Technology Exercises

1. [**Usawa**] Consider an economy where $Y_t = K_t^{1/2}\left[(1-\mu)(N_t\lambda_t)\right]^{1/2}$, the saving rate is *20%*, population is constant, and capital depreciates at the rate $\delta = 0.04$. Technology evolves according to $\lambda_t = e^{\gamma t}$, $\gamma = b\mu$, and b=0.05. (a) Assume for a moment that $\mu = 0$. (a1) Find out the steady state level of per capita income. (a2) Represent in a graph. (a3) In the steady state, per capita income and the interest rate evolve according to the Kaldor stylized facts? (b) (b1) Explain the equation describing the technological change. Examine the implications of an increase in μ to $\mu = 0.2$. In particular: (b2) Describe the change in equilibrium with the help of a graph. (b3) Compute the new equilibrium values of $\tilde{y}$ and $\tilde{k}$. (b4) Describe the time path of per capita income before and after the change. (b5) In the steady state, per capita income and the interest rate evolve according to the Kaldor stylized facts? (b6) was the change in μ welfare improving? (c) Compare the impacts of an increase in the saving rate in cases (a) and (b). (d) Does this model display conditional convergence?

2. [**Usawa**] Consider an economy where the production function is given by $Y_t = (1-\mu)^{1/2} K_t^{1/2} (N\lambda_t)^{1/2}$, were $\mu = 0.75$ is the fraction of time devoted to production. In this economy, the saving rate is 15%, the population is constant and capital does not depreciate. The productivity of labour accumulates at the rate $\gamma = \dot{\lambda}/\lambda = b\mu$, where b=0.02. (a) Explain the equation describing technological progress. (b) Using the equation describing the change in $\tilde{k} = K/L$ (the fundamental dynamic equation), find out the steady state values of $\tilde{y}$ and $\tilde{k}$. (c) Examine the implications of an increase in the saving rate from s=15% to s=18%. In particular, compute the new equilibrium values of $\tilde{y}$ and $\tilde{k}$. Describe the change in a graph and explain what will happen to the interest rate. (d) Returning to the initial figures, examine the implication of an increase in b to b=0.04. In particular, compute the new equilibrium values of $\tilde{y}$ and $\tilde{k}$. Describe the change in a graph and explain what will happen to the interest rate. (e) Compare the effects on the path of per capita income, y=Y/N, of the changes described in c) and in (d).

3. [**Technology Adoption**] In the "Alpha" economy, the typical company's production function is given by $Y_t = 0.5K_t^{1/2}(\lambda_t N_t)^{1/2}$, where K_t e N_t represent the physical capital and the number of workers in each production unit and λ is a term measuring the quality of the work factor. In this economy, the savings rate is 25%, population is expanding at 1%, and the depreciation rate is $\delta = 0.02$. (a) Assume for a moment that $\lambda_t = e^{0.02t}$. d1) Calculate the per capita product in efficiency units in the at steady state, $\tilde{y}$. d2) Describe the evolution of per capita income in steady state.

d3) Graph and discuss the stability of the equilibrium. (b) Now take over that this economy did not produce its own technology, importing ideas from the rest of the world instead. In particular, assume that technology evolves according to $\dot{\lambda}/\lambda = b\mu\left(\bar{\lambda}/\lambda\right)^{0.5}$. f1) Interpret this equation. f2) Calculate the steady state technological gap assuming that $b = \mu = 0.01$. (g) Assume that the country managed to increase parameter b to b=0.125. (g1) Which type of reforms can be captured with this change? (g2) Describe in a graph the evolution of per capita income in that country until the new steady state is reached.

4. **[Technology Adoption**] Consider a small emerging economy with the following production function: $Y = AK^{0.5}\left(\lambda N\right)^{0.5}$, where K includes both human and physical capital and λ measures the efficiency of labour. In this economy the population is constant, the saving rate is equal to s=0.2, the depreciation rate is equal to δ=0.03 and A=0.25. (a) Assume first that technology in this economy expands at 2% per year. Find the steady state in this economy and discuss the stability of the equilibrium. (b) Assume that technology in this economy evolves according to $\dot{\lambda}/\lambda = b\mu\left(\bar{\lambda}/\lambda\right)^{0.5}$. (b1) Find out the steady-state technological gap assuming that $\mu = 0.05$, b=0.1, and $\bar{\gamma}$=0.02. (b2) Departing from (b1), assume that the rate of technological progress at the frontier decelerated to 0.01. Wat would be the implications for the technological gap and per capita income convergence?

5. (**Vertical innovation, neck and neck**) Consider an economy where the production function in the final good sector is given by $Y = 40K^{0.5}x^{0.5}$, where x refers to an intermediate input, and K=1. In the intermediate good sector, the production function of each individual firm is given by $x_i = \lambda_i N_i$. Also assume that this market is small relative to the rest of the economy, with W=5. (a) Find out the demand for the intermediate input, assuming perfect competition in the final good sector. In this market, R&D costs are paid by each firm to their own research departments, but in the form of an *annual royalty* (F). (b) Assume that this market was explored by an incumbent monopolist holding the technology $x = 2N$. What would be its optimal price and operating profit? What would be the net profit, after paying the annual royalty amounting to F=5? (c)Suppose that a second entrepreneur managed to join the frontier and share the profits with the incumbent, at the annual cost F=5. (c1) would that innovation be profitable from the private point of view? (b2) and from the social point of view? (c3) which externalities are involved in this case? (c3) would be profitable for other entrepreneurs to join the frontier? (d) Suppose that the original incumbent estimated at F=15 the (flow) cost of achieving $\lambda = 2.5$. Would this investment be worthwhile? Explain the underlying effect.

6. **[Creative destruction**] Consider an economy where demand for each intermediate product is given by $p_j = 100x_j^{-0.5}$. Initially the market for this product is competitive, being $\lambda_F = 0.5$. It is also known that the interest rate is r=5% and that initially the salary in this economy is W=1. (a) Consider the problem of an entrepreneur who has discovered a new way to produce this good, given by x=2N . (a1) How do you classify this type of innovation? If the innovator becomes a monopolist in this sector, what will be (a2) the optimal production; (a3) the price?

(a4) profit? (a5) Will this innovation be drastic or not drastic? (a6) Represent graphically the welfare gain resulting from this innovation, under monopoly. (a7) Identify in the figure the additional welfare gain when the monopoly is eliminated. $x = 2N$ (b) Assume now that the entrepreneur was still thinking in inventing this product. Analyze the decision of engaging or not in R&D, taking into account a fixed cost F=1500 and the probability of discovery equal to b=10%. In particular, discuss the cases in which: (b1) the innovation becomes immediately available to all competitors; (b2) the probability of arrival of a superior technology each year is 0%; (b3) the probability of arrival of a superior technology each year is 20%. (b4) Which of these situations is more interesting from the firm point of view? Which of these situations is more efficient from the dynamic point of view? Discuss, taking into account the different possible effects.