The basic Solow model

"A thrifty society will, in the long run, be wealthier than an impatient one, but it will not grow faster" [Robert Lucas Jr.]

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Introduction

- The main innovation of the Neo-Classical model in respect to the Malthusian Model is the replacement of land by “capital” in the production function.
- Contrary to land, capital can be produced and accumulated
- This opens an avenue to overcome the diminishing returns to labour: by allowing the capital stock to expand, the Solow model avoids the negative relationship between productivity and the size of population that plagues the Malthus model.
- Still, because capital itself faces diminishing returns, capital accumulation alone cannot generate long term growth. In the Solow model, a higher investment rate leads to a higher level of per capita income, but it does not generate long term growth.

The Kaldor' facts

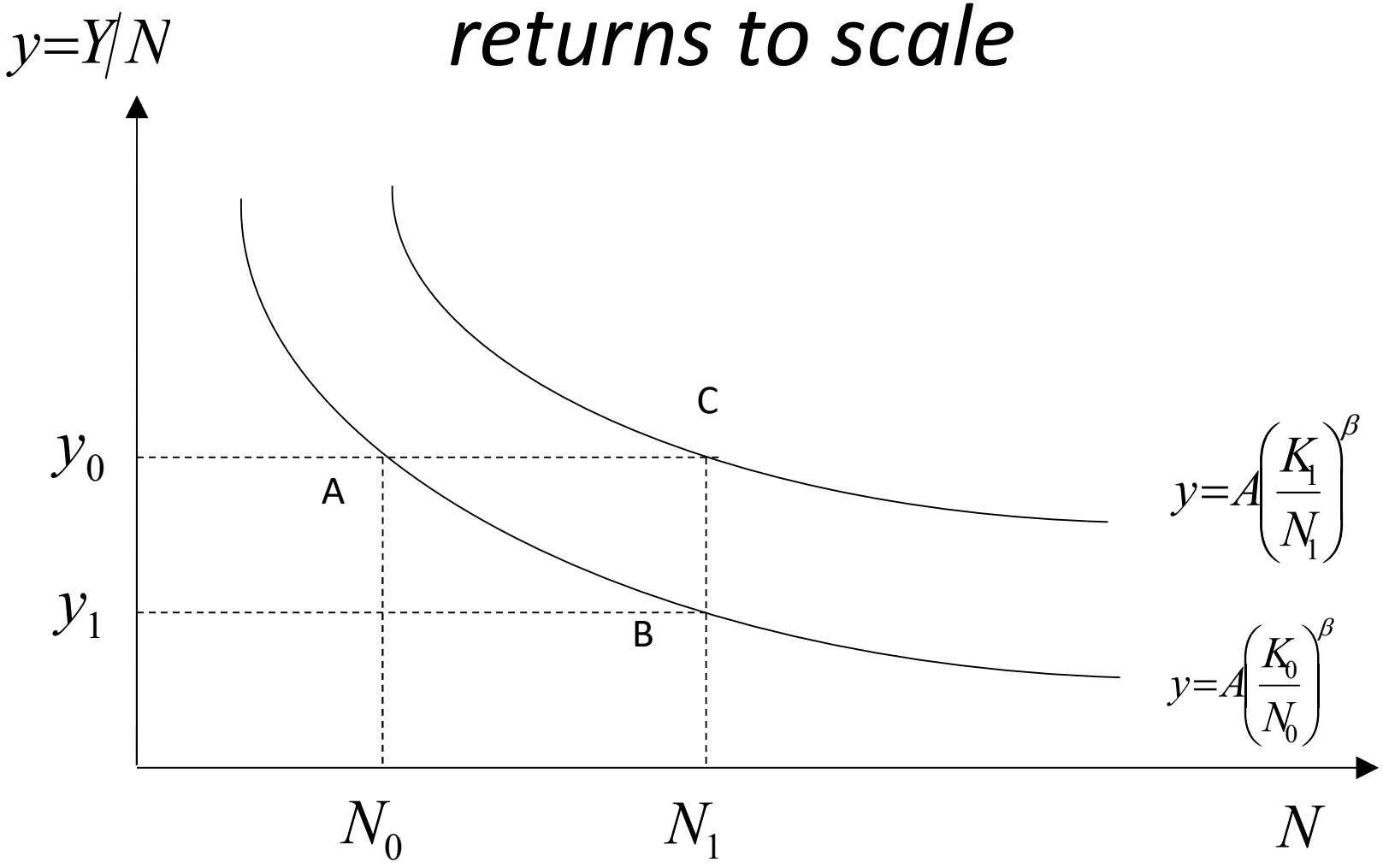
1. Output per worker grows over time at a sustained rate
2. The capital stock per worker grows over time at a sustained rate
3. The capital-output ratio exhibits no clear trend over time
4. The real return to capital is relatively constant over time
5. The shares of labour and of capital on national income are roughly constant over time
6. There are wide differences in the growth rate of productivity across countries

