history of everybody for the last 13,000 years*. Vintage, Surrey, UK.

quality of the natural resources at the very beginning: that is, technology was the same, but parameter T was larger in region R. In term of figure 1.3, region Q would be described by the dashed curve and region R by the solid.

Once the initial geographical conditions determined different carrying capacities, the Malthusian mechanism of population dynamics would imply a larger population in region R (N_1^*) than in region Q (N_0^*). Then, because region R had a larger population, an asymmetric path of technological change would be triggered: region R should have a higher chance of achieving technological progress just because the pool of human beings was larger, which then would bring more population and faster innovation, in a virtuous cycle.

It can be rightly argued that the assumption that regions are completely isolated from each other does not square well with global reality: nowadays, knowledge has the potential to cross borders, so a larger population will not be needed to enjoy technological progress. People in Luxembourg, for example, can enjoy the benefits of technology developed in the United States, just as people in the United States benefit from technological developments in Luxembourg. In other words, what matters for technological expansion should not be the size of the population of each country or region, but rather the total population of a set of interdependent countries or regions. Thus, at the best, the model should be considered valid only for the global economy.

However, for a long period of time in human history, until the 15th century, some regions were in fact completely isolated from each other and, therefore, unable to exchange knowledge. Box 1.5 shows how Harvard's Michale Kremer used this period as a historical experiment to demonstrate that the theory was basically right: regions of the world that started out with a higher carrying capacity ended up having a more advanced technological level.

Box 1.5. Technology and population density: an historical experiment

The Malthusian prediction that the level of technology impacts positively on the size of its population was investigated by Michael Kremer, from Harvard[9]. Kremer augmented the Malthusian model with the hypothesis of causality from population to technology: he argued that regions with larger populations should exhibit faster technological progress than regions with smaller populations. The reason is intuitive: if the probability of inventing something is the same for any single person, then a region with more inhabitants should, in principle, be better endowed to generate ideas and enjoy fast technological progress than a region with less inhabitants.

A problem in testing empirically that theory is that technology does not recognizes borders: if ideas diffuse freely across the space, smaller regions should be able to free ride on the larger regions' discoveries and catch up independently of their population sizes. To rule out such possibility, Kremer focused on particular era of the human history, where populations in different areas were effectively isolated from each other. This period provides a "natural experiment"[10].

The author noted that, before the end of the last ice age (about 10.000 B.C.) ocean levels were so low that humans could easily migrate across continents, including through the Bering Strait, which connects Asia to the Americas. Hence, at that time, technology had the potential to diffuse across regions. It is thus plausible to assume that – say - by 12.000 B.C., the known technologies were similar across all humanity (note that in those times human were basically hunter-gatherers).

With the melting of the polar ice caps, around 10,000 B.C, land bridges were flooded. In consequence, the Old World (Europe, Asia, Africa), the Americas, Australia, Tasmania, and the Flinders Island became isolated from each other. If the theoretical model was right, one

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[9] Kremer, M. 1993. "Population growth and technological change: one million B.C. to 1990". Quarterly Journal of Economics, 108, 681-716.

[10] Natural experiments are studies where the experimenter cannot manipulate the independent variable to access its effect on the dependent variable, but it can observe the effects of exogenous changes in the independent variable.

would expect that, at the time connections were re-established - with the European explorations of the 15th century - technological levels, population densities and land sizes were all positively correlated. Why? Because larger regions would have built bigger populations and therefore technology would have developed and diffuse quicker, causing in turn faster population expansions.

Kremer showed that the historical facts confirm these conjectures. By the year 1500, population densities where much higher in Eurasia-Africa (4.85/km2) - the region with larger area - than in the Americas (0,36/Km2), Australia (0.026/Km2), Tasmania (0,018/Km2 to 0.074/Km2) and the Flinder Island (0,0/Km2). Accordingly, the Old World had the highest level of technological sophistication, followed by the Americas (the Aztec and the Mayan had already discovered agriculture). Australia was in an intermediate stage, having developed some artefacts like the boomerang, but with a population that was still hunter-gatherer. Tasmania registered technological regression: its inhabitants lacked basic tools such as fire-making and lost the ability to make bone tools. Finally, the Flinders Island, with 680 square kilometres of land and only 500 inhabitants initially, lost all its inhabitants by around 4,700 BC.

1.3.3 Race between technology and diminishing returns

The claim that technological improvements impact only on the size of population without improving living standards does not square well with the modern facts of economic development. In today's world, societies are continuously exposed to technological progress and yet living standards are improving rather than returning to a subsistence level. This means that at some stage in history per capita income and population began to grow together, departing from the Malthusian model.

To understand the transition, let's refer again to figure 1.3. In the figure, the improvement in living standards is only temporary because it is given time enough for population to increase and fully match the new "carrying capacity" of the economy, at point R. But now suppose that a second invention tilted the production function while population was still on its way from P to R. And again, before the new equilibrium was reached, a third technological change took place, and so on. Clearly, if the economy was continuously hit by technological improvements and population expansion was slow enough, then per capita income would never fall back to the subsistence level, even if population was expanding according to the Malthusian rule. The economy would be permanently engaged in transition dynamics, with per capita income increasing over time.

Along this reasoning, whether continuous technological progress will be capable of driving per capita income away from the subsistence level or not, it depends on: (a) the pace of technological progress; and (b) how fast the population will respond to changes in living standards. Formally, the change in per capita income is obtained log-differentiating (1.2):

$$\hat{y}_t = \hat{B}_t - \beta n_t \quad , \quad (1.5)$$

where $\hat{y} = \dot{y}/y$, $\hat{B} = \dot{B}/B$, and $n = \dot{N}/N$ refer to the rates of change of per capita income, productivity and of population, respectively. Equation (1.5) describes the change in per capita income as a *race between technological change and diminishing returns*.

1.3.4 Escaping the trap

Figure 1.4 plots the evolution of per capita income and population in the world from year 1 to 2000. As the figure shows, the two variables expanded very slowly over a long period of time. Arguably, the world economy was in the Malthusian regime, with technological progress so slow that population had time to fully adapt to technological change. During this period the change in per capita income was approximately zero, even though technological change was accelerating[11]. However, at a certain point, technological progress began to win the race with increasing returns: per capita income entered an upward trend without any tendency to return to the subsistence level.

This pattern has a natural interpretation in terms of the ideas sketched out above. Thousands of years ago, human societies were dispersed in small bands, with little contact with

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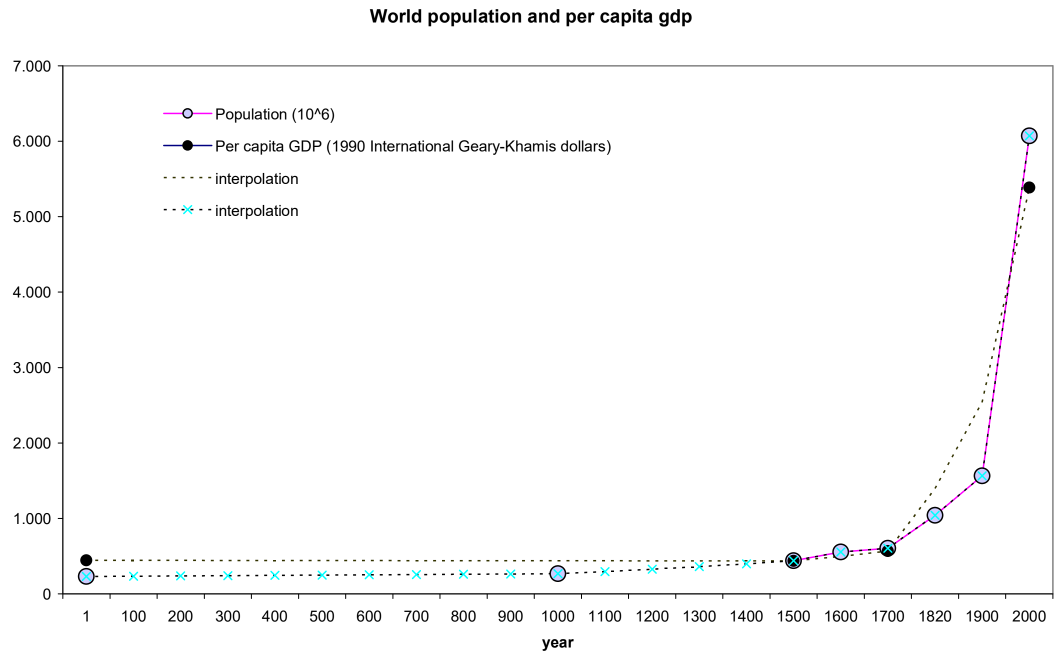
[11] In light of (1.5), population should have expanded during the Malthusian era according to $n_t = \hat{B}_t / \beta$. Kremer (op. cit) combined this equation with the model of technological change (1.5) to contend that during the Malthusian era population growth evolved *explosively* as a positive function of the size of population, that is $n_t = (b/\beta) N_t$. The author found that this equation fits well with the data for most human history..

