

Exogenous growth

“The Solow model did not assume that technical progress was exogenous—that is, determined outside the model. Rather, the model made the assumptions necessary to produce a model of an economy with a dynamic equilibrium, a path to which, in the long run, the economy would settle down. The implication of those assumptions was that technical progress had to be exogenous to the model”. [Lant Pritchett]

Perfect technological diffusion

- Knowledge is non-rival
- With perfect technological diffusion, knowledge becomes non-excludable too
- Public goods face a free riding problem: nobody will spend resources in producing them because there is no way of rising revenues to compensate the effort
- Thus, once we assume perfect technological diffusion (as required in perfect competition), technological change has to be exogenous

Adapting the model

- Technology expands over time at the exogenous rate g

$$A_t = A e^{gt}$$

- Production function

$$Y_t = A_t K_i^\beta N_t^{1-\beta} = A e^{gt} K_i^\beta N_t^{1-\beta} = A K_i^\beta \left(N_t e^{\frac{g}{1-\beta}t} \right)^{1-\beta} = A K_i^\beta L_t^{1-\beta}$$

- Effective labour input per worker

$$\lambda_t = e^{\frac{g}{1-\beta}t} = e^{\gamma t}$$

- Labor in efficiency units

$$L_t = N_t \lambda_t = \left(N_o e^{nt} \right) e^{\gamma t}$$

Solving the model

- Define output per worker and capital per worker in efficiency units

$$\tilde{k} = K/L \quad \tilde{y} = Y/L$$

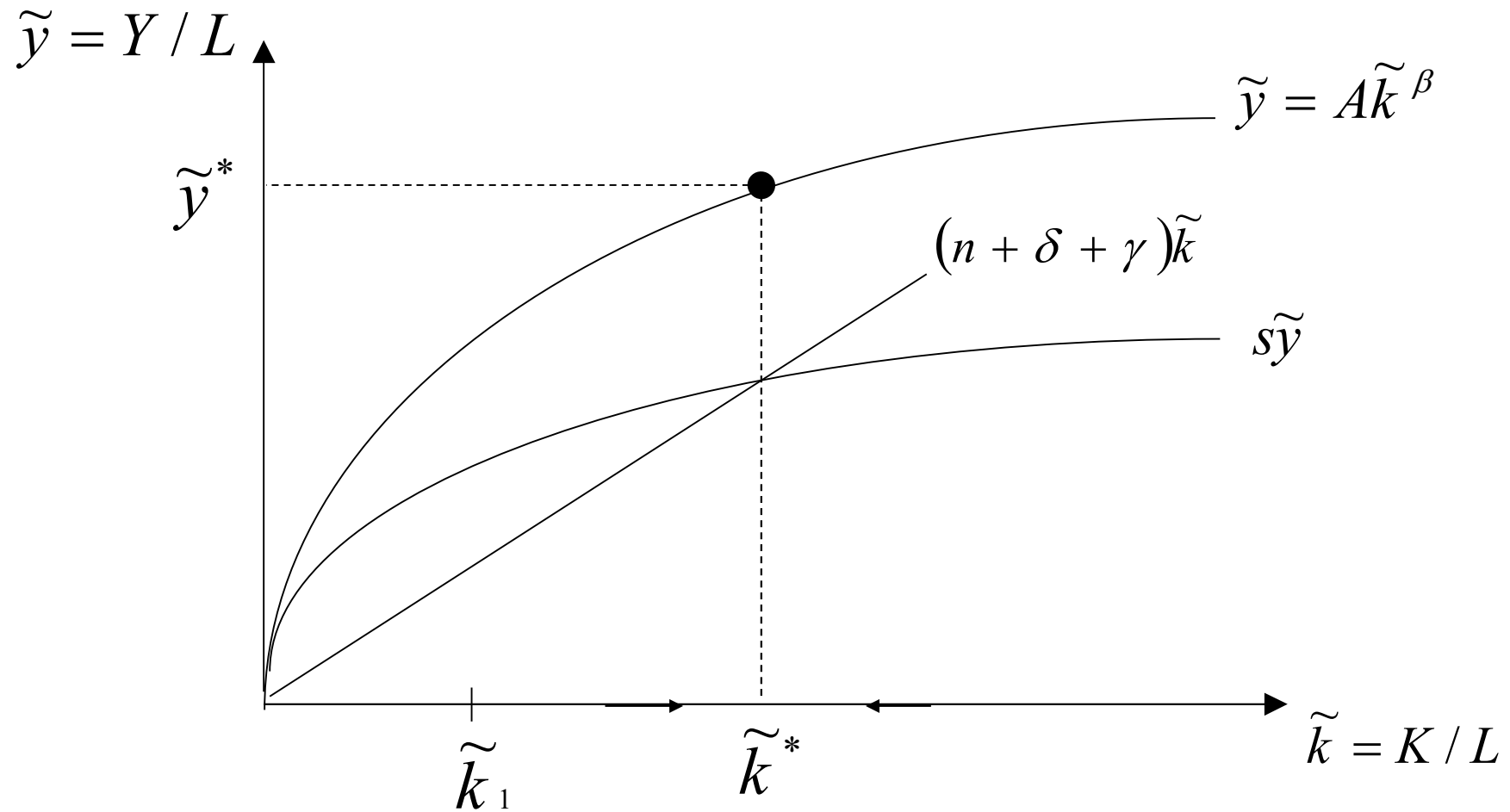
- Production function in the intensive form