Capital output ratios

Table 3.2 Capital–Output Ratios (*K*/*Y*), 1913–2009

	1913	1950	1973	1992	2009
France	1.6	1.7	1.8	2.3	2.6
Germany	2.3	2.1	2.4	2.3	2.6
Japan	1.0	1.8	1.7	3.0	3.9
UK	1.0	1.1	1.7	1.8	1.9
USA	2.9	2.3	2.1	2.4	2.3

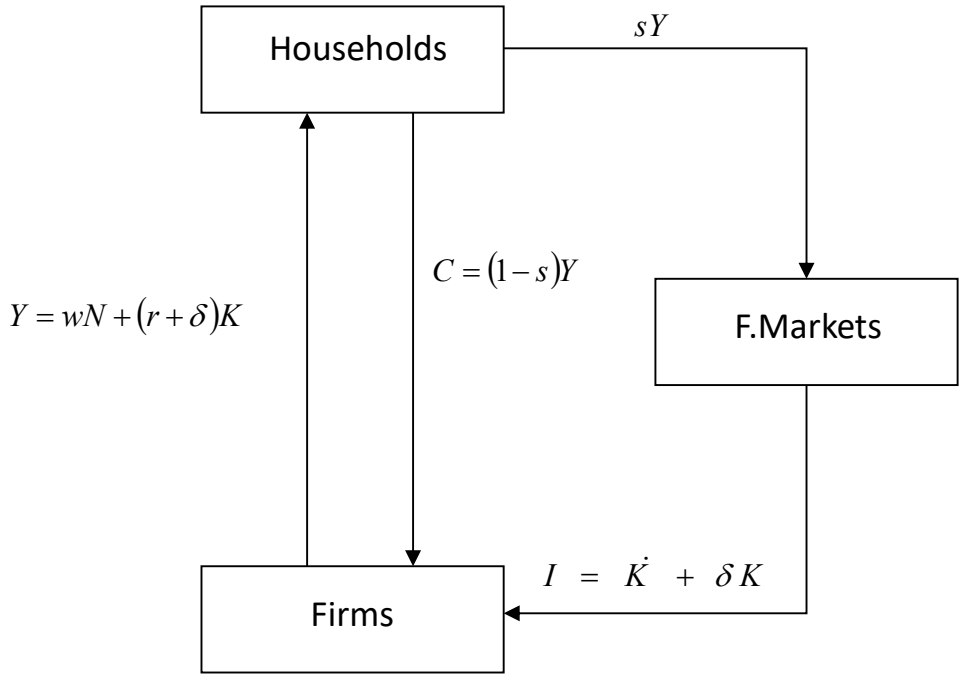
Diminishing returns versus constant returns to scale