each other. Since the arrival of new ideas was slow, populations had time to expand and fully match the technological change. New ideas translated to larger populations, but not to significant improvements in living standards. At some stage, when populations became large enough, the arrival rate of new ideas accelerated so much that productivity growth outpaced population growth: in the figure, this is captured by the increase in per capita income after the 15th century. At that time, it looks like *population lost the race* with technology.

Figure 1.4 Population and per capita GDP over the last two thousand years



The figure displays the evolution of per capita income and of population along the last twenty centuries. Source: Angus Maddison, 2001. The World Economy: a Millennial Perspective. Development Centre, Paris.

Box 1.6: The case with non-renewable natural resources

The race between technology and diminishing returns can be extended to account for the possibility of natural resources (T) eroding over time. So far, we have assumed that the availability of land is constant. Arguably, any eventual erosion on the quality of the arable land could be fixed with simple techniques, such as rotation of cultures. But if T is thought to include non-renewable natural resources (those that are depleted when used in production such as oil, natural gas, and climate), then T shall me modelled as decreasing over time. Defining $\delta = \dot{T}/T$ as the rate of erosion of the natural resources, then equation (1.5) becomes:

$$\hat{y}_t = \hat{B}_t - \beta\left(n_t + \delta_t\right) > 0 \qquad (1.5a)$$

Equation (1.5a) extends (1.5) requiring the pace of technological progress to be fast enough not only to overcome the population expansion but also the erosion of natural resources along time. This version of the model is much more relevant in our days, where new challenges are arising, namely excess fishery, deforestation, and carbon emissions. Optimistic say that these restrictions will not challenge our living standards, because they will be mitigated by technological progress.

1.3.5 Pitfalls on the relationship between technology and population

Modelling the pace of technological progress as a function of the size of population is not an easy task. First, because technology is something that we don’t know how to measure: shall we count ideas? Shall all ideas be counted as valuing the same? If not, how to measure each idea? Second, there is an element of risk: researchers may not succeed in inventing something. Third, the likelihood of an agent discovering something may depend on the existing stock of already invented ideas: are previous ideas helpful for new discoveries? Or will new discoveries become more difficult as the number of ideas already discovered increases? Fourth, the positive relationship between population and technological progress may be mitigated by the possibility of overlapping efforts by independent inventors. Finally, even if the key ingredients of a “knowledge production function” were well known, a question remained as to the choice of its functional form: shall output knowledge vary linearly with each ingredient or shall it exhibit diminishing returns? That is, to sustain a given rate of technological progress will it be sufficient to have a constant population or instead an increasing pool of potential ingenious men over time?

All these questions mean that the relationship between technology and the size of population is too complex to be described by a simple equation. And yet, the shape of such a “technology production function” is an essential ingredient in our model. With no surprise, the choice of an appropriate specification for the relationship between technology and the size of population became a matter of dispute in the research arena.

Charles Jones criticized specification (1.4) on the grounds that it implies that the growth rate of technology is a positive function of population size[12]: With a growing population ($n_t > 0$), that would imply an ever-accelerating rate of technological expansion, a problem that became known as "the scale effect".

To fix get rid of the scale effect, Jones removed the linearity implied by the "standing on shoulders effect" from the production function of knowledge, assuming that new ideas are increasingly more difficult do discover, a mechanism that he coined as the "Fishing out effect" (Box 1.7):

$$\dot{B} = bN_t^{\theta} B_t^{\varepsilon} \quad \text{, with } 0<\varepsilon<1,\ 0<\theta<1 \qquad (1.6)$$

In this formulation, the sign and magnitude of parameter ε captures the net balance of two opposing externalities on productivity growth: the "standing on shoulders" effect (positive), whereby productivity of current research increases with the accumulated knowledge in the society and the "fishing out effect" (negative) whereby past discoveries turn new ideas more difficult. Jones conjectured that the net effect of these two externalities leads to $0<\varepsilon<1$: that is, new researchers benefit from previous ideas, but there are diminishing returns to knowledge in knowledge production.

Another novelty in Jones's formulation is that it accounts for overlapping discoveries: This happens when the same piece of knowledge is invented independently by different inventors. This waste of time is accounted for $\theta<1$ in (1.6) (that is, doubling the number of researchers less than doubles the production of new ideas). This negative externality - labelled "stepping on shoes" - weakens the relationship between population size and technological change.

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[12] Jones, C., 1995. "Time series tests of endogenous growth models", Quarterly Journal of Economics, 110, 495-525.

When the knowledge production function takes the form (1.6), the rate of technological progress becomes:

$$\hat{B} = \frac{\dot{B}}{B} = bN_t^{\theta} B_t^{\varepsilon - 1} . \qquad (1.7).$$

With $0<\varepsilon<1$, equation (1.7) implies a negative relationship between the growth rate of B and the level of B. This means that the model converges to a steady state, with a constant rate of technological progress, $\hat{B}$. The steady state growth rate may be obtained by log-differentiating both sides of (1.7) and setting the result equal to zero. This gives:

$$\frac{\dot{B}}{B} = \theta n / (1-\varepsilon) \quad . \qquad (1.8)$$

In (1.8), the growth rate of technology is not a direct function of the population size, so the scale effect is removed. Still, a *weak scale effect* shows up: the growth rate of per capita income is a direct function of the population growth rate. This is different however from the original *strong* scale effect whereby the growth rate of technology was a function of the size of the population.

In (1.8), population growth is *necessary* to achieve sustained technological progress: the assumption that new ideas become increasingly difficult to discover implies that the growth rate of B falls to zero over time when the population is constant (in other words, once linearity is removed from the knowledge production function, a constant population will no longer be sufficient to sustain the continuing proportional increase in the stock of knowledge). Thus, only with an ever-increasing population will be possible to maintain a constant rate of technological progress.

The proposition that the rate of technological progress depends on population growth does not fit well with cross-country evidence: we do not observe a general tendency for larger economies to experiment faster technological progress than smaller economies. The idea can however be rescued considering that it applies to the World economy, and not to a single country: in the global economy, all countries have the opportunity to benefit from a common pool of technological progress. Thus, for each country, the size of its population does not matter: what matters is the size of the global population, which has been increasing over time. Given that technology has improved along with the size of world’s population, the claim that the growth rate of technology depends on the growth rate of population is not easy to refute.

Box 1.7. The "Fishing out" effect

In the 18th and 19th centuries, many breakthrough inventions were achieved by hobbyists or by single individuals. Thomas Edison, for instance, invented alone the light bulb, the phonograph, and the motion picture. For today's standards, such achievements are impressive. In today's world advances in technology are mostly achieved by scientists engaged in research teams and focusing on very narrow problems.

This pattern suggests that new discoveries are increasingly difficult to find. That is, as the stock of accumulated knowledge increases, researchers will find it more difficult to invent new technologies, because the easiest ideas have already been discovered. Then, as technology becomes more complex, it takes more time and effort for new researchers to learn everything they need just to catch up with cutting hedge.

This idea was coined as the Fishing out effect". The label "fishing out" arises from the classical example of the fishing pound for the Tragedy of the Commons: if the pound is stocked with a *fixed* number of fish, then it becomes increasingly difficult to catch each new fish.

