Exercises Malthus model

1.1. Consider a closed economy with no government and basically devoted to agriculture. Output takes the form of a single homogeneous good (Y), which is produced using labour (N) and land (T). The relationship between inputs and output is described by an aggregate production function of the form: $Y_t = BT_t^{0.5} N_t^{0.5}$. Assume that the availability of land is fixed, with T=1. The dynamics of population (*N*) is described by the following equation: $\dot{N} = \upsilon[y - \bar{y}]$. (a) Where ν is a positive parameter, y=Y/N and $\bar{y} = 2$ is the subsistence level of per capita income. Assume initially that B=18. (b) Explain the equation that describes the dynamic of population in this economy. (c) Find out the steady state of the model and represent it in a graph. Is this steady-state stable? (d) Suppose now that the discovery of a new fertilizer improves B from 18 to 20. Following this change, will the population expand indefinitely? Why? What happens to the population density, N/T? € Suppose instead that B was expanding continuously at a rate of 2% per year? Would population expand at 2% per year as well? Why? What if ν was very small?

1.2. Consider an isolated Malthusian economy (Alfa), where $Y_t = BT_t^{0.5} N_t^{0.5}$, T=4, $\bar{y} = 10$, and B=100 (exogenous). (a) (a1) Assuming that, initially N=256, how much will be per capita income in that year? (a2) How will the economy evolve onwards? Explain the theory for the dynamics of population. (a3) Describe the steady state for population and population density. (a4) Represent in a graph. (a5) Discuss the stability of the equilibrium.(b) Consider an economy (Beta), also isolated, identical to alfa except in that T=1. (b1) With B=100 and N=256, will per capita incomes in the two economies converge? Quantify. (c) Suppose that technology evolved very slowly, as a function of the size of population? Would the two economies converge? Discuss.

1.3. Consider a closed economy devoted to agriculture, where the aggregate production is $Y_t = BT_t^{0.75} N_t^{0.25}$, where initially T=4, and B=8. The population dynamics can be described by the following equation $N_t - N_{t-1} = 100[y_{t-1} - \bar{y}]$ where $\bar{y} = 1$ is the subsistence level of per capita income and t is a time subscript for centuries. (a) Find out the stead state in this economy, (N*, y*) and represent in a graph. Is it a stable steady state? Explain. (b) Suppose that, at moment *t=1,* some swamps were drained, so that the arable land expanded by 4%. How much would be per capita income in that century? And population in the century after? How much would be population, per capita income and population density in the long run? Represent in a graph. (c) Assume that, instead of exogenous, technology was a function of the last century' population $B_t = 0.125 N_{t-1}$. (c1) Explain the intuition; (c2) Explain what would happen to technology, per capita income and population in the years that followed the swamp drainage. (c3) would the economy face any Malthusian barrier again? (d) To which extend does this model helps explain real world facts?