

Exogenous Growth Exercises

1. Consider an economy where the production function is given by: $Y_t = A_t K_t^{1/3} N^{2/3}$, where $A_t = 16e^{0.02t}$ describes the technology and N is the (constant) number of workers. In this economy, 25% of income is saved the capital depreciation rate is 1%.
 - a) Describe the main equations of the model and find out the fundamental dynamic equation for K/L , where L is labour in efficiency units.
 - b) Find out the equilibrium values of K/L , Y/L and K/Y .
 - c) Describe the time-paths of per capita income (Y/N), the wage rate, the interest rate and the factor income shares in the steady state. Are these paths in accordance to the real-world facts?
 - d) Suppose that a war destroyed part of the stock of capital of that economy. Describe the subsequent evolution of per capita income (Y/N).
 - e) How did the growth rate of per capita income and the interest rate evolved during the transition path? Explain.

2. In economy W , the aggregate production function is given by $Y_t = A_t K_t^{1/4} N_t^{3/4}$, where N refers to population. In this economy, $s=24\%$, $n=1\%$, $\delta=0$, and $A_t = 8e^{0.015t}$.
 - a) Find out the steady state values of K/L and Y/L , where L is labour in efficiency nits. Represent the equilibrium in a graph and explain why it is stable.
 - b) Compute the interest rate, the capital and labour income shares, and explain the steady state patterns of wages and per capita income. Are these results in accordance to the Kaldor facts?
 - c) Assume that a benevolent planner managed to increase the saving rate in this economy to $s=27.78\%$. (f1) Would the growth rate of per capita income change? (f2) What about the interest rate? (f3) Consumers would be better off? Explain, quantifying when possible.

3. Consider an economy (Oldland) where the production function is given by $Y = A_t K_t^{1/3} N_t^{2/3}$, where N measures the number of workers. It is known that, in this country 25% of income is saved, population is expanding at 0.5% per year, the capital stock depreciates at 3% and $A_t = 20e^{0.01t}$.
 - a) Find out the equilibrium levels of K/L , Y/L and K/Y of this economy, where L represents labour in efficiency units. Discuss the stability of the equilibrium and represent it in a graph.
 - b) Describe the short and long run effects of a rise of the saving rate in the following variables: per-capita income, growth rate of per-capita income, per-capita consumption and interest rate.

- c) Admit that in Oldland per-capita income was ten times higher than in Newland. In what conditions could you state that Newland was growing faster than Oldland?
- d) Knowing that technology was the same in both countries, find out what the interest rate in Newland should be. Would the two economies converge? Discuss.