1.4 The demographic transition

According to the Malthus theory of population, a higher per capita income should come along with faster population expansion. This proposition looks like squaring well with the real-world facts before the industrial era, but it no longer applies to *modern societies*: in our days, wealthier people tend to have less children, not more. So, a question arises as why there has been such a change in humans' attitude towards fertility. This section briefly reviews the theories of demographic transition, establishing a bridge between the Malthusian theory of population, which applies to pre-industrial societies, and theories focusing on modern economic growth. The discussion will help understand why some countries departed from the Malthusian regime sooner than others giving rise to increasing disparities in the cross-country distribution of per capita income.

1.4.1 From the Malthusian regime to modern growth

Table 1.1 describes the evolution of GDP, population, and per capita income in West Europe along that last twenty centuries. The table also displays a rough estimate of *technological progress* $\dot{B}/B$ in line (4), using equation (1.5) and postulating a value for β equal to 1/3 (details in the table).

According to the table, output and population evolved very slowly in Western Europe from year 1 to 1.000 (lines 1 and 2), reflecting an almost stagnant technology (line 4). During this period, technological change was fully matched by population expansion. This phase can be viewed as roughly in line with the Malthusian model.

From 1000 to 1820 technological progress was slow but steady. During this period, per capita income was increasing over time, meaning that population was already losing the race with technology. This pattern was reinforced between 1820 and 1900, with both productivity and population accelerating significantly, but with the proportion of output growth that was matched by increasing population declining sharply (line 5)[13].

Finally, the positive relationship between per capita income and population growth vanished in the twentieth century: after 1990, population growth rates in Europe started declining, even though GDP per capita was growing faster than ever. This means that the Malthusian theory of population was no longer applying. Europe was already engaged in *modern economic growth*.

Table 1.1. GDP, Population and per Capita GDP, 1-2000

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[13] Galor and Weil (2000) coined this intermediate stage, where the Malthusian (positive) relationship between population and per capita income still holds, but technology takes a clear lead in the race with population as "the post-Malthusian regime". Galor, O. and Weil, D., 2000. "Population, Technology and Growth: From Malthusian Stagnation to the Demographic Transition and Beyond". The American Economic Review 90(4), 806-828

	1	**1000**	**1500**	**1600**	**1700**	**1820**	**1900**	**1960**	**2000**
29 Western European Countries									
(1) GDP									
Billions of 1990 International Geary-Khamis dollars	11	10	44	66	81	160	676	2,251	7,430
Growth Rate (% per annun)		*-0.01*	*0.29*	*0.40*	*0.21*	*0.57*	*1.82*	*2.02*	*3.03*
(2) Population									
Millions	25	25	57	74	81	133	234	326	391
Growth Rate (% per annun)		*0.00*	*0.16*	*0.25*	*0.10*	*0.41*	*0.71*	*0.56*	*0.45*
(3) Per Capita GDP									
1990 International Geary-Khamis dollars	450	400	771	890	998	1,204	2,893	6,896	19,002
Growth Rate (% per annun)		*-0.01*	*0.13*	*0.14*	*0.11*	*0.16*	*1.10*	*1.46*	*2.57*
Memo:									
(4) Total Factor Productivity (% per annun)		*-0.01*	*0.19*	*0.23*	*0.15*	*0.29*	*1.35*	*1.65*	*2.73*
(5) Population growth divided by GDP growth			0.55	0.64	0.46	0.72	0.39	0.28	0.15

Source: (1) and (2): Maddison (2001), op. cit.. (3)=(1)/(2). Total Factor Productivity growth (4) is a measure of technological progress and is computed as a residual using $\dot{B}/B = \dot{y}/y + \beta(\dot{N}/N)$, and postulating β=1/3, that is: (4)=(3)+(1/3)*(2). Note that, since only labour is accounted for in this decomposition, the term B captures the contribution of all other eventual inputs to production.

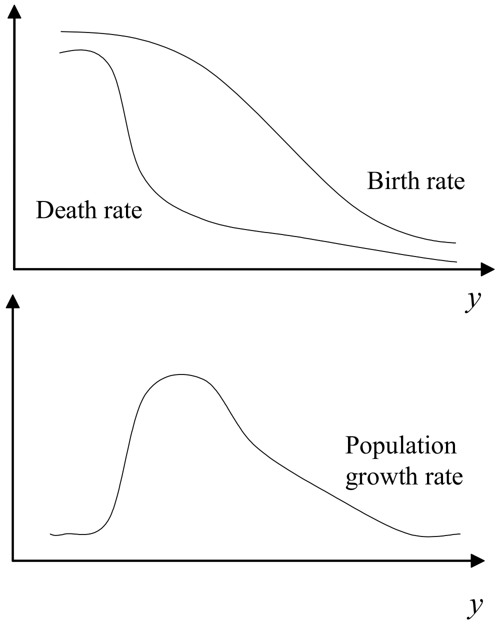
1.4.2 Birth rates and death rates

A question that naturally arises is what fundamental changes have occurred after the Industrial Revolution to reverse the relationship between per capita income and population, tilting economies towards the Modern Growth Regime. The process by which a country' demographic characteristics are transformed as it develops is labelled "demographic transition". To understand the process of demographic transition, it is useful to introduce two basic demographic indicators: the birth rate and the death rate. The birth rate is defined as the number of new-borns each year per thousand of inhabitants. The death rate is defined as the number of people that die each year per thousand of inhabitants. The difference between birth rates and death rates is the growth rate of population.

Figure 1.5 describes the stylized facts of the demographic transition in terms of these basic indicators. The first stylized fact is that death rates decline monotonically with economic development. The explanation is straightforward: when per capita income is very low, death rates are high, especially among children, reflecting malnourishment, deficient sanitation, and disease. When income increases, better nutrition, improvements in housing, public health, modern sewage, clean water, etc. cause death rates to decline.

The second stylized fact is that the decline in death rates is not accompanied with an equally fast decline in the birth rate: there is an intermediate stage, where the gap between birth rates and death rates widens, giving rise to an increase in the growth rate of population (bottom panel)[14]. At the later stage of development, the birth rate catches the death rate, and the growth rate of population declines again.

Figure 1.5. The stylized facts of demographic transition



The upper panel displays the stylised relationship between birth rates and death rates and per capita income. The lower panel displays the relationship between the rate of population growth (given by the difference between birth rates and death rates) and per capita income. According to this stylized view, the fact that birth rates take more time to decline than death rates implies that, in the intermediate stage, the population growth rate accelerates.

Since death rates decline with economic development, they are not a good candidate to explain why the Malthusian relationship between income and population growth vanishes in the Modern Growth Regime: the explanation for the demographic transition must lie on *birth rates*: the decline in birth rates with economic development is the most important feature of the demographic transition.

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[14] In England, for instance, the decline in mortality rate preceded that of the birth rate by 140 years. Coale, A., Treadway, R., 1986. A summary of the changing distribution of overall fertility, marital fertility, and the proportion married in the provinces of Europe. In: Coale, A., Watkins, S. (eds), The Decline of Fertility in Europe. Princeton University Press. Princeton.

Recent theories aiming to understand the decline in birth rates have investigated the *economic incentives* underlying fertility choices[15]. A common aspect of these theories is that parents decide the number of offspring according to some optimal criteria. As in any economic model, that choice is formulated in terms of benefits and costs. Below, we briefly review some of these ideas.

1.4.3 Missing institutions and risk premium

In primitive societies, social security and financial institutions were absent. People were therefore more reliant on their own children to obtain protection at the old age or in case of misfortune. Thus, just like in our times people buy financial assets to transfer income across time or states of nature, in traditional societies people "invested" in children as a form of saving and insurance.

Of course, the "investment in children" was not absent of risk: children could die, migrate, or fail to generate a decent income. In response, parents in the Malthusian regime tended to rise more children than they would need, as a matter of caution. Through this "risk premium effect", death rates played an *indirect influence* on birth rates: when mortality rates were high, parents decided to have more children to increase the chance of reaching a minimum number of survivors; when mortality rates declined the risk premium in terms of excess births also declined, helping to explain the demographic transition.