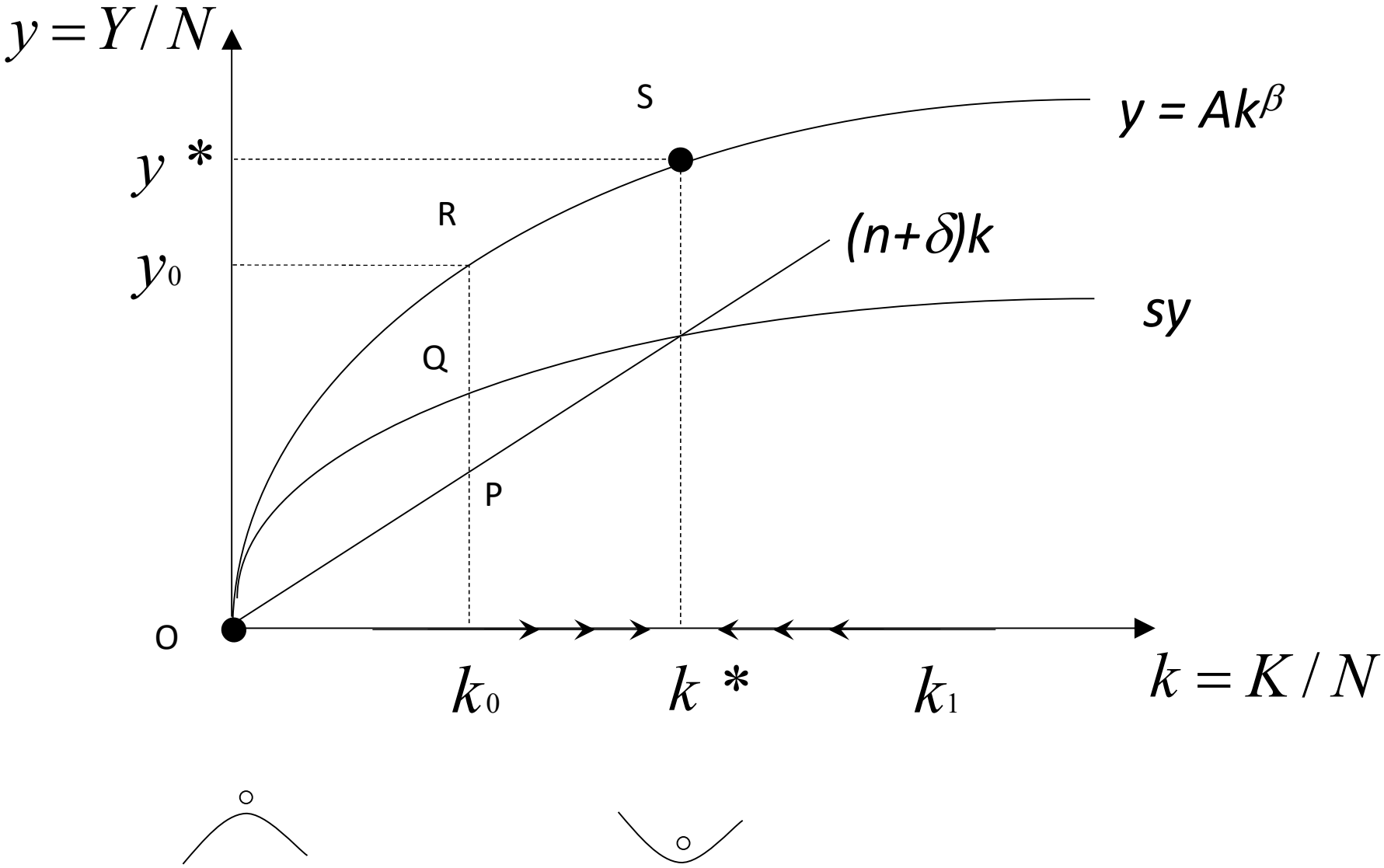


The basic model

- Production function with CRS on capital (K) and labour (N)
- TFP (A) constant
- Perfect competition
- Keynesian consumption function with exogenous savings rate
- Population expanding at a constant rate
- Constant depreciation rate

