

Solow Model Exercises

1. Consider an economy where the aggregate production function $Y=AF(K,N)$ exhibits Constant Return to Scale, positive and decreasing marginal productivity and unit elasticity of substitution between factors. Admitting that the saving rate, the population growth rate, technology and the rate of capital depreciation are all constant and exogenous:
 - a) Describe in a graph the steady state of this economy. Is it a stationary steady state? Why?
 - b) Describe in a graph the effects of the following changes on the long run level of per capita output: An increase in the population growth rate; an earthquake that destroys part of the capital stock.
 - c) Describe the effects of a rise in the saving rate in the time paths of the following variables: capital per worker; per capita income; per capita consumption.
 - d) Describe the effects of a rise in the level of technology on the time paths of the following variables: per capita output; capital per worker; interest rate.
 - e) In light of the Solow model, is there a tendency for per capita output levels in different countries to approach each other in the long term? Why?

2. Consider an economy where the production function is given by: $Y_t = 20K_t^{1/3}N_t^{2/3}$, where N_t is the number of workers in period t . In this economy, 25% of income is saved, the labour force grows at 2.5% and capital depreciates at 2.5%. We also know that in this economy there is perfect competition, and wages and prices are fully flexible.
 - a) Compute the steady state values of capital and output per worker. Represent in a graph and describe the stability of the equilibrium.
 - b) Suppose this economy was affected by a hurricane, which reduced its capital stock. Discuss the subsequent dynamic adjustment of this economy with the help of a graph.

3. Consider two economies, A and B, sharing the same technology, given by $Y = K^{0.5}N^{0.5}$. Assume that the saving rates in A and B are, respectively 10% and 20% and that the sum $n+\delta$ is equal to 10% in both countries.
 - a) Suppose that initially the capital-labour ratio was equal to 2 in both countries. What will be the corresponding initial levels of per capita consumption and per capita income?
 - b) Starting from the position described in a), compare the evolution of per capita income in both economies as time goes by. Discuss.

4. Consider an economy where the production function is given by $Y_t = 0.2K_t^{1/3}N_t^{2/3}$. In that economy, 25% of income is saved, capital depreciation is 5% and population is constant and equal to 1000 inhabitants.

- a) Find out the steady state values of per capita income, per capita consumption, real wages and the interest rate.
- b) Find out the saving rate that would maximize C/N in *steady state*, where C is consumption. Illustrate with the help of a graph the adjustment dynamics of Y/N and C/N admitting that the saving rate actually changed to that level.
- c) Suppose you were a benevolent planner with power alter the economy's saving rate to the level found in b). Would such policy be welfare improving?
5. Consider an economy where the aggregate production function is given by $Y_t = A_t K_t^{1/2} N_t^{1/2}$. In this economy, the saving rate is 20%, capital depreciates at 5% per year, and population is constant and equal to $N=100$.
- a) Assume that $A_t = 1$. (Find out the steady state values for: (a1) per capita income; (a2) interest rate; (a3) capital and labor income shares; (a4) wage rate. (a5) To what extent does this model comply with the Kaldor stylized facts? (a6) Represent the equilibrium in a graph and discuss its stability.
- b) Sticking with $A_t = 1$, analyze graphically and quantify, when possible, the short term and long-run effects of a fall in the saving rate to $s=2.5\%$ on:
- (b1) per capita income;
- (b2) per capita consumption;
- (b3) the interest rate.
- Considering your findings, would the saving rate be a good candidate to explain: (b4) why some countries are much richer than others? (b5) long term growth? Elaborate.
- c) Departing from $s=20\%$, analyze graphically and quantify, when possible, the implications of a decrease in A to $A_t = 0.125^{0.5}$, namely on:
- (c1) per capita income;
- (c2) the interest rate.
- Based on your findings, could A be a good candidate to explain:
- (c3) why some countries are much richer than others?
- (c4) long term growth?
- (c5) What would be the theoretical problems with this explanation?
6. Consider an economy where the labour income share is 75%. What would be the Solow residual, if both output and capital were growing at 3% per year and the labour force was expanding at 1.5%?