In modern societies, the "asset role" of children has vanished. Economic development comes along with institutions that specialize in covering risks, such as insurance and social security, and in protection at the old age, such as pension funds. When all these institutions are

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[15] Becker, G., 1960. An economic analysis of fertility. In Easterlin, R. (ed), Demographic and economic change in developed countries. Universities-National Bureau Conference Series nº 11, Princeton: Princeton University Press, pp. 209-40. Becker, G., Murphy, K and Tamura, R., 1990. "Human Capital, fertility and economic growth". Journal of Political Economy 98 (5): S12-S37. Schultz, T., 1997. The demand for children in low income countries. In Rosenzweig, M., Stark, O. (eds.) The Handbook of Population and family Economics, Amsterdam Elsevier.

available, children will be an expensive (dominated) substitute. Not surprisingly people responded to financial development reducing the number of offspring.

A similar argument shows up in terms of the defence needs in small communities. Prior to the modern growth regime, a small group of people could not defend a large territory from outside enemies. Hence, societies had to choose between lower per capita income (due to diminishing returns) and the risks of being invaded. Social norms then emerged, creating incentives for populations to expand considering the defence needs. In modern regimes, the emergence of unified states shifted the burden of defending the territory from local communities to the central government. In response social norms in local communities evolved to prioritize labour productivity[16].

1.4.4 Income and substituting effects

A theory of fertility alternative to the "asset view" assumes that parents derive "intrinsic pleasure" in rearing children[17]. In this approach, children enter in the households' utility function as normal goods. Hence, when parents' income increases, everything else constant, fertility increases. However, fertility also depends on the cost of rearing children: children need to be fed, clothed, and schooled. When these costs increase relative to other goods, the demand for children declines, all else constant.

In light of this approach technological progress has impacted on fertility decisions through two different channels:

- On one hand, it has eased the household's budget constraints, allowing parents to spend more resources in raising children (positive income effect). This is the pure Malthusian mechanism.

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[16] Parente, S. and Prescott, P. , 2005. A unified theory of the evolution of international income levels" in Aghion, P., and Durlauf, S. (eds), Handbook of Economic Growth, Volume 1, Chapter 21, pp. 1371-1416, Elsevier.

[17] Becker, G., 1981. A Treatise on the Family. Harvard University Press.

- On the other hand, it has increased the *cost of rearing children* (negative substitution effect).

The increase in the relative costs of rearing children came from two sources:

First, looking after children consumes parents' time, and time involves an opportunity cost. In traditional societies, where job opportunities for women were scarce and social norms dictated that women should stay at home, the opportunity cost of upbringing children was naturally low. Along the development process, women became more progressively more engaged in the labour force, and the opportunity cost of raising children has increased.

Second, children must be schooled, and education requirement have increased along the development process. In the Middle Ages, people were basically devoted to agriculture, using simple technologies. At that time, children could learn by helping their parents and observing what their parents were doing. In these societies, the returns to schooling were low. In plus, in an environment plagued by high infant mortality, parents would be naturally reluctant in spending valuable resources to educate a single child: they would rather invest in *quantity*: whenever a productivity improvement eased the household budget constraints, parents would tend have more children. In the Modern Growth Regime, in contrast, the knowledge required to operate complex machinery cannot be acquired observing what parents are doing. Technological sophistication creates a demand for technical skills, raising the returns to formal education. At the same time, child labour became less attractive and socially banished. In response, parents tend to invest more in children *quality*, instead as on quantity. It shall not be surprising that the departure from the Malthusian relationship between income and fertility at the turn of the 19th century in Western Europe was accompanied by an increase in the average number of years of schooling.

This reasoning provides an interpretation for the demographic transition based on the changing balance between the income and substitution effects along time: in earlier stages of development, the first effect dominated, so population growth responded positively to the income generated by technological change, as Malthus had predicted. However, the arrival of more demanding technologies gradually changed the balance between the two effects: with returns to human capital increasing, parents gradually shifted from child quantity to child quality. Then, more educated people become more likely to develop and adopt new technologies, accelerating the pace of technological progress in a virtuous cycle. At a certain stage, returns to education become so high that the substitution effect became dominant in fertility decisions, explaining the entry in the Modern Growth regime[18].

1.4.5 Why are birth rates so persistent?

A stylised fact of the demographic transition is that birth rates are more persistent than death rates, favouring demographic explosions (figure 1.5). In the literature, various explanations have been proposed for this phenomenon:

First, *economic incentives* for birth rates to decline may arrive with a lag relative to the initial improvements in living standards that cause the death rates to fall in the first place. According to this view, the arrival of a welfare state, the deepening of financial markets, the integration of women in the labour force, the raise in the value of education tend to materialize only after critical improvements in nutrition and in health care take place.

Second, family level fertility decisions are not entirely driven by private considerations: fertility decisions are also influenced by the need to conform with *social norms*: if societies demand families to give a large number of births, families desiring to conform to what is socially acceptable will refrain from reducing fertility, even if there are economic incentives to do so. Social norms evolve along time in response to economic incentives, but this process is inherently slow.

Third, the demographic dynamics is slow by nature, because it depends on the age structure of population, which is pre-determined each moment in time. To better understand this, note that birth rates (number of new-borns each year per thousand of inhabitants) are jointly determined by *fertility rates* (number of children per women in the reproductive age) and the structure of population (the percentage of women in the reproductive age per thousand

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[18] Galor and Weil (2000), op. cit.

inhabitants). So, even if fertility rates start to decline, the birth rate may remain high for generations just because there are still many girls entering childbearing age.

The implication for the demographic transition is straightforward: suppose that a given economy starts out with high birth rates and high death rates. Then, suddenly living standards improve causing infant mortality to decline. This means that more babies will survive childhood, causing the age structure of the population to change. With a younger age structure, the number of potential mothers in the future exceeds the existing number of mothers today. Thus, even if each new mother decides to have less offspring (say, responding to the low risk of child mortality), the economy' birth rate will not decline immediately because more women are still entering in the reproductive age. This phenomenon is known as the *population momentum*: whatever a country does, the future growth rate of the population is largely determined by the existing age structure, and this takes generations to change.

Box 1.8: Lant Pritchett and the Theory of States and Transitions

Along this chapter, we referred to a Malthusian Regime and a Modern Growth Regime. We distinguished the different attitudes towards fertility across in these two regimes and we outlined some theories attempting to explain the conditions under which a society can move from one regime to the other (Demographic Transition).

Lant Prichett (2006) offers a nice analogy for this way of structuring our thinking[19]:

“Suppose you have a pot of water, and you pick it up and turn it over. Where will the water go? The answer, that it will spill out onto the ground, is so obvious that the astute reader already realizes it is a trick question. If the water is frozen, it may stay right in the pot. If the water is vapor, then turning the pot over will trap the steam in the pot. The obvious point is that the equations of motion of water (or any other substance) depend on the state—solid, liquid, or vapor—it is in. What determines the transitions of water between states? Well, applying heat

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[19] Pritchett, L. 2006, The Quest Continues, Finance and Development, March.

will cause water to change states, but only in a discontinuous way—water at 35° F and water at 95° F behave almost the same, while water at 32° F and at 102° F behave nothing like each other. The equations of motion of water in one state do not work at all when water is in another state, and the response of water to heat applied within a state does not work at all well when applied to transitions from one phase to another".

Likewise – Prichett argues – "If France and Nepal can both be treated as water in a liquid state, then it is conceivable that a theory and empirics of growth that treat France and Nepal as both generic countries is adequate. I regard it as much more likely that growth dynamics are characterized as equations of motions within states and equations that determine transitions across states" (…). "The key idea in my proposal is that economies are in different "states," and, therefore, the dynamics of output are different for economies in different states, and the dynamics of transitions between states are different from the dynamics within states".

1.5 Globalization, fertility decisions and the Great Divergence

In the section above, we saw that under certain conditions, societies engage in a demographic transition, whereby parents' attitudes towards fertility shift from quantity to quality. In the modern growth regime, the relationship between per capita income and the number of children is reversed, allowing living standards to improve along with technological progress. Moreover, as parents move towards child quality, the population becomes more educated and therefore more likely to experiment fast technological progress.