The flow income chart



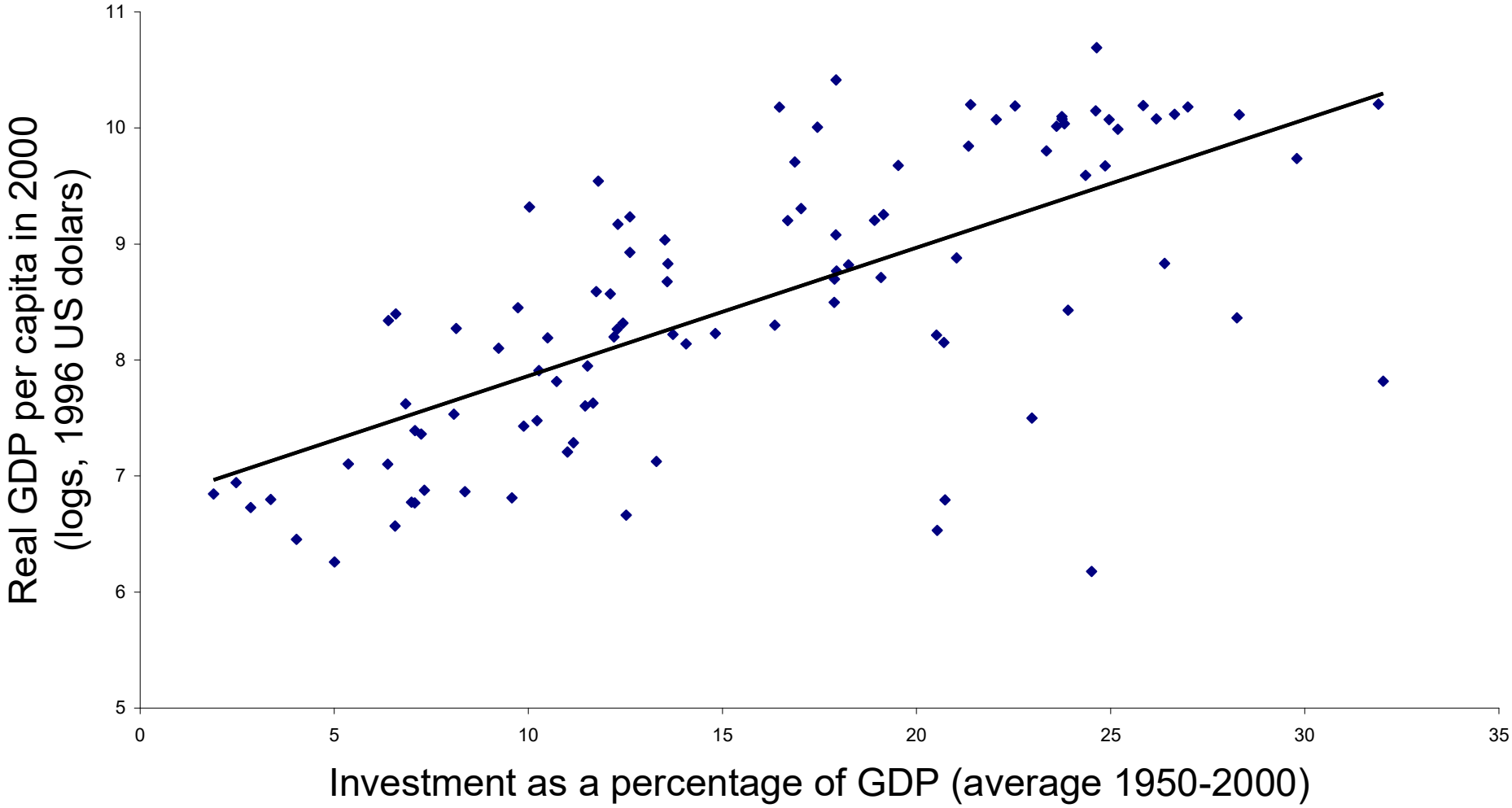


Dynamics and Equilibrium

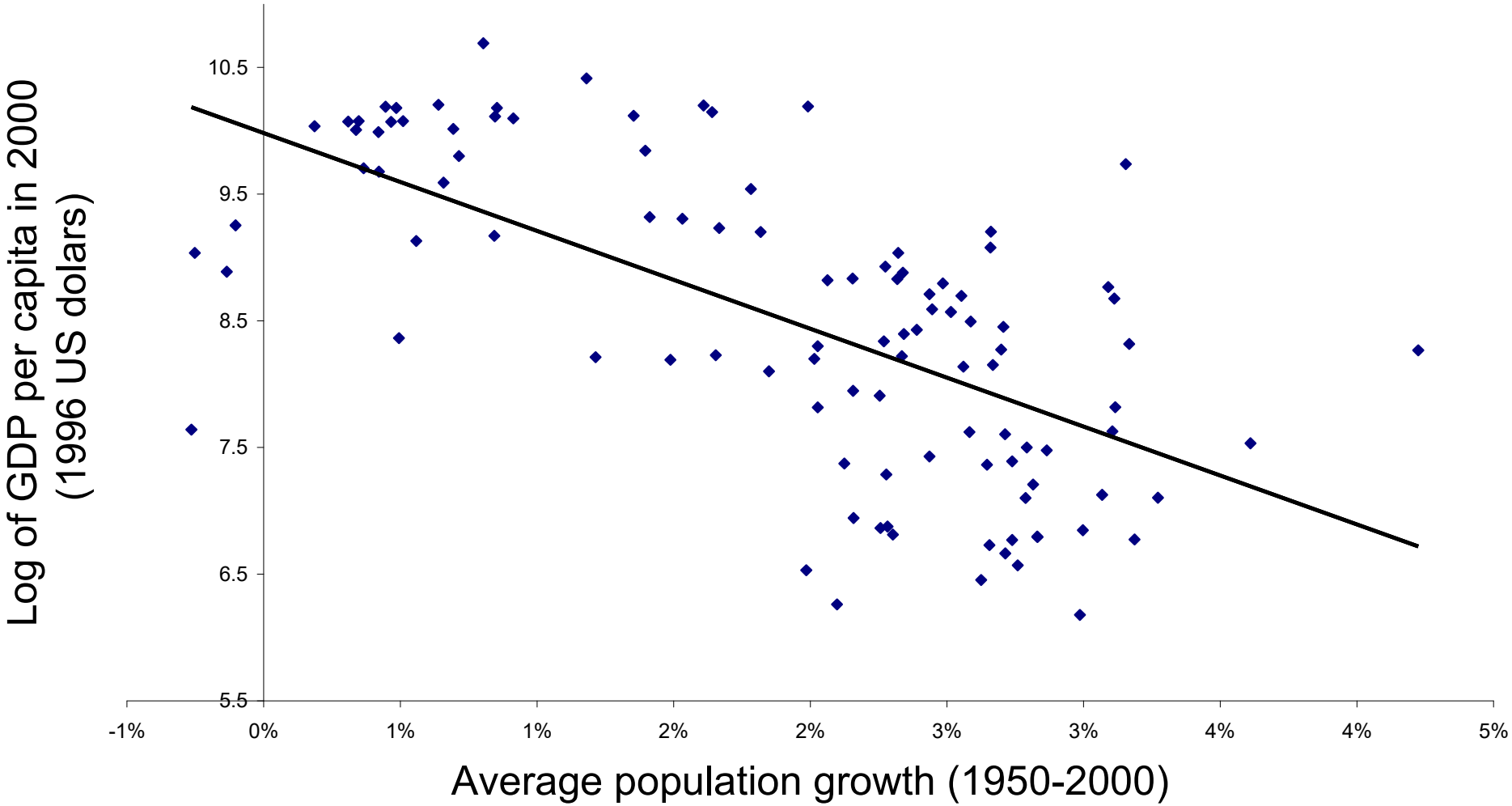
The Solow model and the Kaldor' facts

1. Output per worker grows over time at a sustained rate - Not OK
2. The capital stock per worker grows over time at a sustained rate – Not OK
3. The capital-output ratio exhibits no clear trend over time – OK
4. The real return to capital is relatively constant over time – OK
5. The shares of labour and of capital on national income are roughly constant over time - OK
6. There are wide differences in the growth rate of productivity across countries – Not OK.

