**Expectations and Phillips curve exercises** 

## **Review** questions

## 3.1.

- **3.2. (Worker misperception model)** In a given economy, the labour supply is given by  $W/P^e = 8$ . The output market operates under perfect competition, with the representative firm having the following production function:  $Q = zN^{0.5}$ . In most of the exercise, consider Z=4.
- a) (Labour demand) Assuming perfect competition, find out the demand for labour.
- b) (Labour market) Explain the labour supply function. Describe the labour market equilibrium in the short-run and in the long run. According to this model, real wages are pro-cyclical or counter-cyclical?
- c) (Aggregate Supply) Find out the short-run (SS) and the long run (LS) supply curves.
- d) (Baseline) Suppose that the real money demand in this economy was given by m=M/P=Q. Find out the equilibrium, assuming that M has been constant at M=1, so that  $P = P^e = 1$ .
- e) (Long run neutrality) Now suppose that at t=1 the money supply increased to M=4. In the long run, what would be the price level and the level of output? What about wages? Represent in a graph.
- f) (Adaptive expectations). Assume that workers form their expectations according to  $P^e = P_{-1}$ . Describe the short-run equilibrium at the time of the monetary expansion (t=1). Namely: find out the price level and the level of output and describe this in the SS/AD space. Find out the real wages and the level of employment and describe this in the W/P, N space. In the following period (t=2) will workers meet their labour supply curve? Explain.
- g) (Accelerationist hypothesis) Now assume that the government wanted the economy to operate continuously at Q=2. Given the law that governs expectations, would that be possible? How? How reasonable would that case be?
- h) (**Rational expectations**). Instead of postulating adaptive expectations, consider the case in which workers were rational. Describe the short-term adjustment to an unanticipated change in the money supply from M=1 to M=4. Comparing to f) what are the implications for the duration of the business cycle?
- i) (**RBC**) Finally, assume that  $P^e = P$  each moment in time (perfect foresight). Examine, in this case, the implications of changes in the parameters Z and M. Which one will produce an output change?
- j) (**Phillips curve**): taking logs in c), find out the expression of the Philips curve in this model.

- **3.3. (Rules versus discretion)** Consider an economy where the aggregate demand and the aggregate supply are given by the following expressions  $q_t^d = m_t p_t$  and  $q_t^s = p_t p_t^e$ , where q refers to the log of output, p refers to the log of the price level,  $p^e$  stands for the log of the "expected" price level and  $\mu$  is the log of the money supply. Assume that the money supply is revealed *after* public expectations are formed.
- a) (Aggregate demand) Explain the equation describing the aggregate demand: what is the implied theory? Plot the aggregate demand schedule in the (q,p) space, considering the cases in which m = 0 and m = 1.
- b) (Aggregate supply) Plot the aggregate supply schedule in the (q,p) space, considering the cases in which  $p^e = 0$  and  $p^e = 1/2$ .
- c) (Price level) Find out the equilibrium levels of q and p as functions of  $p^e$  and m.
- d) (**Perfect foresight**): If workers knew for sure that the central bank would increase the money supply once-and-for-all to  $m_t = 1$ , how much should be  $p_t$  and  $q_t$ ?
- e) (Adaptive expectations): Suppose that workers formulate their expectations according to the following rule:  $p_t^e = p_{t-1}$ , and that initially  $p_{t-1} = m_{t-1} = 0$ . If the central bank surprised with a and once-and-for-all increase in the money supply to  $m_t = 1$ , how much would be  $p_t$  and  $q_t$ ? And  $p_{t+1}$  and  $q_{t+1}$ ? Describe the adjustment process in graph. Explain why these economic agents cannot be rational.
- f) (**Rational expectations**): Now assume that agents in this economy know the law governing the price level, as found in (c). Expectations, however, are formed one period ahead, before the actual money supply is revealed. In that case, what will be the best forecast for p? Assuming that the past behaviour of money supply could be described by  $m = \rho m_{-1} + \varepsilon$ , would there be any future deviations from the natural rate?
- g) (Central bank preferences): From now on, assume that the central bank has the following utility function:  $U = q p^2$ . Explain this utility function and represent the corresponding indifference curves in the space (q, p). In particular, plot the indifference curves corresponding to U = 0, U = -1/4 and U = 1/4. Find out the slope of these curves when p = 1/2.
- h) (Discretion): Assume that the central bank is unconstrained in its monetary decisions. Show that in this case its optimal price target will be  $p_t = 0.5$ .
- i) (Inconsistency): Suppose that the central bank announced a zero inflation target for moment *t*. In case the public was convinced, would it be optimal for the central bank to stick with the promise? Explain the central bank incentives, with the help of a graph. Compute the corresponding utility level. (A: U = 1/4).
- j) (Consistency): If the public was aware of the central bank preferences, how much would be  $p^e$ ? In that case, what would be the central bank optimal choice for m? Explain why such equilibrium would be consistent. Compute the central bank utility level in this case. (A: U = -1/4).
- k) (Rule) Finally, assume that the central bank was forced to follow the rule m = 0. Assuming that this rule is fully enforced by law and that the public is aware of this,

what would be the equilibrium? Compare the central bank utility level in this case to the case with discretion. (A: U = 0).

3.4.

3.5.

3.6.

**3.7.**(Slope) Consider an economy where each period a fixed proportion  $1-\sigma$  of firms drawn randomly from the population is able to reset their prices according to  $p = \overline{p} + \alpha (q - q_N)$ , where p is the price of the firm and  $\overline{p}$  is the average price level in this economy (in logs). In this economy, the proportion  $\sigma$  of firms cannot reset prices, being instead tied to a previous expectation regarding the general price level:  $p = P^e$ . Find out the expression of average price level and the corresponding Phillips curve. Which factors influence its slope the slope of the Phillips curve in this model? How shall it be related to policy?

3.8.

**3.9.**Consider the following Central Bank problem: max  $B = 2(\overline{u} - u) - \frac{1}{2}\pi^2$ 

s.t. 
$$u - \overline{u} = 0.02(\pi - \pi^e)$$
 where  $\overline{u} = 0.05$ .

- a) Suppose the central bank announces an inflation rate of zero percent, but it has discretionary power to change the policy after expectations are formed. If the public believed in the central bank target, would it still be optimal for the central bank?
- b) What will be the equilibrium rate of inflation and unemployment if the public has rational expectations?
- c) Explain, in light of this framework, why tying the central bank hands may improve the macroeconomic performance.
- d) Discuss the pros and cons of using rules to restrict the policy options of the central bank.