$$\tilde{y} = A\tilde{k}^\beta$$

- Fundamental dynamic equation

$$\dot{\tilde{k}}_t = sA\tilde{k}_t^\beta - (n + \delta + \gamma)\tilde{k}_t$$

The steady state



Solving back for y

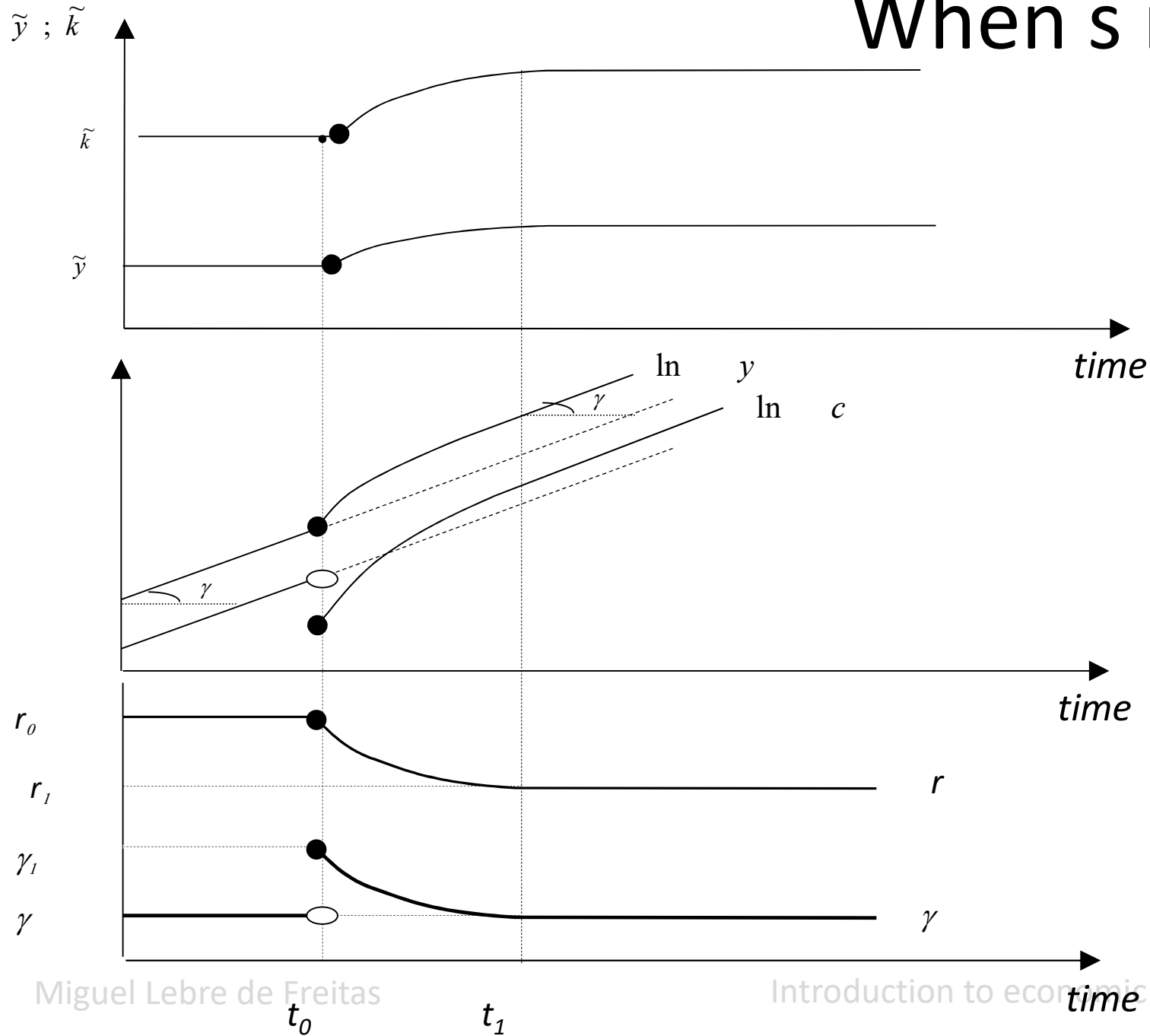
- Once the steady state is found

$$\tilde{y}_t^* = A^{\frac{1}{1-\beta}} \left(\frac{s}{n + \delta + \gamma} \right)^{\frac{\beta}{1-\beta}}$$

- Solve back for y to obtain a growing per capita GDP

$$y_t^* = A^{\frac{1}{1-\beta}} \left(\frac{s}{n + \delta + \gamma} \right)^{\frac{\beta}{1-\beta}} e^{\gamma t}$$