Saving rate and per capita GDP

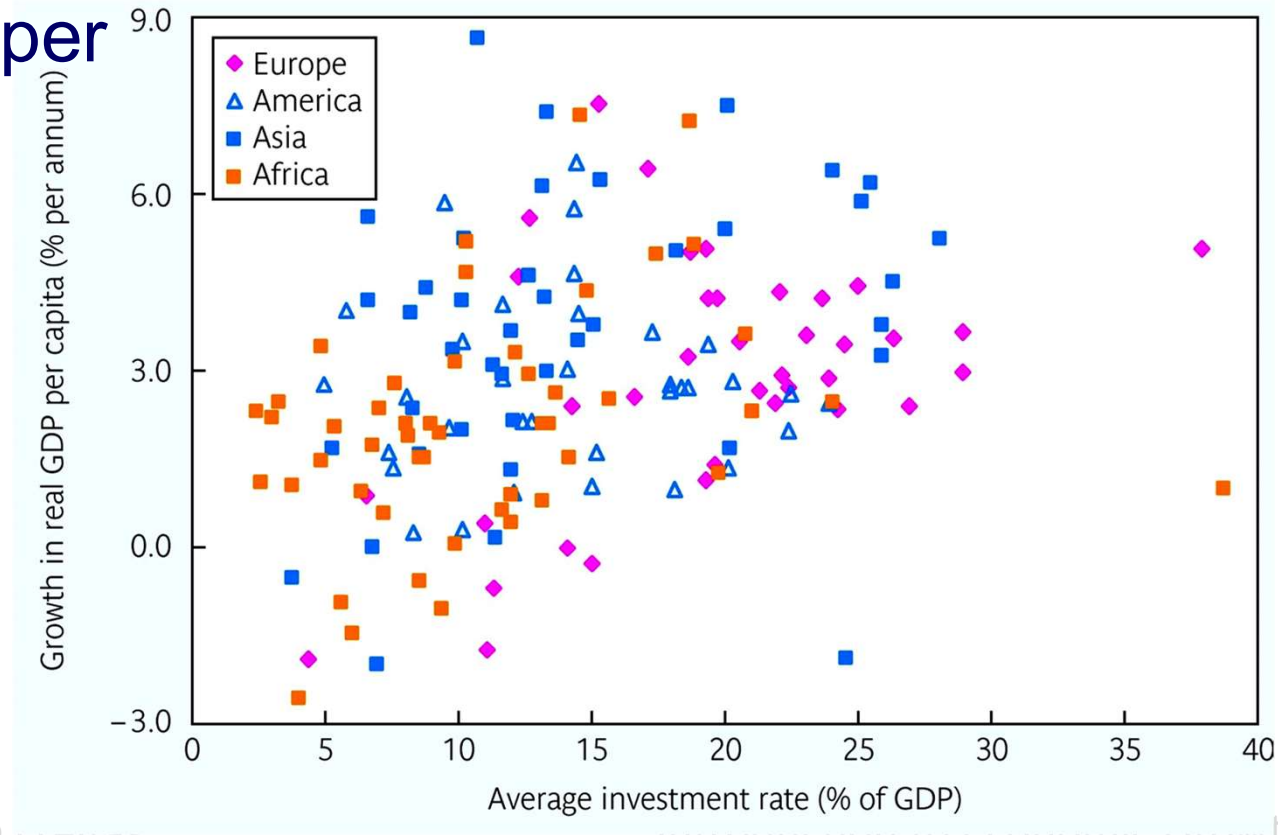


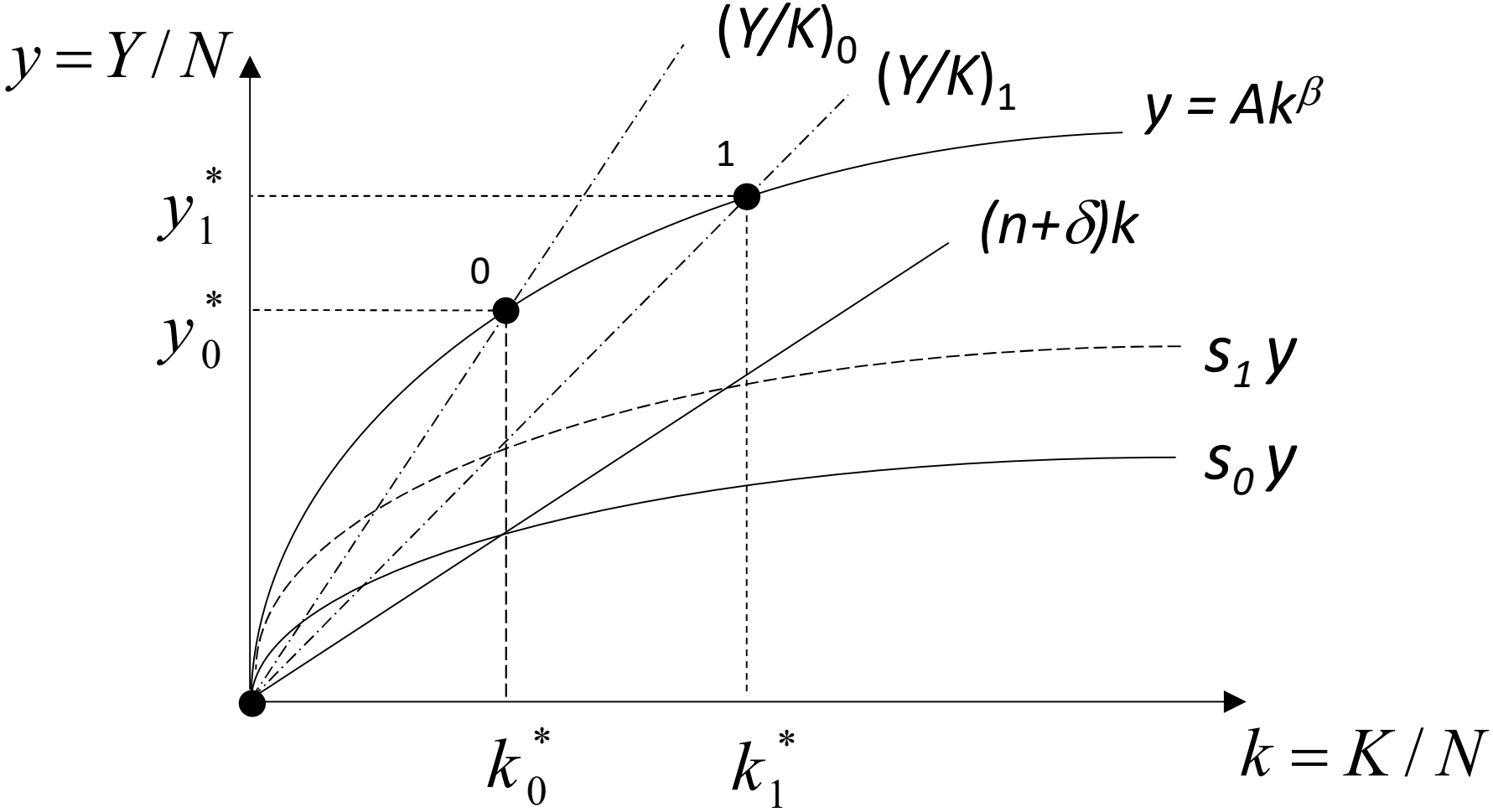