In the World economic history, we observe that the timing of demographic transitions differed considerably across countries and regions. At the time the Western nations was entering in Modern Growth, laggard regions were still engaged in Malthusian stages, meeting fast population expansions and sluggish per capita income. The fact that different countries engaged in demographic transitions at different timings helps explain the dramatic increase in cross-country income disparities that characterized the 19th and 20th century, a phenomenon that was coined "Big divergence". A question that naturally arises is why some countries entered in the modern growth regime sooner than others. This section reviews some theories that have been proposed to explain these historical facts.

1.5.1 The Great Divergence

Along the 19th and 20th centuries, there was a dramatic divergence in living standards across the globe. Lant Pritchett, from Harvard University, dubbed this period as of "Divergence, Big Time"[20]. The author observed that between 1870 and 1994, one small set of countries - consisting in 12 West European countries plus 4 Western offshoots (United States, Canada, Australia and New Zealand) and Japan - managed to sustain fast economic growth, leaving the remaining regions behind.

Table 1.2: The Great Divergence

	Per Capita GDP (1990 International Geary-Khamis dollars)								Average growth rates:			
	1	1000	1500	1600	1700	1820	1960	2000	1000-1700	1700-1820	1820-1960	1960-2000
Western Europe	450	400	771	890	998	1.204	6.896	19.002	0,13	0,16	1,25	2,57
Western Offshoots	400	400	400	400	476	1.202	10.961	27.065	0,02	0,77	1,59	2,29
Latin America	400	400				692	3.133	5.838			1,08	1,57
Former USSR	400	400	499	552	610	688	3.945	4.351	0,06	0,10	1,26	0,24
7 East European Countries	400	400	496	548	606	683	3.070	5.804	0,06	0,10	1,08	1,61
Asia	449	449	568	572	571	581	1.029	3.817	0,03	0,01	0,41	3,33
16 Asian countries						581	962	3.794			0,36	3,49
26 East Asian countries						556	862	1.467			0,31	1,34
15 West Asian countries						607	2.492	5.706			1,01	2,09
Africa	430	425	414	422	421	420	1.066	1.464	0,00	0,00	0,67	0,80
World	445	436	566	595	615	667	2.777	6.012	0,05	0,07	1,02	1,95
Western Europe/ Africa	1,0	0,9	1,9	2,1	2,4	2,9	6,5	13,0				

Source: Maddison, 2001.

Notes: Western Offshoots: Australia , New Zealand, Canada, United States; 7 East European Countries: Albania, Bulgaria, Czechoslovakia, Hungary, Poland, Romania, Yugoslavia; 16 Asian countries: China, India, Indonesia, Japan, Philippines, South Korea, Thailand, Taiwan, Bangladesh, Burma, Hong Kong, Malaysia, Nepal, Pakistan, Singapore, Sri Lanka; 26 East Asian countries: Afghanistan, Cambodia, Laos, Mongolia , North Korea, Vietnam, and 20 other Small Asian Countries; 15 West Asian countries: Bahrain, Iran, Iraq , Israel , Jordan , Kuwait , Lebanon , Oman, Qatar, South Arabia , Syria , Turkey , United Arab Emirates , Yemen , Palestine and Gaza.

Table 1.2 illustrates the Great Divergence. The table describes the evolution of per capita incomes in some regions of the world between year 1 and year 2000. According to this data, between year 1 and year 1000, per capita income disparities remained relatively small. Starting in the 10th century, per capita income in Western Europe decoupled from those in the other regions. Still, by 1700, the ratio of per capita incomes between Western Europe and Africa was 2.4, only. Between 1820 and 2000, regional income disparities increased dramatically: per capita GDP increased 23-fold in the Western Offshoots and only 3-fold in

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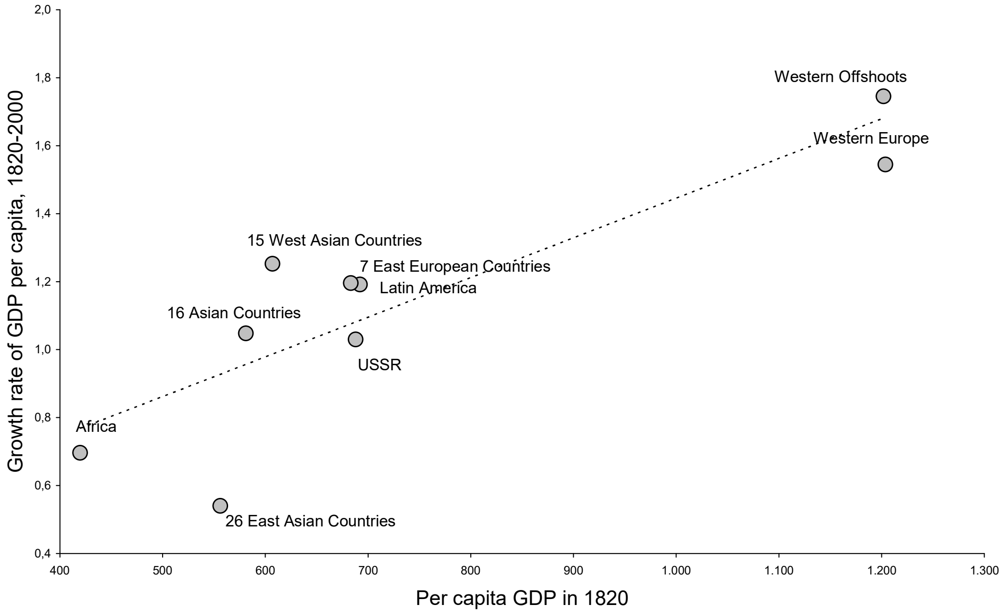
[20] Pritchett, L., 1997. "Divergence, Big Time". Journal of Economic Perspectives 11(3), 3-17.

Africa. By the year 2000, per capita income in Western Europe was 13 times higher than in Africa.

Figure 1.6 provides a graphical illustration of the Great Divergence. The figure relates the growth rates of per capita GDP to the initial levels of per capita GDP, for the period 1820-2000 (the data is the same as in Table 1.2). The positive correlation between growth and per capita incomes indicates that, along this period, initially rich countries tended to grow faster than poor countries.

The Great Divergence has vanished along the last decades. Starting in the second half of the twentieth century, a set of highly populated economies in Asia managed to accelerate their rates of economic growth, above the levels observed in the developed world. As shown in Table 1.2, between 1960 and 2000, Asia expanded much faster than Europe and the Western Offshoots. Given the population size of the converging countries in Asia, the global picture became of convergence.

Figure 1.6 The Great Divergence



Source: same as Table 1.2.

1.5.2 The timings of the demographic transitions

Figure 1.7 compares the population growth rates in the different regions of the world along the last three centuries. As shown in the figure, population growth rates started declining in the Western Offshoots in the 19th century, followed by Europe in the early 20th century, and then by Asia and Latin America in the last quarter of the 20th century. In Africa, population growth rates were still increasing by the end of the 20th century.

This evidence suggests that the timing of demographic transition differed considerably across the regions in the World. Within each region, there was also significant heterogeneity. For instance, Mexico started the transition to modern economic growth during the first half of the 19th century; Japan initiated the transition in the second half of the 19th century; Brazil started in the early twentieth century, and India started its transition sometime between 1950 and 1980[21].

A characteristic of the latest demographic transitions is that they have been much faster than they had been in Europe. In the 19th century Europe, birth and death rates fell gradually, accompanying the slow progresses in medicine and in sanitation. Birth rates declined later than death rates, but without creating significant population explosions. In more recent transition processes, the fall in death rates tended to be more abrupt, whenever the local conditions favored the adoption of healthcare and sanitary practices already discovered elsewhere. In result, more recent demographic transitions have been more painful in terms of population explosions.