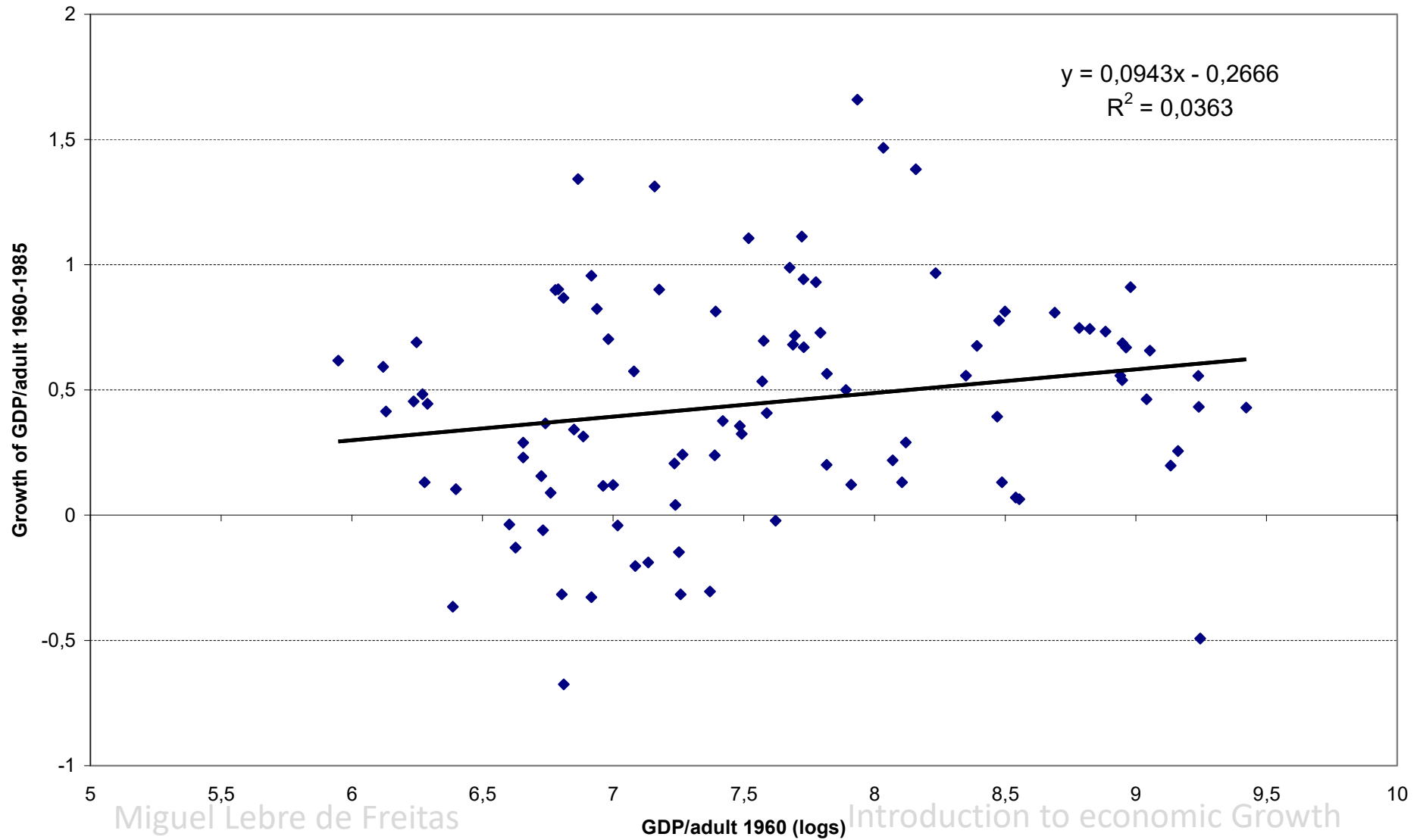
When s rises



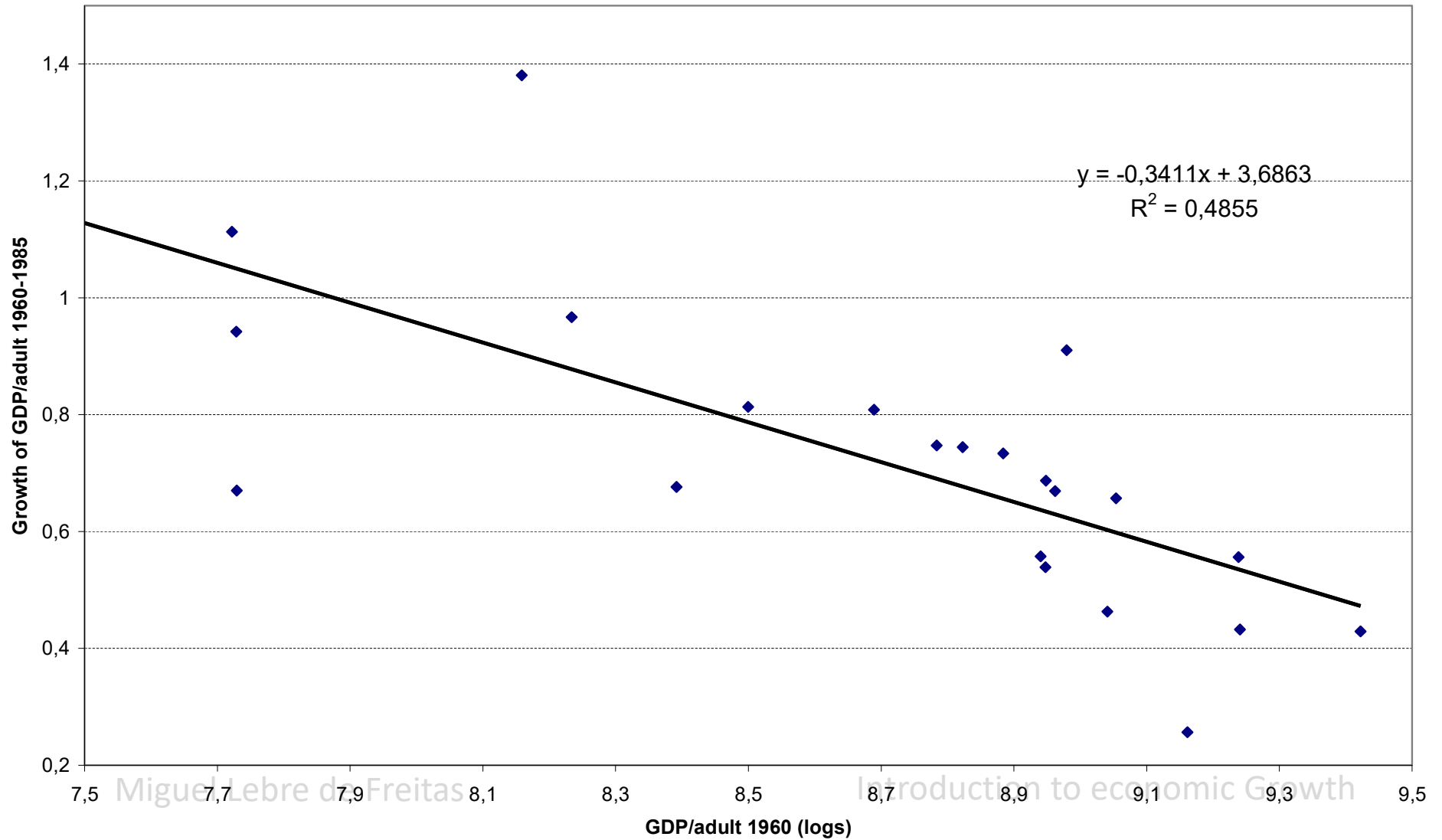
Revisiting the Kaldor' facts

1. Output per worker grows over time at a sustained rate - OK
2. The capital stock per worker grows over time at a sustained rate – OK
3. The capital-output ratio exhibits no clear trend over time – OK
4. The real return to capital is relatively constant over time – OK
5. The shares of labour and of capital on national income are roughly constant over time - OK
6. There are wide differences in the growth rate of productivity across countries – Not OK.