Population growth and per capita GDP



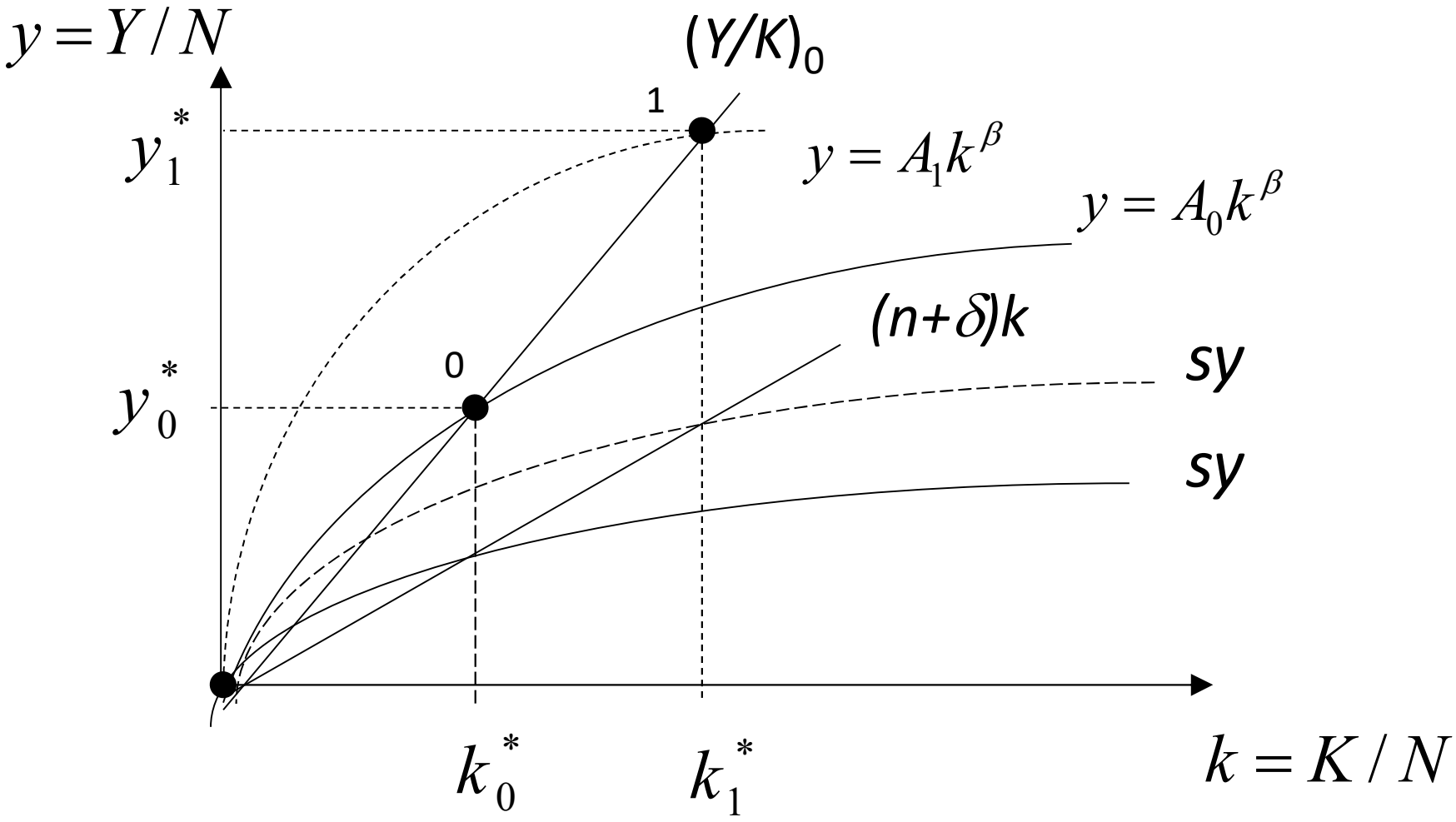
Growth effects ?