Figure 1.7.- Population growth along 1700-2000

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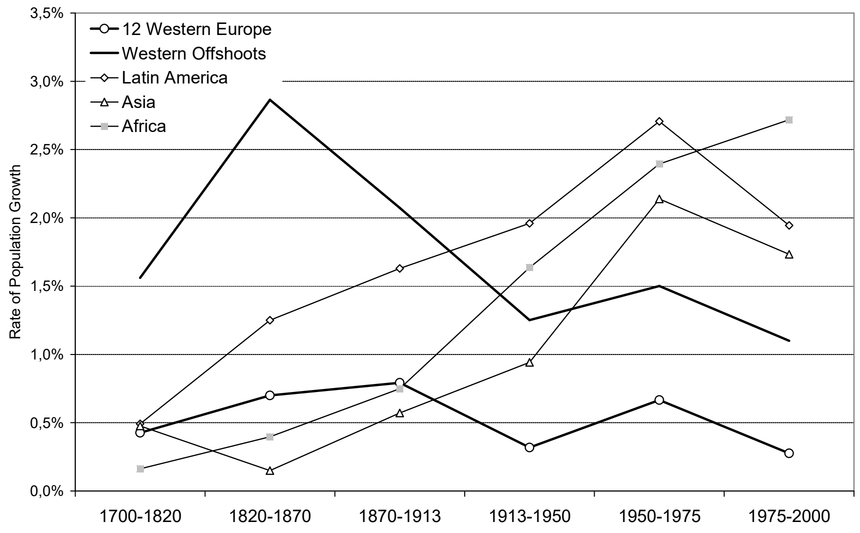
[21] Parente and Prescott (2005), op. cit.



Source: Maddison (2001), op cit. Notes: The 12 Western European Countries are Austria, Belgium, Denmark, Finland, France, Germany, Italy, Netherlands, Norway , Sweden, Switzerland , United Kingdom. The Western Offshoots are Australia, New Zealand, Canada and the United States.

1.5.3 Industrialization and the demographic transition

A stylized fact of economic development is that the entry in Modern Growth Regime has been associated to a reallocation of resources from agriculture to manufactures. In the cases of Western Europe and the Western Offshoots (Australia, Canada, New Zealand, United States), this coincided with the Industrial Revolution, which began at the end of 18th century in England and spread to other countries along the 19th century. Other countries that managed to catch up also experienced fast industrialization (for instance, Japan, South Korea, and Singapore)[22].

The move towards manufactures has played a key role in the demographic transition. The acceleration of technological progress that comes along with industrialization generates an

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[22] Authors reporting the declining share of agricultural employment in total employment along the process of economic development include: Clark, C., 1940. The conditions of Economic Progress. London: McMillan. Kuznets, S., 1966. Modern Economic Growth. New Haven, CT: Yale University Press. Chenery, H., Syrquin, 1975. “Patterns of Development, 1950-1970. London: Oxford University Press.

increased demand for skilled labour and theoretical knowledge, raising the returns to education and inducing parents to alter their choices over their children education. In response, societies press their governments to introduce universal schooling and to ban child labour. As educational reforms succeeded in inducing more children to engage in formal educational, fertility rates decline, and technological change accelerated.

1.5.4 From the Agriculture Revolution to the Industrial Revolution

A natural question that arises is why some countries were able to industrialize first than others. This question is, of course, too complex to be answered with a simple theory. At this stage, however, it is useful to introduce a conventional wisdom, largely inspired in the 18th century England, according to which the modernization of agriculture played a key role in the industrialization process.

Positive links between agriculture and industry may arise for different reasons: first, rising productivity in agriculture makes it possible to release workers from agriculture to industry; second, food surpluses are necessary to feed a large urban population; third, a raising income in agriculture creates a natural market for industrial products; finally, savings generated in agriculture may be used to finance investment in industry. Under this reasoning, classical development theorists defended that agricultural (green) revolution is a *precondition* for industrialization[23]. Although in our days this is no longer true, in was true for that particular period.

The link between technological change in agriculture and industrialization can be illustrated with the help of a simple model[24]. Assume that there are two goods, Manufactures (Y) and Agriculture (Z) and that (homogeneous) labour is the only input to production. To

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[23] Nurkse, R., 1953. “Problems of capital formation in underdeveloped countries”. New York: Oxford University Press. Rostow, W., 1960. “The stages of economic growth: a non communist manifesto”. Cambridge: Cambridge University Press.

[24] The model adapts from Matsuyama, K., 1992. “Agriculture productivity, comparative advantage and economic growth”. Journal of Economic Theory 58, 317-334.

simplify the algebra, it is assumed that both production functions are linear, and the total labour force in the economy is equal to 1:

$$Y = AN_Y \qquad (1.9)$$

$$Z = BN_Z = B(1 - N_Y) \qquad (1.10)$$

where N_Y and N_Z denote for the fractions of the labour force employed in manufactures and agriculture, respectively. Wages are assumed flexible.

The last term in equation (1.10) incorporates the resource constraint of the economy and stresses the trade-off between agricultural production and manufactures production: given the productivity parameters, the only way to expand manufacture output in the amount B is by reallocating employment from agriculture, implying a loss of agriculture output by the amount A . The opportunity cost of expanding the production of Y by one unit is B/A units of Z.

From the production function (1.10), we know that a productivity improvement in agriculture (raise in B) causes an expansion of agricultural output, Z. A different question is how productivity change in agriculture interacts with employment in manufactures, Y. The argument that modernization of agriculture is a pre-condition for industrialization relies on the assumption that technological improvement in agriculture favours the reallocation of workers away from agriculture towards manufactures. This will be the case if the demand for manufactures increases faster with income than the demand for agricultural goods.

In fact, there is an indisputable statistical regularity, which tells us that, as households' income increases, the fraction of households' income spent in food tends to decline. This is the Engel's Law, owing its name to a 19th century German statistician called Ernst Engel. Formally, the Engel law can be incorporated in the model, postulating a utility function of the form:

$$U = \ln\left(Z - \bar{Z}\right) + \ln Y \quad , (1.11)$$

where $\bar{Z}$ refers to a minimum subsistence level of agricultural consumption. When the actual consumption of agricultural products Z falls close to the lower bound $\bar{Z}$, the household utility tends to minus infinity regardless the consumption of manufactures. Substituting (1.9) and (1.10) in (1.11) and maximizing in respect to N_Y, one obtains the level of employment in manufactures in the competitive equilibrium:

$$N_Y = \frac{1}{2}\left(1 - \frac{\bar{Z}}{B}\right) \quad . \qquad (1.12)$$

This equation states that a productivity increase in agriculture (B) leads to an increase in the share of employment in manufactures. This is the result one wanted to obtain. The intuition is the following: an exogenous increase in agricultural productivity leads to an increase in per capita income; then, because of the Engel law, the relative demand for manufactures rises, implying a reallocation of labour away from agriculture towards manufactures. This captures the claim that “modernization in agriculture is a precondition for industrialization”.

Note that the share of employment in manufactures is unaffected by changes in manufactures’ productivity: an increase in A leads to an increase in Y, but the relative price of manufacture goods falls proportionally, so the demand for Z remains unchanged. Hence, technological improvements in manufactures do not cause agricultural employment to increase[25].

1.5.5 International trade and industrialization

The model (1.9)-(1.12) implies that an increase in agriculture productivity leads to an expansion of employment in manufactures. This is indeed what happened in Britain prior to the Industrial Revolution: the innovations introduced in agriculture caused agriculture output to expand, giving rise to positive income effects that led consumers to spend higher fractions of their income in manufactures, allowing the economy to industrialize and undergo the demographic transition.

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[25] Hansen and Prescott (2002) proposed an alternative theory, according to which the increase in productivity in manufactures makes employment in manufactures progressively more attractive for workers, inducing them to reallocate and change the attitude towards fertility [Hansen, G. and Prescott, E., 1999. “Malthus to Solow”, American Economic Review 92 (2002), 1205-17].

History is however plenty of examples of countries with strong agriculture, such Argentina, that failed to industrialize, as well as of countries with poor natural resources, such as Japan, that successfully industrialized. An explanation for this apparent paradox is that the experiences of these countries differed from that of Britain in respect to the *timing relative to globalization*[26]. According to this theory, from the Seven Years War in 1756 until the beginning of the nineteenth century, England could be seen mainly as a closed economy. In that case, the equilibrium (1.12) applies. During the nineteenth century, however, there was a significant expansion of international trade: the expansion of agriculture in Argentina and the industrialization of Japan occurred at a time where globalization was already under way. The key idea is that, when an economy is open to international trade, the relationship between agricultural productivity and employment in manufactures is the inverse of that in a closed economy: high productivity in agriculture increases the likelihood of a country having *comparative advantages* in agriculture, in which case trade openness implies a specialization in agriculture goods and a reallocation of the labour force away from manufactures.