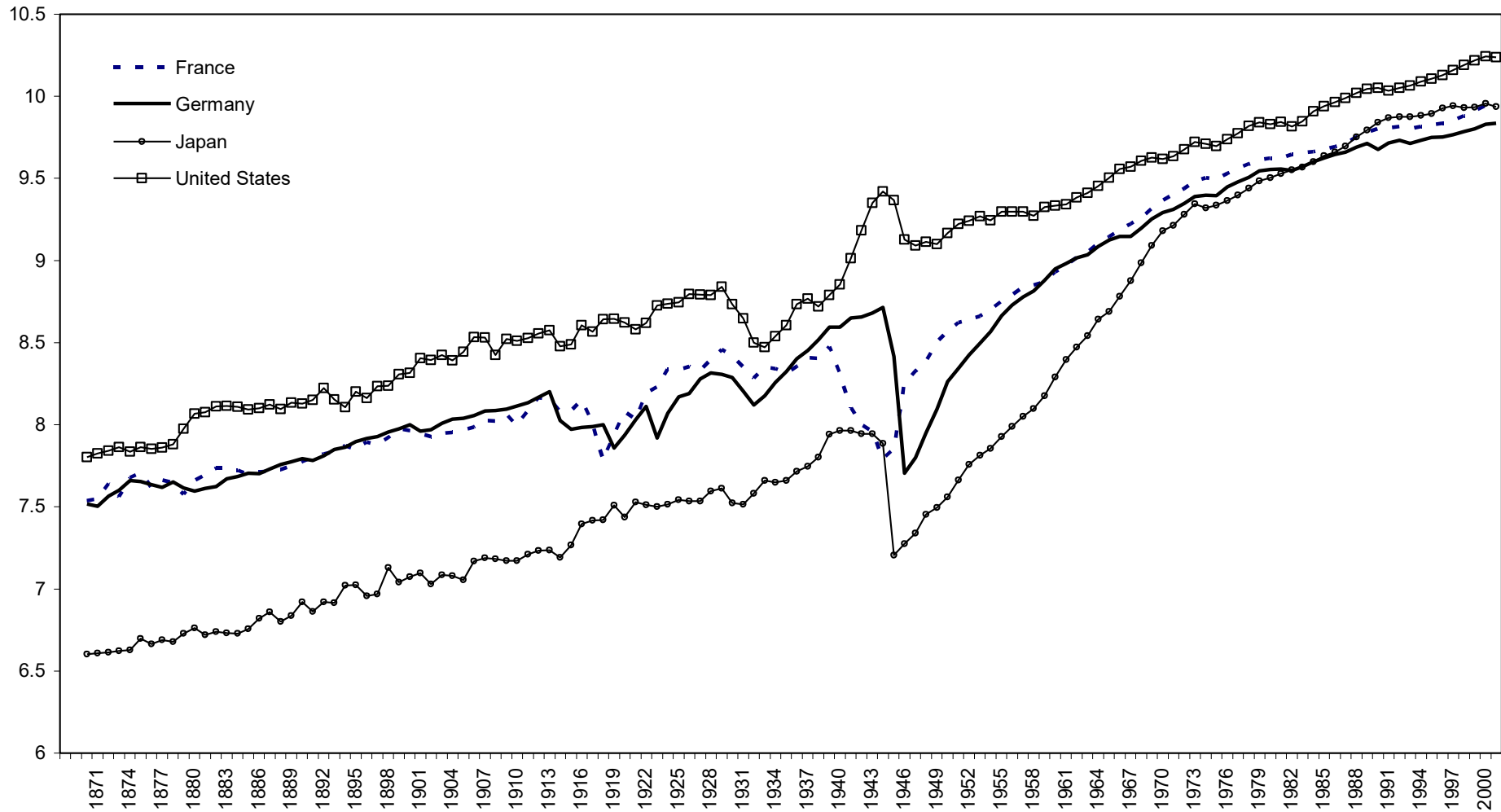
Absolute convergence



Absolute convergence - OECD



Interpreting growth patterns



Miguel Lebre de Freitas

Introduction to economic Growth

Discussion

- The extended model accounts for the Kaldor facts
- But it basically accounts for economic growth by assuming it
- The model is not capable of explaining why technological progress takes place