Investment Rate and Growth of Real GDP per capita



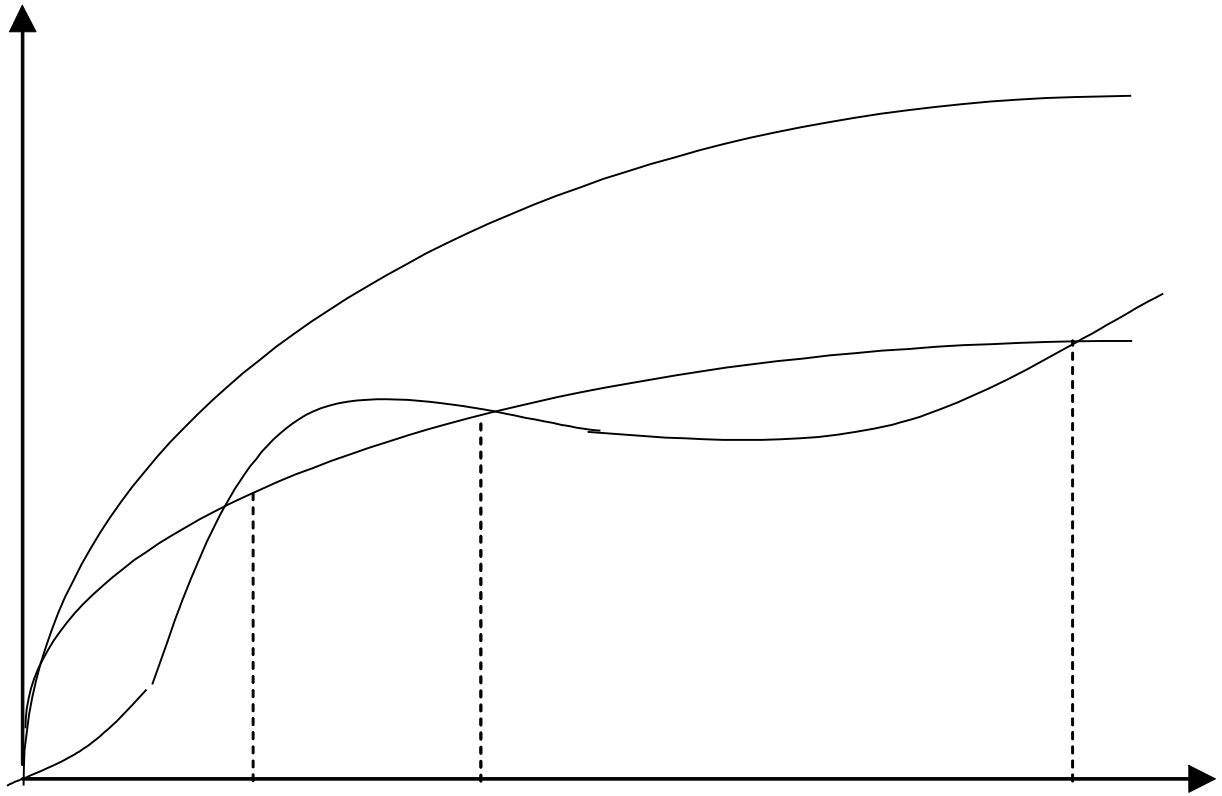


What happens when s increases?