To see the argument formally, note that the main difference between the closed economy and the open economy concerns the determination of relative prices: in the open economy case, prices are determined in the global economy instead of locally. In terms of our model, let $p = P_Z / P_Y$ be the relative price of agriculture goods in terms of manufactures in the global economy. Under free trade, the specialization pattern will be determined according to the Law of Comparative Advantages. This is equivalent to choosing the allocation of workers that maximizes the value of domestic income at world prices, given by:

$$pZ + Y = pB(1 - N_Y) + AN_Y \qquad (1.13)$$

With a simple derivative with respect to N_Y you may verify that this expression is an increasing function of N_Y when $p < A/B$ and a decreasing function of N_Y when $p > A/B$. Interpreting, this means that: if the relative price of the agriculture good in the world economy

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[26] Matsuyama (1992), op. cit.

is lower than the opportunity cost of producing the agriculture good in the domestic economy (that is, the domestic economy has *comparative advantages* in manufactures), then it will be optimal to expand the employment in manufactures until $N_Y = 1$; when instead the relative price of the agriculture good in the world economy is higher than the opportunity cost of producing the agriculture good domestically (the domestic economy has comparative advantages in agriculture), then it will be optimal to reduce employment in manufactures until $N_Y = 0$.

In light of this model, it is easy to understand why a country like Argentina failed to industrialize: in a context of trade openness, the Argentinean high productivity in agriculture induced the country to specialize in agriculture goods, retarding the industrialization process. In contrast, for a country like Japan, where the quality of the land is poor, it became profitable to specialize in manufactures: the low productivity in agriculture endowed the country with an abundant supply of "cheap labour" that the manufactures sector could use[27].

1.5.6 Trading population for productivity

Putting the pieces together, the rapid expansion of international trade in the 19th century lead some countries to specialize in manufactures while others specialized in agriculture goods. This, in turn, may have influenced the different timing of demographic transition, affecting persistently the distribution of the world population, human capital and technology. In other words, this provides an explanation for the Great Divergence.

The argument runs as follows: By the end of 19th century England and Northwest Europe became net exporters of manufacture goods and net importers of primary products, whereas the exports of the other regions were overwhelming composed of primary products. In Western countries, the increasing demand for skilled labour induced by the specialization

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[27] Note that what matters for comparative advantages is relative productivities, A/B, not their absolute levels. That is, a country with lower productivity in manufactures than the rest of the world (low A) may still have the chance of industrializing just because it is even less productive in agriculture.

pattern caused the respective societies to press governments for educational reforms, expediting the demographic transition. With more educated populations, Western countries met faster technological progress, which further enhanced their comparative advantages in skilled-intensive industries. Once these countries escaped the Malthusian trap, rapid technological development resulted in improved living standards.

In non-industrial economies, on the contrary, international trade induced specialization in unskilled-intensive goods. This generated incentives to invest in child quantity, delaying the demographic transition. In these countries, the gains from trade were channelled towards population expansions – a la Malthus -, without impacting significantly on living standards. Moreover, the growing abundance of unskilled labour reinforced the comparative advantages in unskilled-intensive products, in a vicious cycle. Galor and Mountford dubbed the emergence of North-South trade in this period as "trading population for productivity" [28].

The case of India provides a real-life example of how specialization according to comparative advantages may have end up retarding the demographic transition. Between 1813 and 1850, India increased its trade relations with England. This opening process turned India from an exporter of manufactured goods (mainly textiles) into a supplier of primary commodities (between 1800 and 1913, industrialization in India declined by 2/3[29]). In India, this implied a low demand for skilled workers, reducing the incentives for investment in education and delaying the demographic transition. The gains from trade were mostly channelled towards increasing population, without a significant impact on living standards. In England, on the contrary, the gains from trade were channelled towards investment in education stimulating faster technological change and faster economic growth.

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[28] Galor, O., Mountford, A., 2006. "Trade and the Great Divergence: The Family Connection," American Economic Review, American Economic Association, vol. 96(2), pages 299-303, May. Galor, O., Mountford, A., 2008. "Trading Population for Productivity: Theory and Evidence," Review of Economic Studies, Blackwell Publishing, vol. 75(4), 1143-1179, October.

[29] Bairoch, P., 1982. International Industrialization levels from 1750-1980. Journal of European Economic History 11 (2), 269-333.

1.5.7 Static and dynamic gains from international trade

The argument above suggests that the taking opportunity of comparative advantages achieving the highest possible efficiency in the short-run may not go along with welfare improving in the long run. This raises the question as to whether a poor country with initial comparative advantage in agriculture should instead impose trade restrictions on manufactures imports, rather than to engage in free trade. If that strategy helped the country to industrialize, the policy could eventually accelerate the demographic transition and tilt the economy away of the Malthusian trap. This reasoning is an incarnation of a well-known proposition in the theory of international trade and development, that "static efficiency and dynamic efficiency do not necessarily go along".

1.6 Key ideas of Chapter 1

- The Law of Diminishing Returns (LDR) has an important role in the theory of economic growth. The Malthusian model provides a simple illustration of that role. In this model, a growing labour force leads to a more intensive use of land and thereby to a decline in per capita income. At the moment household incomes fall short a minimum subsistence level, both population and output stop growing. Furthermore, any technological improvement will be offset, in the long run, by an increase in the size of the population, without delivering any positive impact on living standards.
- The Malthus prediction that technological improvements should translate into higher population densities without much impact on living standards provides a reasonable description of the real-world facts prior to the modern era.
- The model also provides a useful tool to think about contemporaneous problems of depletion of natural resources and environmental sustainability. These problems can also be formulated in the context of a race between technology and the limitations imposed by resource constraints.
- The Malthusian model fails, however, to describe modern economic growth. On one hand, the model conflicts seriously with the stylised fact that, in modern economies, per capita incomes exhibit a tendency to growth over time, not to remain constant at the very low subsistence level. On the other hand, its predictions regarding the relationship between population growth and per capita income no longer hold in contemporaneous societies.
- The change in the human behaviour towards fertility along the process of economic development is labelled "Demographic Transition". The fall in birth rates is the most important feature of demographic transition. Microeconomic theories explaining the changing attitude towards fertility view the number of offspring as being determined by individual optimization. Along time, the cost of rearing children increased relative to benefits, resulting in a reduction in the number of optimal offspring.

- Social norms and the population momentum help explain why changes in birth rates tend to be very persistent, leading to demographic explosions during the transition process.
- The interaction between globalization, industrialization and attitudes towards fertility may help explain the Great Divergence: countries with comparative advantages in agriculture remained basically trapped in the Malthusian regime. In countries with comparative advantages in manufactures, societies felt the pressure to switch from child quantity to child quality, investing more in education and achieving faster technological change, in a virtuous cycle.

1.7 Review questions and exercises

Key concepts

Positive checks vs. preventive checks. Stable vs. unstable steady state. Race between technology and diminishing returns. Stepping on shoes, standing on shoulders, fishing out effects. *Demographic transition. Asset role of children. Population momentum. The Great Divergence. Static versus dynamic efficiency. Trading population for productivity.*

Essay questions:

- Comment: “The most decisive mark of the prosperity of any country is the increase in the number of its inhabitants”.
- Comment: “Technology and population reinforce each other”.
- By the 15th century, population density in Eurasia was much higher than in Australia. Explain.
- What drives the fall in fertility rates in the transition to the Modern Growth regime?
- Explain why birth rates exhibit a long persistence, declining much slower than death rates.
- Explain why high productivity in agriculture favoured the demographic transition in England but not in Argentina.

Exercises

1.1. Consider a closed economy with no government and basically devoted to agriculture. Output takes the form of a single homogeneous good (Y), which is produced using labour (N) and land (T). The relationship between inputs and output is described by an aggregate production function of the form: $Y_t = BT_t^{0.5}N_t^{0.5}$. Assume that the availability of land is fixed, with T=1. The dynamics of population (*N*) is described by the following equation: $\dot{N} = \upsilon\left[y - \bar{y}\right]$. (a) Where ν is a positive parameter, y=Y/N and $\bar{y} = 2$ is the subsistence level of per capita income. Assume initially that B=18. (b) Explain the equation that describes the dynamic of population in this economy. (c) Find out the steady state of the model and represent it in a graph. Is this steady-state stable? (d) Suppose now that the discovery of a new fertilizer improves B from 18 to 20. Following this change, will the population expand indefinitely? Why? What happens to the population density, N/T? € Suppose instead that B was expanding continuously at a rate of 2% per year? Would population expand at 2% per year as well? Why? What if ν was very small?

1.2. Consider an isolated Malthusian economy (Alfa), where $Y_t = BT_t^{0.5}N_t^{0.5}$, T=4, $\bar{y}=10$, and B=100 (exogenous). (a) (a1) Assuming that, initially N=256, how much will be per capita income in that year? (a2) How will the economy evolve onwards? Explain the theory for the dynamics of population. (a3) Describe the steady state for population and population density. (a4) Represent in a graph. (a5) Discuss the stability of the equilibrium.(b) Consider an economy (Beta), also isolated, identical to alfa except in that T=1. (b1) With B=100 and N=256, will per capita incomes in the two economies converge? Quantify. (c) Suppose that technology evolved very slowly, as a function of the size of population? Would the two economies converge? Discuss.

1.3. Consider a closed economy devoted to agriculture, where the aggregate production is $Y_t = BT_t^{0.75}N_t^{0.25}$, where initially T=4, and B=8. The population dynamics can be described by the following equation $N_t - N_{t-1} = 100[y_{t-1} - \bar{y}]$ where $\bar{y}=1$ is the subsistence level of per capita income and t is a time subscript for centuries. (a) Find out the stead state in this economy, (N*, y*) and represent in a graph. Is it a stable steady state? Explain. (b) Suppose that, at moment *t=1,* some swamps were drained, so that the arable land expanded by 4%. How much would be per capita income in that century? And population in the century after? How much would be population, per capita income and population density in the long run? Represent in a graph. (c) Assume that, instead of exogenous, technology was a function of the last century' population $B_t = 0.125N_{t-1}$. (c1) Explain the intuition; (c2) Explain what would happen to technology, per capita income and population in the years that followed the swamp drainage. (c3) would the economy face any Malthusian barrier again? (d) To which extend does this model helps explain real world facts?

1.4. Consider a closed economy in the Malthusian regime, where the aggregate production is $Y_t = BT_t^{0.5}N_t^{0.5}$. In this economy, technology is a function of the last century' population $B_t = 0.01N_{t-1}$. The population dynamics can be described by the following equation $N_t - N_{t-1} = 250\left[y_{t-1} - \bar{y}\right]$ where $\bar{y}=1$ is the subsistence level of per capita income and t is a time subscript for centuries. Iinitially, T=100, and N=100. Describe what will be the impact of an increase in the availability of land to T=125. Complete the following table:

Time	t-1	t	t+1	t+2
Land (T)	100,00	125,00	125,00	125,00
Technology (B)	1,00			
Population (N)	100			

Per capita income (y)

1.5. Consider an economy where people live two periods. In the first period, people are young, they work, they have children and they support their parents. In the second period, people are too old to work or to have children, so they need assistance from their children to sustain their consumption needs. Each family is only concerned with its lifetime utility function, given by $U = \ln c_t + \ln c_{t+1}$. Further assume that: family income during the working age period is equal to 10 monetary units; the cost of rising children is equal to 1; each child delivers 1 unit of its income to his parents in the old age. There are no social security or capital markets. (a) Formalize the utility maximization problem of a period-1 individual. Write down the intertemporal budget constraint.(b) Describe the welfare gains associated to the fact that people can have children. Use a graph to illustrate your answer. (c) What happens to the optimal choice when the family income increases from 10 to 12? Show in a graph. Explain how this relates to the Malthusian theory. (d) What happens if the cost of rising children increases from 1 to 1.25? € Show that the problem above is equivalent to that of a static optimization in which individuals derive an intrinsic utility from having children. (e) Suppose now that banking services become available, so that households could borrow or lend any amount of money at a zero interest rates. Would children still be a profitable investment?

1.6. The following table illustrates the "demographic momentum". Initially, the population is stable with a fertility rate equal to 2. The number of fertile women in each generation is equal to half of the new-borns 30 years before, and the death rate is stable at 1/3. In year zero the fertility rate jumps temporarily from 2 to 3. (a) Explain how the birth rate is determined in the model, (b) Explain why the temporary shock in fertility at year 0 produces lasting effects in the birth rate for more than two centuries.

Year	-60	-30	0	30	60	90	120	150	180	210	240
Initial Population	100.0	100.0	100.0	116.7	127.8	135.2	140.1	143.4	145.6	147.1	148.0
Fertile women	16.7	16.7	16.7	25.0	25.0	25.0	25.0	25.0	25.0	25.0	25.0
Fertility Rate	2	2	3	2	2	2	2	2	2	2	2
Birth Rate	33%	33%	50%	43%	39%	37%	36%	35%	34%	34%	34%
Death Rate	33%	33%	33%	33%	33%	33%	33%	33%	33%	33%	33%
Population growth	0.0%	0.0%	16.7%	9.5%	5.8%	3.7%	2.3%	1.5%	1.0%	0.7%	0.4%

1.7. Consider an economy where two goods, Manufactures (Y) and Agriculture (Z), are produced using labour input, only. For simplicity, assume that the total labour force in the economy is equal to 100 and that both production functions are linear in labour: $Y = BN_Y$; $Z = AN_Z$. (a) Find out the expression for the production possibilities frontier. Display it in a graph, assuming that the productivity parameters are A=1/2 and B=1. In that case, what will be the opportunity cost of expanding one unit of manufactures output? (b) Assume now that we are dealing with a closed economy and that the utility function is given by $U = \ln(Z - \bar{Z}) + \ln Y$, with $\bar{Z} = 40$. (b1) Interpret the parameter $\bar{Z}$. (b2) Find out an expression relating the equilibrium level of employment in manufactures with the parameter A. (b3) Assume that this country experiments an agricultural revolution, with the productivity parameter A shifting from A=½ to A=1. Explain what happens to employment in manufactures. (c) Suppose that productivity in manufactures evolves according to $B_t = 0.1N_{Y,t-1}$. (c1) Interpret this rule.(c2) What happens to productivity in manufactures after the agricultural revolution? (c3) Does employment in manufactures change at all? Why? (d) Assume now that two countries, say England and India, engage in international trade. Assume that, before openness, England experimented an agrarian revolution as described in b) and the implied transformation, as described in c). India, on the contrary, was still in the first stage (A=1/2 and B=1): (d1) Has England absolute advantages in agriculture? (d2) Has England comparative advantages in agriculture? (d3) Admitting that both economies open up to international trade, how will employment evolve in both countries? (d4) Taking into account rule (c), are comparative advantages likely to change in the future?

1.8. Consider the following production function for the World economy: $Y_t = B_t T$, where T refers to a fixed amount of land, and $\dot{B} = bN_t^{\theta} B_t^{\varepsilon}$, and $0 \leq \varepsilon \leq 1$, $0 \leq \theta \leq 1$. Further assume that population expands whenever per capita income $y = Z/N$ increases above the subsistence level, $\bar{y}$. Choose one: (a) A small ε means: (a1) a sizeable "stepping on shoes" effect; (a2) a negligible fishing out effect; (a3) a large standing on shoulders effect; (a4) none of the above. (b) This model will display a strong scale effect when: (a1) $\varepsilon = 0$, and $\theta = 1$; (a2) $\varepsilon = 1$ and $\theta = 0$; (a3) $\varepsilon = 1$ and $\theta = 1$ (a4) $\varepsilon = 0$ and $\theta = 0$. (c) In the long term, the model will display exogenous growth when: (a1) $\varepsilon = 0$, and $\theta = 1$; (a2) $\varepsilon = 1$ and $\theta = 0$; (a3) $\varepsilon = 1$ and $\theta = 1$ (a4) $\varepsilon = 0$ and $\theta = 0$