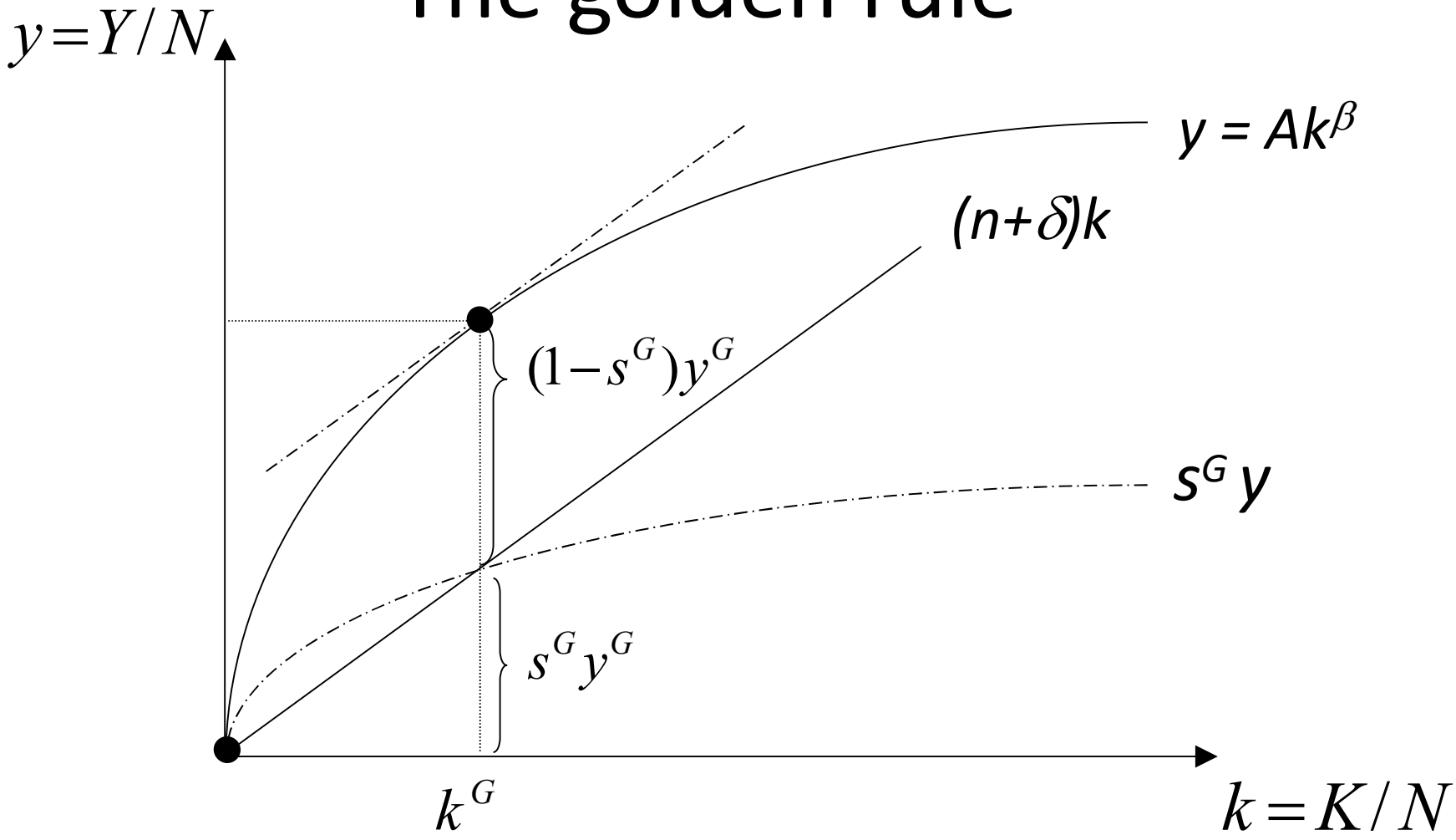


What happens when A increases?

Demographic transition and poverty trap



The golden rule



How efficient is a move towards the golden rule?

- If $s>s_G$ a free lunch is available (dynamic inefficiency)
- But if $s<s_G$, the move towards the golden rule obeys to a trade off. Without a social welfare function, one cannot evaluate this move.
- Why would a decentralized economy deliver too much savings? Distortions?

The Solow residual

- Measures the contribution of factors other than labour and physical capital to the change in per capita GDP
- The contribution of capital and labour is assessed taking into account the corresponding income shares
- TFP change is computed as the difference between the actual growth of output and the growth implied by factor accumulation.

Accounting for the growth of British GDP per head, 1340s to 1860s

(% per annum)

	Output Growth	Population growth	Output per worker	Contributions:	
				Capital Deepening	TFP Growth
1340s-1400s	-0.73	-1.27	0.54	0.34	0.20
1400s-1450s	-0.21	-0.13	-0.08	-0.13	0.05
1450s-1640s	0.5	0.53	-0.03	-0.01	-0.02
1640s-1690s	0.84	-0.04	0.88	0.22	0.67
1690s-1830s	1.08	0.74	0.34	0.01	0.26
1830s-1860s	2.28	1.17	1.11	0.68	0.43

Notes: Output and capital stock are in constant 1700 prices. Source: Broadberry, S and A M de Pleijt (2021), "Capital and Economic Growth in Britain, 1270-1870: Preliminary Findings", CEPR Discussion Paper 15889.

Discussion

- The Solow model accounts for the fact that historical ratios of capital to output and real interest rates tend to be relatively stable in the long run.
- It also offers the credible suggestion that countries with high savings rates and low population growth rates should enjoy higher per capita incomes.
- In its current formulation the model fails, however, to explain the most basic fact of modern economic growth: that per capita income tends to increase over time.
- Continuous growth of per capita income could be obtained in the context of the Solow model if saving rates rose continuously over time. But then, interest rates should exhibit declining trends, and this does not happen in reality.
- The obvious solution is to allow technology to expand over time: as we just saw, with technological progress, per capita output expansion does not imply a declining interest rate.